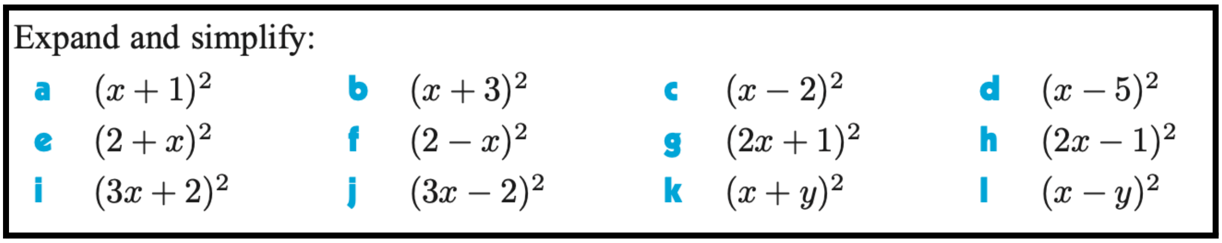
Do now



Expand and simplify:

a $(x+1)^2$

b $(x+3)^2$

c $(x-2)^2$

d $(x-5)^2$

e $(2+x)^2$

f $(2-x)^2$

g $(2x+1)^2$

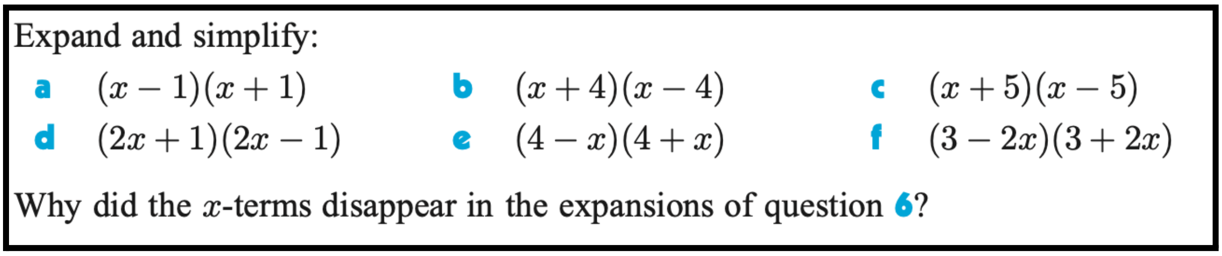
h $(2x-1)^2$

i $(3x+2)^2$

j $(3x-2)^2$

k $(x+y)^2$

l $(x-y)^2$



Expand and simplify:

a $(x-1)(x+1)$

b $(x+4)(x-4)$

c $(x+5)(x-5)$

d $(2x+1)(2x-1)$

e $(4-x)(4+x)$

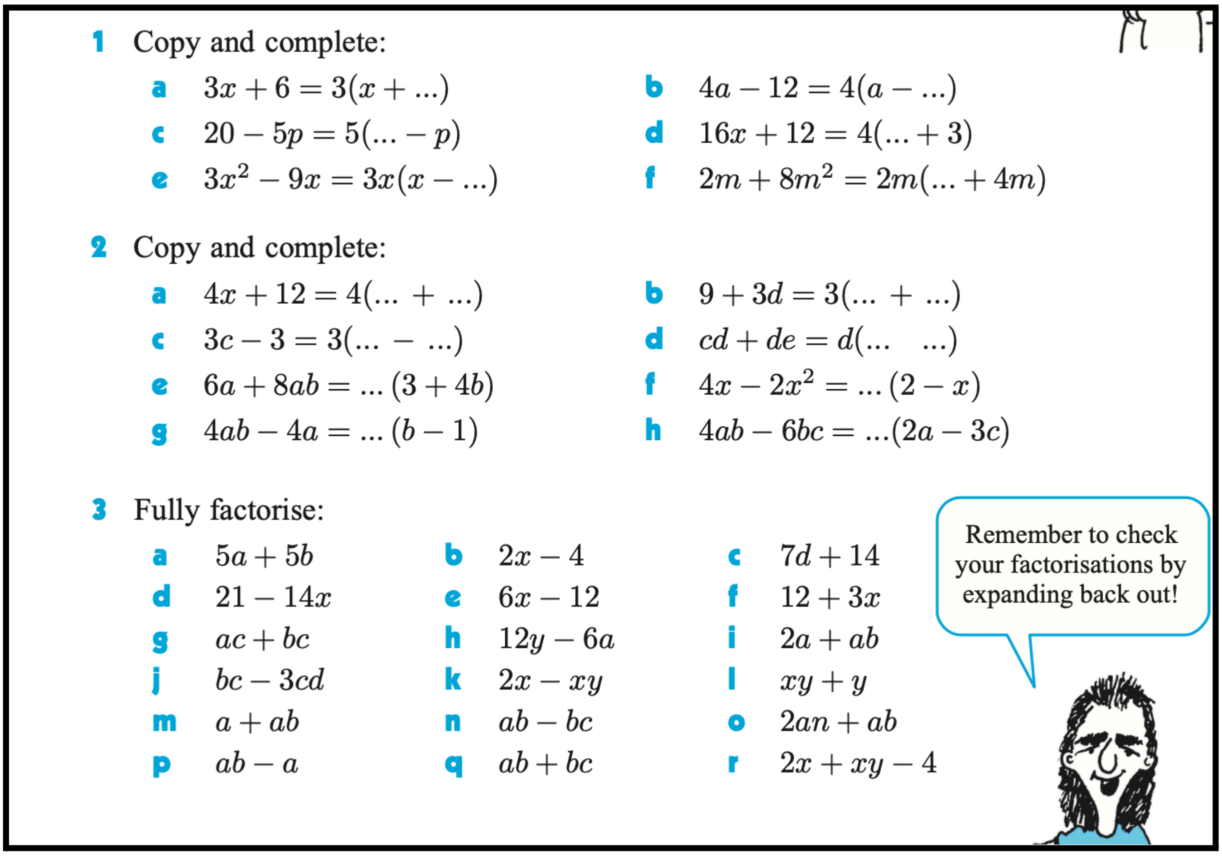
f $(3-2x)(3+2x)$

Why did the x-terms disappear in the expansions of question 6?

WALT: Factorise algebraic expressions

Success Criteria: I know in expansions we have to remove brackets whereas in factorisation we have to insert brackets

Video Dr Frost



1 Copy and complete:

a $3x+6=3(x+...)$

b $4a-12=4(a-...)$

c $20-5p=5(...-p)$

d $16x+12=4(...+3)$

e $3x^2-9x=3x(x-...)$

f $2m+8m^2=2m(...+4m)$

2 Copy and complete:

a $4x+12=4(...\ +\ ...)$

b $9+3d=3(...\ +\ ...)$

c $3c-3=3(...\ -\ ...)$

d $cd+de=d(...\quad ...)$

e $6a+8ab=...\,(3+4b)$

f $4x-2x^2=...\,(2-x)$

g $4ab-4a=...\,(b-1)$

h $4ab-6bc=...(2a-3c)$

3 Fully factorise:

a $5a+5b$

b $2x-4$

c $7d+14$

d $21-14x$

e $6x-12$

f $12+3x$

g $ac+bc$

h $12y-6a$

i $2a+ab$

j $bc-3cd$

k $2x-xy$

l $xy+y$

m $a+ab$

n $ab-bc$

o $2an+ab$

p $ab-a$

q $ab+bc$

r $2x+xy-4$

Remember to check your factorisations by expanding back out!

Factorising Quadratics - Challenge

Example 17

Factorise: $x^2 + 11x + 24$

We need to find two numbers which have sum $= 11$, product $= 24$.
Pairs of factors of 24:

Factor product	1×24	2×12	3×8	4×6
Factor sum	25	14	11	10

↑ this one

The numbers we want are 3 and 8.

So, $x^2 + 11x + 24$

$= (x + 3)(x + 8)$

Note: Only the last two lines of this example need to be shown in your working.

2 Factorise:

a $x^2 + 4x + 3$

b $x^2 + 11x + 24$

c $x^2 + 10x + 21$

d $x^2 + 15x + 54$

e $x^2 + 9x + 20$

f $x^2 + 8x + 15$

g $x^2 + 10x + 24$

h $x^2 + 9x + 14$

i $x^2 + 6x + 8$

j $x^2 + 11x + 18$

k $x^2 + 9x + 18$

l $x^2 + 13x + 42$

m $x^2 + 11x + 24$

n $x^2 + 15x + 26$

o $x^2 + 29x + 100$

Example 15

Fully factorise: ${}^{-}2a + 6ab$

$$
\begin{aligned}
& {}^{-}2a + 6ab \\
&= 6ab - 2a && \{\text{Rewrite with } 6ab \text{ first. Why?}\} \\
&= 2 \times 3 \times a \times b \;-\; 2 \times a \\
&= 2a(3b - 1) && \{\text{as } 2a \text{ is the HCF}\}
\end{aligned}
$$

5 Fully factorise:

a ${}^{-}2a + 2b$
b ${}^{-}3 + 6b$
c ${}^{-}4a + 8b$
d ${}^{-}3c + cd$
e ${}^{-}a + ab$
f ${}^{-}7x^2 + 14x$
g ${}^{-}6x + 12x^2$
h ${}^{-}4b^2 + 2ab$
i ${}^{-}a + a^2$

Example 16

Fully factorise: ${}^{-}2x^2 - 4x$

$$
\begin{aligned}
& {}^{-}2x^2 - 4x \\
&= {}^{-}2 \times x \times x + {}^{-}2 \times 2 \times x \\
&= {}^{-}2x(x + 2) && \{\text{as HCF is } {}^{-}2x\}
\end{aligned}
$$

6 Fully factorise:

a ${}^{-}3a - 3b$
b ${}^{-}4 - 8x$
c ${}^{-}3y - 6b$
d ${}^{-}5c - cd$
e ${}^{-}x - xy$
f ${}^{-}5x^2 - 10x$
g ${}^{-}4y - 12y^2$
h ${}^{-}6a^2 - 3ab$
i ${}^{-}8x^2 - 24x$

Example 18

Factorise: $x^2 - 7x + 12$

sum $= {}^{-}7$ and product $= 12$

$\therefore$ numbers are ${}^{-}3$ and ${}^{-}4$

So, $x^2 - 7x + 12$

$= (x-3)(x-4)$

As the sum is negative but the product is positive, both numbers must be negative.

3 Factorise:

a $x^2 - 3x + 2$
b $x^2 - 4x + 3$
c $x^2 - 5x + 6$
d $x^2 - 14x + 33$
e $x^2 - 16x + 39$
f $x^2 - 19x + 48$
g $x^2 - 11x + 28$
h $x^2 - 14x + 24$
i $x^2 - 20x + 36$
j $x^2 - 7x + 12$
k $x^2 - 17x + 30$
l $x^2 - 11x + 30$
m $x^2 - 13x + 36$
n $x^2 - 13x + 42$
o $x^2 - 17x + 60$

Example 19

Factorise: **a** $x^2 - 2x - 15$ **b** $x^2 + x - 6$

a sum $= {}^{-}2$ and product $= {}^{-}15$

$\therefore$ numbers are ${}^{-}5$ and 3

So, $x^2 - 2x - 15$

$= (x-5)(x+3)$

b sum $= 1$ and product $= {}^{-}6$

$\therefore$ numbers are 3 and ${}^{-}2$

So, $x^2 + x - 6$

$= (x+3)(x-2)$

Notice that as the product is negative, the numbers are opposite in sign.

4 Factorise:

a $x^2 - 7x - 8$
b $x^2 + 4x - 21$
c $x^2 - x - 2$
d $x^2 - 2x - 8$
e $x^2 + 5x - 24$
f $x^2 - 3x - 10$
g $x^2 + 3x - 54$
h $x^2 + x - 72$
i $x^2 - 4x - 21$
j $x^2 - x - 6$
k $x^2 - 7x - 60$
l $x^2 + 7x - 60$

Example 20

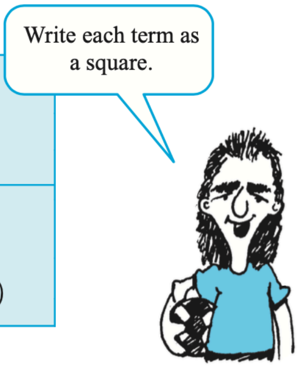
Fully factorise by first removing a common factor: $3x^2+6x-72$

$$\begin{aligned} &3x^2+6x-72 && \{\text{first look for a } \textbf{common factor}\} \\ =\;&3(x^2+2x-24) && \{\text{sum} = 2, \quad \text{product} = {}^{-}24 \quad \text{i.e., 6 and } {}^{-}4\} \\ =\;&3(x+6)(x-4) \end{aligned}$$

5 Fully factorise by first removing a common factor:

a $2x^2+10x+8$ **b** $3x^2-21x+18$ **c** $2x^2+14x+24$
d $2x^2-44x+240$ **e** $4x^2-8x-12$ **f** $3x^2-42x+99$
g $2x^2-2x-180$ **h** $3x^2-6x-24$ **i** $2x^2+18x+40$
j x^3-7x^2-8x **k** x^3-3x^2-28x **l** $x^4+2x^3+x^2$

"**the difference of two squares**".



Example 21

Fully factorise:

a x^2-4 **b** $1-25y^2$

a
$$\begin{aligned} &x^2-4 \\ =\;&x^2-2^2 \\ =\;&(x+2)(x-2) \end{aligned}$$

b
$$\begin{aligned} &1-25y^2 \\ =\;&1^2-(5y)^2 \\ =\;&(1+5y)(1-5y) \end{aligned}$$

EXERCISE 11E

1 Fully factorise:

a c^2-d^2 **b** m^2-n^2 **c** n^2-m^2 **d** m^2-x^2
e x^2-16 **f** x^2-81 **g** a^2-9 **h** $4x^2-1$
i $4x^2-9$ **j** $9y^2-25$ **k** $64-x^2$ **l** $16-9a^2$
m $9x^2-1$ **n** $4a^2-9b^2$ **o** $16a^2-x^2$ **p** $9x^2-16b^2$

F PERFECT SQUARE FACTORISATION (EXTENSION)

Recall that

$$\begin{aligned} &(a+b)^2 \\ =\;&(a+b)(a+b) \\ =\;&a^2+ab+ab+b^2 \\ =\;&a^2+2ab+b^2 \end{aligned}$$

and

$$\begin{aligned} &(a-b)^2 \\ =\;&(a-b)(a-b) \\ =\;&a^2-ab-ab+b^2 \\ =\;&a^2-2ab+b^2 \end{aligned}$$