2.0 Algebra

.1 Patterns and Relationships



Patterns and Relationships

mathematical pattern is a relationship that follows rule. It is an important problem solving skill to be ole to recognise a pattern, find specific terms of a attern as well as identify the rule that generates the attern.

*l*e can represent a pattern graphically, in a table or y a rule or equation.

onsider the first of three T pattern designs, each of hich are made up of a series of dots.

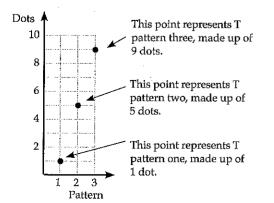


Pattern 1 Pattern 2 Pattern 3

epresenting the pattern number and number f dots used for each design in a table we get the bllowing

Pattern	Dots
1	1
2	5
3	9

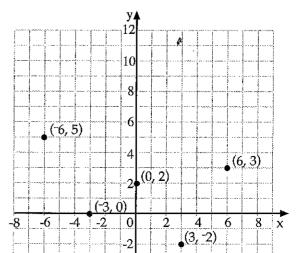
epresenting the relationship between the pattern umber and the number of dots graphically we get he following



Extra Notes	normanning room on the areas is a short when the	·····
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To plot points on a Cartesian graph we always go across first (x axis) and then up or down (y axis). A good way to remember this is that an aircraft always goes along before it goes up. Points on a Cartesian graph are written (x, y).

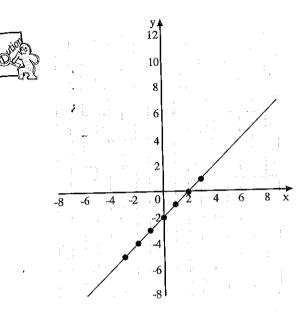




Example

Plot the points in the table on the axes provided. Join the points with a straight line and then describe the relationship that relates the x coordinate to the y coordinate.

_	
x	у
-3	-5
-2	-4
-1	-3
0	-2
1	-1
2	0
3	1

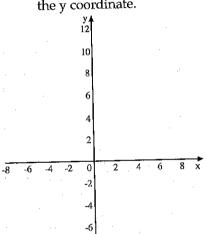


Relationship: Subtract 2 from x to get the y value.

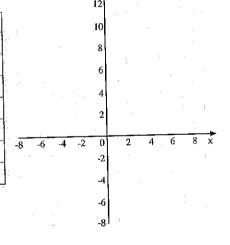


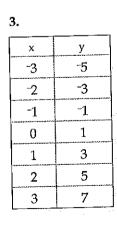
Achievement – Plot the points in the tables on the axes provided. Join the points with a straight line and then describe the relationship that relates the x coordinate to the y coordinate.

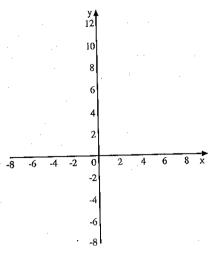
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1.		
х	у	ŀ.
-3	-2	
-2	-1	
-1	0	
0	1	
1	2	
2	3	
3	4	

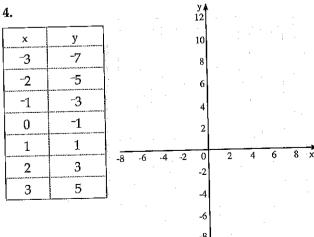


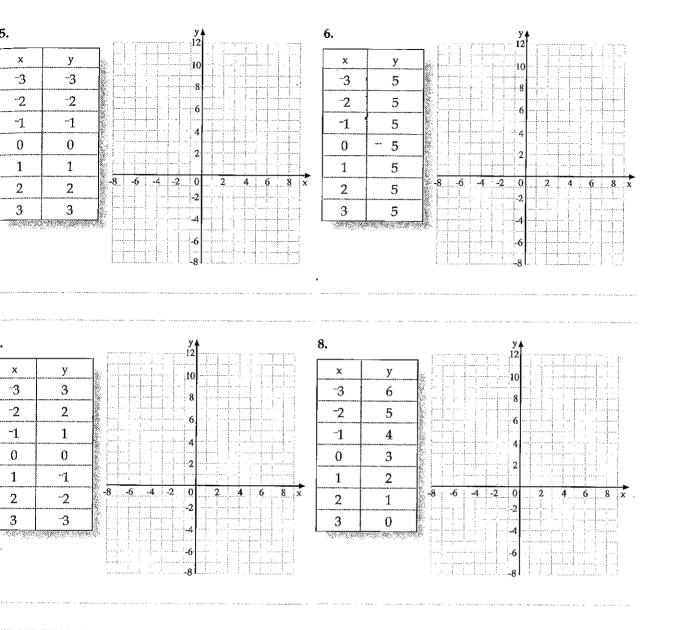
у
- 6
-4
-2
0
2
4
6











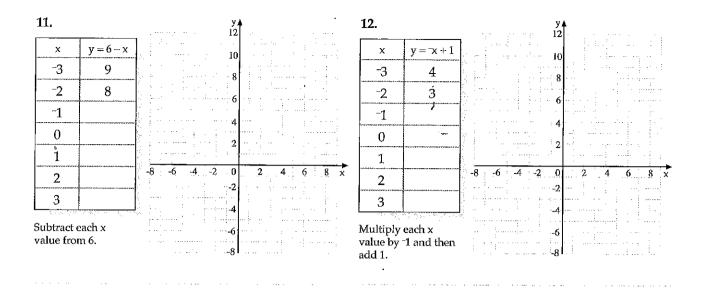


Achievement – Complete the table using the rule provided, then plot the points and join them with a straight line.

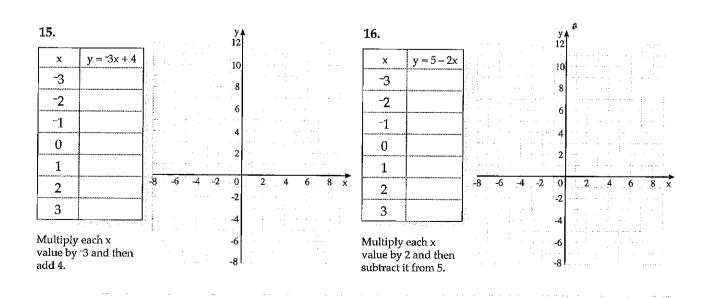
L	
x	y = 3x
-3	-9
-2	-6
-1	
0	
1	
2	
3	

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			 			2										
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х	y = x + 4	36		i					10	. 			<u>.</u>	ļ				
-3	1	SAME					• • •	-		٠		+		<u></u>				
-2	2	35.00	1											<u>:</u> :				
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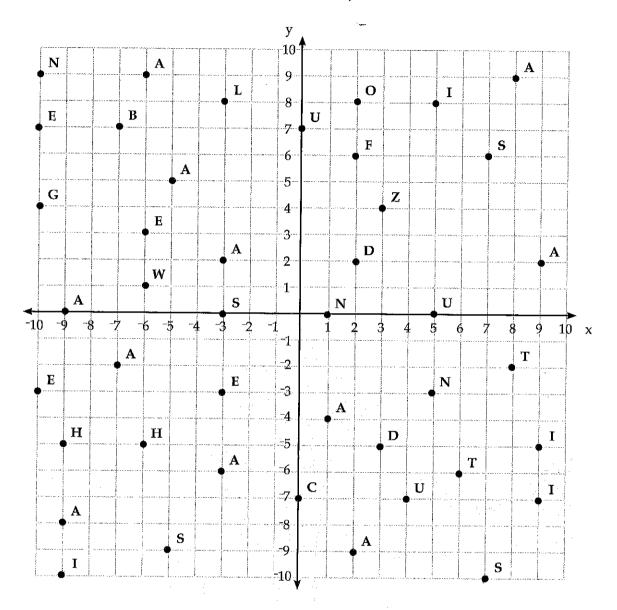


13.		у 12		14.	у х
х	y = ~2]		$x \qquad y = x - 5$	10
-3		8	in the second second	-3	8
-2		6		-2	6
-1				-1	
0		2		0	
1				1	2
2		-8 -6 -4 -2 0	2 4 6 8 x	2	-8 -6 -4 -2 0 2 4 6 8
3	n agraei mag	-2 -4		3	4
For each x y = 2.		-6		Multiply each x value by -1 and then subtract 5.	-8





Find the answer to the riddle at the bottom of the page by identifying which letter corresponds to the applicable coordinate. For example the coordinate (3, 4) corresponds to the letter *Z*.



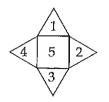
Why did the scarecrow win the Nobel Prize?

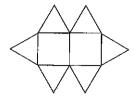
(~7,7)	(-6, 3)	(0, ~7)	(-3, -6)	(5,0)	(7,6)	(-10, -3)	(-6,			(-6, 1)	(8,9)	(7,-10)
	(2, 8)	(4,-7)	(8,-2)	(7, -10)	(6, -6)	(-9, -8)	(5, -3)	(2, 2)	(5,8)	(1,0)	(-10,4)	
	(9,-5)	(-10, 9)	(-9,			, 0)	(2, 6)		(-10, 7)	(-3, 8)	(3, ~5)	



Achievement – Answer the following questions.

17. Study the pattern of 'flowers' drawn below.





Pattern 1

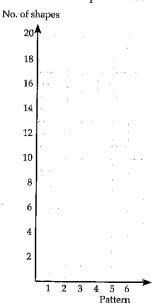
Pattern 2

a) Draw the next 'flower' pattern in the space provided below.

b) Complete the table below for the pattern number and the number of shapes required to create each pattern.

Pattern	No. of shapes
1	5
2	8
3	
4	
5	
6	A Final straining and

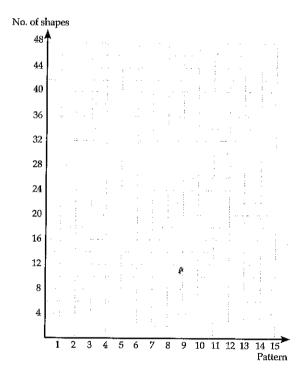
c) From the table above and using the axes below plot the relationship between the pattern number and the number of shapes used.



- d) Which of the following rules describes the relationship between the pattern number and the number of shapes used.
 - A Add 4 to the pattern number.
 - **B** Multiply the pattern number by 2 and add 3.
 - C Multiply the pattern number by 3 and add 2.
 - **D** Multiply the pattern number by 4 and add 1.

Rule:

- e) How many shapes would be required for pattern number 10? Use your rule from part d).
- f) Redraw your graph from part c) on the axes below, but this time join the points with a straight line and extend the line as far as possible.



- g) How many shapes would be required for pattern number 12? Use your graph from above.
- h) What pattern number is made up of 47 shapes? Use your graph from f).

18. Study the pattern of 'S's drawn below.

Pattern 1

Pattern 2

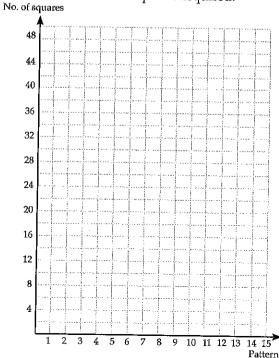
Pattern 3

a) Draw the next 'S' pattern in the space provided below.

b) Complete the table below for the pattern number and the number of squares required to create each 'S' pattern.

Pattern	No. of squares
1	8
2	11
3	14
4	
5	
6	

From the table above and using the axes below plot the relationship between the pattern number and the number of squares required.



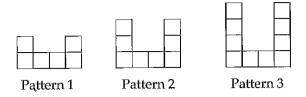
- d). Which of the following rules describes the relationship between the pattern number and the number of squares used.
 - $\mathbf{A} \cdot Add$ 7 to the pattern number.
 - **B** I Multiply the pattern number by 3 and add 5.
 - C Multiply the pattern number by 5 and add 3.
 - **D** Multiply the pattern number by 6 and add 2.

- Rule: How many squares would be required for 'S' pattern number 10? Use your rule from part d) above.
- How many squares would be required for 'S' pattern number 100? Use your rule from part ď).
- In your graph in part c) join the points with a straight line and extend the line as far as possible.
- What pattern number is made up of 29 squares? Use your graph from part c).
- h) What pattern number is made up of 44 squares? Use your graph from part c).
- Copy your table values from part b) and then complete the differences column. Note: The difference column is made up of the difference between successive terms. The first one is done for you (i.e. 11 - 8).

Pattern	No. of	Differences
1	squares 8	
2	11	3 4 11 − 8 14 − 11
3	14	14-11
4		
5	***************************************	
6		

What do you notice about the differences column?

19. Study the pattern of blocks drawn below.

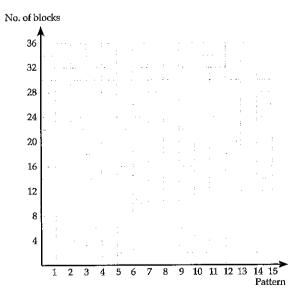


a) Draw the next block pattern in the space provided below.

b) Complete the table below for the pattern number and the number of blocks required to create each pattern.

Pattern	No. of blocks
1	6
2	8
3	
4	
5	
6	

c) Using the axes below plot the points for the relationship between the pattern number and the number of blocks used. Join the points with a straight line and extend the line as far as possible.



 d) How many blocks would pattern number 9 have? Use your graph from part c).

- e) How many blocks would pattern number 15 have? Use your graph from part c).
- f) Which of the following rules describes the relationship between the pattern number and the number of blocks used.
 - **A** Add 5 to the pattern number.
 - **B** Multiply the pattern number by 5 and add 1.
 - C Multiply the pattern number by 4 and add 2.
 - **D** Multiply the pattern number by 2 and add 4.

Rule:

- g) What pattern number is made up of 30 blocks? Use your graph from c).
- h) Copy your table values from part b) and then complete the differences column.

 Note: The difference column is made up of the difference between successive terms.

 The first one is done for you (i.e. 8 6).

Pattern	No. of blocks	Differences	
1	6		8-6
2	8		0-0
3			1
4			
5			·
6			<u>.</u>

- i) What do you notice about the differences column?
- j) What do you notice about the column of differences and the rule you chose for describing the relationship in part f)?
- k) How do you think the column of differences can help you find the rule that describes the relationship between the pattern number and the number of blocks used?



Table of Differences

If we are given a pattern or sequence of numbers in table form, we can identify whether the pattern or relationship is linear by looking at the table of differences. If the differences are constant (i.e. the same) then we know that the rule that describes the sequence must be linear.

A linear relationship is one that when plotted on a graph gives a straight line.

Consider the table below. The sequence is listed in the first column and the difference between successive terms is calculated in the second column, called the difference column. These differences are called the first differences.

Sequence	Differen	ces	٠
4			7 - 4 = 3
<i>H</i>	3	*	
	3	4	10 - 7 = 3
10	2		4
13	3	-	13 - 10 = 3
	3	K	4
16			16 - 13 = 3
170 11 11 11 11		1 10 10 10	Y.1

Since the first differences are the same (constant) the rule that generates the sequence must be linear.

To find or describe the rule that generates the sequence we can use the constant difference as our starting point. Study the example on the right.





Example

Complete the difference column for the sequence 6, 11, 16, 21, 26, 31, 36 given below and then find the rule that describes the sequence.

Term	Sequence	Differences
1	6	
2	11	
3	16	
4	21	
5	26	
6	31	
7	36	



When we complete the difference column we get the constant value, 5.

Term	Sequence	Differences
1	6	
2	11	5
3	16	5
4	21	5
5	26	5
		[*] 5
6	31	5
7	36	

Because the constant difference is 5, it means our rule involves multiplying each term number by 5.

To find out the amount we have to add or subtract after multiplying by 5, to get the correct sequence value, we try one or two terms.

For term 1, $5 \times 1 = 5$, but we require 6 so we need to add 1.

For term 2, $5 \times 2 = 10$, but we require 11 so we need to add 1.

So the rule that describes or generates this sequence is 'multiply the term number by 5 and add 1' or $S = 5 \times T + 1$, where T is the term number and S the sequence.



Achievement – Fill in the difference table for each of the following sequences then find the rule that generates the sequence.

20. 2, 6, 10, 14, 18, 22, 26

Sequence	Differences
2	
6	4
10	
14	
18	·
22	
26	
	2 6 10 14 18 22

21. 3, 8, 13, 18, 23, 28, 33

Term	Sequence	Differences
1	3	
2	8	5
3	13	
4	18	Ť
5	23	
6	28	
· 7	33	

22. 1, 4, 7, 10, 13, 16, 19

Term	Sequence	Differences
1	1	
2	4	3
3		
4		
5		
6		**************************************
7		

23. 7, 6, 5, 4, 3, 2, 1

Term	Sequence	Differences]
1	7		
2	6	-1	
3		**************************************	
4			
5		***************************************	
6		***************************************	
7	and the second s		

24. 3, 9, 15, 21, 27, 33, 39

Term	Sequence	Differences
1		
2		
3		
4		
5	THE PERSON NAMED AND PARTY OF THE PE	
6		
7		

25. -6, -10, -14, -18, -22, -26, -30

Term	Sequence	Differences	l
1			l
2		**************************************	ľ
3			
4			
5		***************************************	
6		**************************************	
7			

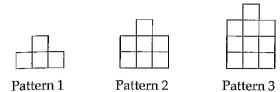
26. 10, 7, 4, 1, -2, -5, -8

Term	Sequence	Differences
1		
2		
3		
4		
5		
6		4-5-7-45
7		

27. 10.5, 15, 19.5, 24, 28.5, 33, 37.5

Term	Sequence	Differences
1		
2	The state of the s	
3		
4		
5		***************************************
6		
7		

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a) Draw the next tiling pattern in the space provided below.

b) Complete the table below for the number of tiles in each pattern as well as the column of differences.

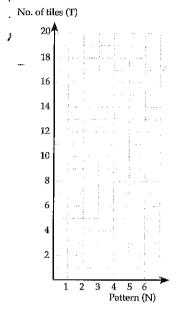
Pattern	Number of tiles	Differences
1	4	
2	7	3
3		~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
4		
5		
6		

c)	With reference to the table above confirm that
	the sequence of tiles is a linear relationship.
	Justify why?

d)	In words describe the rule that generates the
	number of tiles given the pattern number.

e)	Find the rule that generates the number of tiles
	(T) in terms of the pattern number (N).

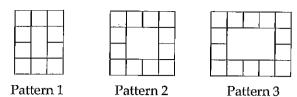
f) Using the axes below draw the graph for the relationship between the pattern number and the number of tiles used.



- g) Using your rule from part d) or otherwise find the number of tiles in pattern number 10.
- h) Using your rule from part d) or otherwise find the number of tiles in pattern number 50.
- i) If 46 tiles are used in a pattern, what pattern number must it be?

j) If 76 tiles are used in a pattern, what pattern number must it be?

29.	Study	the	tilino	patterns	drawn	helow
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a) Draw the next tiling pattern in the space provided below.

b) Complete the table below for the number of tiles in each pattern as well as the column of differences.

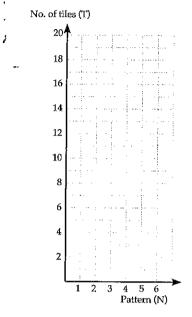
Pattern	Number of tiles	Differences
1	10	
2	12	2
3		
4		
5		
6		

c)	With reference to the table above confirm that
	the sequence of tiles is a linear relationship.
	Justify why?

d)	In words describe the rule that generates the number of tiles given the pattern number.

e)	Find the rule that generates the number of tiles
	(T) in terms of the pattern number (N).

f) Using the axes below draw the graph for the relationship between the pattern number and the number of tiles used.



- g) Using your rule from part d) or otherwise find the number of tiles in pattern number 10.
- h) Using your rule from part d) or otherwise find the number of tiles in pattern number 50.
- i) If 30 tiles are used in a pattern, what pattern number must it be?

) If 78 tiles are used in a pattern, what pattern number must it be?