

WALT Expand quadratic expressions

Success criteria

I know when multiplying two pairs of brackets I use distributive law.

Combine like terms to simplify the expanded form.

Recognize the structure of a quadratic expression in standard form

A **quadratic expansion** is one where terms in two pairs of brackets are multiplied together.

Example

Expand $(x + 2)(x + 3)$.

Answer

If we consider the first brackets $(x + 2)$ to be a single expression, it multiplies each of the terms in the second brackets.

$$(x + 2)(x + 3) = (x + 2)x + (x + 2)3$$

Then expand brackets in the usual way:

$$= x^2 + 2x + 3x + 6$$

$$= x^2 + 5x + 6$$

Note: it is not necessary to show as much working as presented in the above example.

In quadratic expansions each term in the first set of brackets multiplies each term in the second set. This gives four terms. Finally, simplify by adding like terms.

Example

Expand $(x - 4)(x + 7)$.

Answer

$$x \times x \rightarrow x^2$$

$$x \times 7 \rightarrow 7x$$

$$-4 \times x \rightarrow -4x$$

$$-4 \times 7 \rightarrow -28$$

$$(x - 4)(x + 7) = x^2 + 7x - 4x - 28$$

$$= x^2 + 3x - 28$$



The mnemonic FOIL reminds you which terms are multiplied:

(first _____)(first _____) $(x - 1)(x - 6)$

(outer _____)(_____ outer) $(x - 1)(x - 6)$

(_____ inner)(inner _____) $(x - 1)(x - 6)$

(_____ last)(_____ last) $(x - 1)(x - 6)$

$$(x - 1)(x - 6) = x^2 - 6x - 1x + 6$$
$$= x^2 - 7x + 6$$

Expand and simplify these quadratic expressions.

1 $(x + 2)(x + 6)$

6 $(x - 6)(x - 3)$

11 $(x - 9)(x - 8)$

16 $(x - 7)(x + 12)$

2 $(x + 8)(x + 1)$

7 $(x - 3)(x + 1)$

12 $(x - 10)(x + 12)$

17 $(x - 6)(x - 15)$

3 $(x + 3)(x + 5)$

8 $(x + 3)(x - 8)$

13 $(x + 6)(x - 6)$

18 $(x + 15)(x - 4)$

4 $(x + 4)(x - 1)$

9 $(x - 4)(x + 4)$

14 $(x + 8)(x - 10)$

19 $(x + 3)(x - 11)$

5 $(x - 2)(x + 8)$

10 $(x + 12)(x + 10)$

15 $(x - 5)(x + 13)$

20 $(x - 19)(x + 2)$

Follow the examples

The examples in Exercise 8.02 are all very similar. Each bracket begins with a single x . Other types are more challenging!

Example

Expand and simplify:

a $(2x + 5)(3x - 2)$

b $(4 - x)(7 - 3x)$

Answer

a $(2x + 5)(3x - 2) = 6x^2 - 4x + 15x - 10$
 $= 6x^2 + 11x - 10$

b $(4 - x)(7 - 3x) = 28 - 12x - 7x + 3x^2$
 $= 28 - 19x + 3x^2$ or $3x^2 - 19x + 28$

EXERCISE 8.03

1–36 Expand and simplify these quadratic expressions.

- | | |
|------------------------------|-------------------------------|
| 1 $(5 - x)(x + 2)$ | 18 $(4x + 3)(x - 7)$ |
| 2 $(x + 3)(4 - x)$ | 19 $(8x - 3)(x + 1)$ |
| 3 $(x - 1)(1 - x)$ | 20 $(3x + 2)(3x - 2)$ |
| 4 $(3 - x)(x + 2)$ | 21 $(2x - 5)(4x + 1)$ |
| 5 $(2 - x)(x - 7)$ | 22 $(8 - x)(2x - 9)$ |
| 6 $(2 - x)(3 - x)$ | 23 $(1 - x)(3x + 2)$ |
| 7 $(8 + x)(5 - x)$ | 24 $(4x - 1)(4x + 1)$ |
| 8 $(10 - x)(4 + x)$ | 25 $(x + 3)(1 - 2x)$ |
| 9 $x(x + 2)$ | 26 $(2x + 3)(5 - 3x)$ |
| 10 $x(x - 3)$ | 27 $(6 - 2x)(1 - 2x)$ |
| 11 $x(4 - x)$ | 28 $(1 - 2x)(2x + 1)$ |
| 12 $2x(x + 1)$ | 29 $(5 - 3x)(1 + 2x)$ |
| 13 $3x(1 - 2x)$ | 30 $(5 + 2x)(2x - 5)$ |
| 14 $x(5 + 2x)$ | 31 $(3x + 2y)(x + y)$ |
| 15 $(x + 2)(5x + 1)$ | 32 $(x - y)(2x + 3y)$ |
| 16 $(2x + 3)(3x + 4)$ | 33 $(3x - y)(2x + y)$ |
| 17 $(6x - 1)(x - 2)$ | 34 $(4y - x)(3x + 2y)$ |

35 $(2x - y)(2x + y)$

36 $(5x + 3y)(5x - 3y)$

37 Explain why $2x + 3(x - 2)$ and $(2x + 3)(x - 2)$ do not give the same answer when expanded.

38–42 Expand and simplify each expression in the table.

| | | |
|-----------|----------------------------|---------------------------|
| 38 | a $5 + x(x + 3)$ | b $(5 + x)(x + 3)$ |
| 39 | a $(x + 3)(2x - 4)$ | b $x + 3(2x - 4)$ |
| 40 | a $(x + 2)(x - 8)$ | b $(x + 2)x - 8$ |
| 41 | a $1 + x(x - 1)$ | b $(1 + x)(x - 1)$ |
| 42 | a $(2x - 3)(x + 4)$ | b $2x - 3(x + 4)$ |

157 1.2 Algebraic methods

Squaring brackets

When the expressions in the brackets are identical the bracket is being multiplied by itself, and is therefore being **squared**.

Examples

$(x - 3)(x - 3)$ can be written as $(x - 3)^2$.
 $(2x + 1)^2$ written in full is $(2x + 1)(2x + 1)$.

To expand and simplify, write a squared expression in full first.

Example

Expand and simplify $(x + 8)^2$.

Answer

$$(x + 8)^2 = (x + 8)(x + 8)$$

$$= x^2 + 8x + 8x + 64$$

$$= x^2 + 16x + 64$$

EXERCISE 8.04

Expand and simplify.

- | | | | |
|----------------------|-----------------------|------------------------|-------------------------|
| 1 $(x + 4)^2$ | 5 $(x + 11)^2$ | 9 $(3x - 1)^2$ | 13 $(4 - x)^2$ |
| 2 $(x + 1)^2$ | 6 $(x - 9)^2$ | 10 $(2x - 9)^2$ | 14 $(2x + 3y)^2$ |
| 3 $(x - 2)^2$ | 7 $(2x + 3)^2$ | 11 $(1 + 3x)^2$ | 15 $(4x - y)^2$ |
| 4 $(x - 5)^2$ | 8 $(4x - 5)^2$ | 12 $(2 - 5x)^2$ | 16 $(6x - 5y)^2$ |