**Walt** calculate volume to describe the amount of space inside a three dimensional object
**Success Criteria**: I know we use Cubic kilometres for the volume of concrete poured at a building
**Units for Capacity for liquids and gases** - Millilitres, litres, kilolitres, and Megalitres

# 6.8 Volume

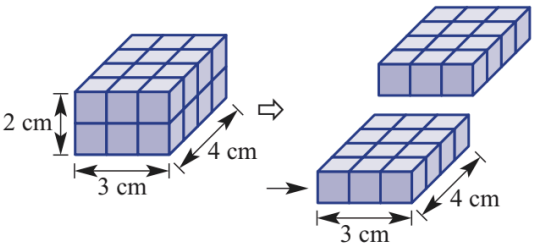
We use volume to describe the amount of space inside a three-dimensional object. We use metric units, such as:

- cubic kilometres for the volume of water in the sea
- cubic metres for the volume of concrete poured at a building site
- cubic centimetres for the volume of space occupied by this book
- cubic millimetres for the volume of metal in a pin.

Units for capacity (millilitres, litres, kilolitres and megalitres) are used for liquids and gases.

## Let's start: Why length × width × height?

For most people, the first thing that comes to mind when dealing with volume is length × width × height. But this rule only applies to finding the volume of rectangular prisms.

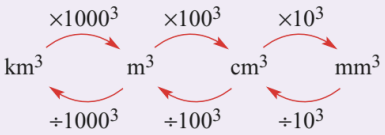



Let's look at a rectangular prism split into two layers.

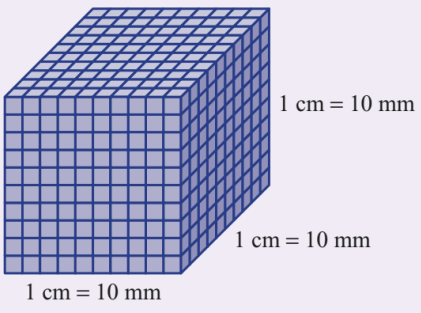
- How many cubes sit on one layer?
- What is the area of the base? What do you notice?
- What is the height and how many layers are there?
- Why is the volume rule given by $V = lwh$ in this case?

## Key ideas

- Common metric units for **volume** include cubic kilometres ($km^3$), cubic metres ($m^3$), cubic centimetres ($cm^3$) and cubic millimetres ($mm^3$).




$1000^3 = 1\,000\,000\,000$
$100^3 = 1\,000\,000$
$10^3 = 1000$

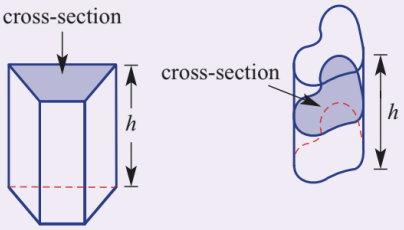



$1\text{ cm}^3 = 10 \times 10 \times 10$
$= 10^3\text{ mm}^3$

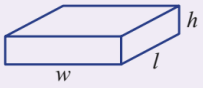
**Volume** The amount of three-dimensional space inside an object

- For **capacity**, common units include:
  - Megalitres (ML) 1 ML = 1000 kL
  - Kilolitres (kL) 1 kL = 1000 L
  - Litres (L) 1 L = 1000 mL
  - Millilitres (mL)

  Also: 1 $cm^3$ = 1 mL so 1 L = 1000 $cm^3$ and 1 $m^3$ = 1000 L
- Volume of solids with a uniform **cross-section** is equal to area of cross-section ($A$) × height ($h$).
  $V = A \times h$



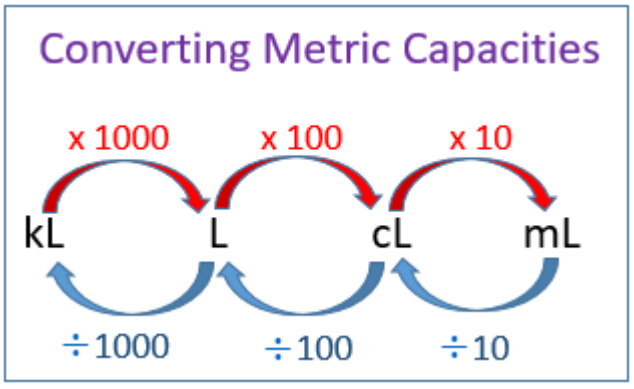

- Volume of a rectangular prism:
  $V = l \times w \times h$

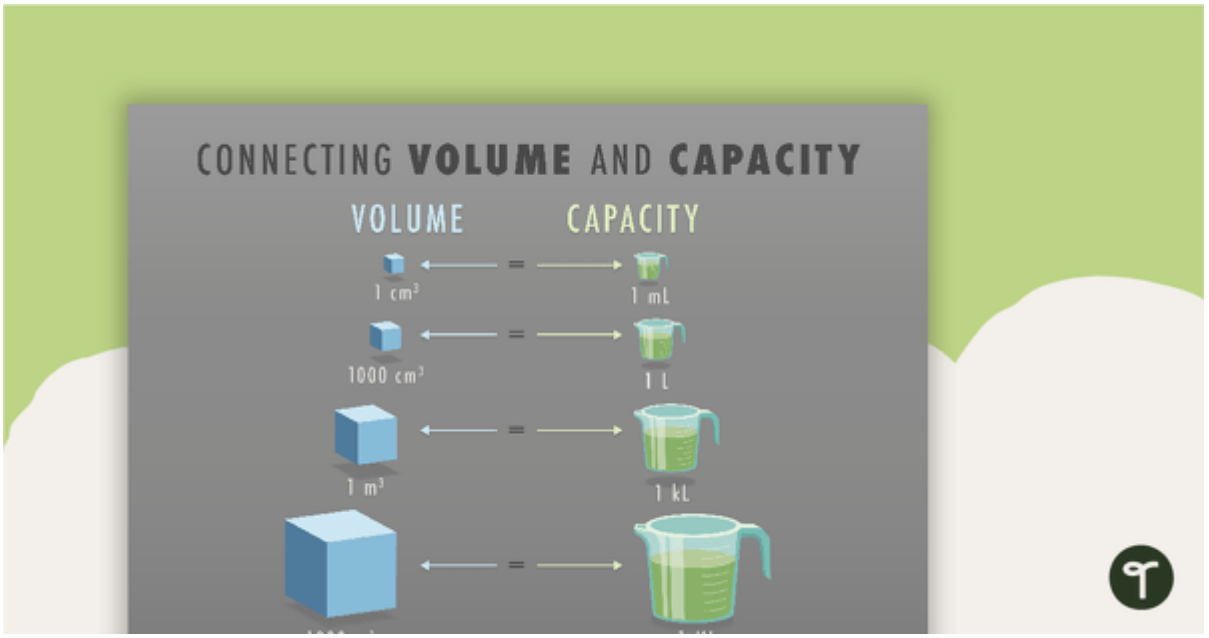



- The 'height' is the length of the edge that runs perpendicular to the cross-section in any solid.

**Capacity** The amount of liquid a container can hold

**Cross-section** The plane figure formed when you slice a solid figure parallel to one of its surfaces



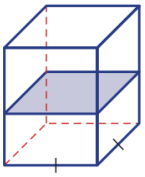



# Exercise 6H

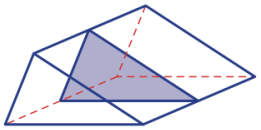
Understanding

**1** What is the name given to the shape of the shaded cross-section of each of the following solids?

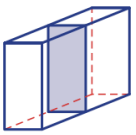
**a**



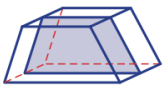
**b**



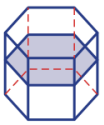
**c**



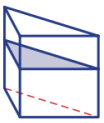
**d**



**e**



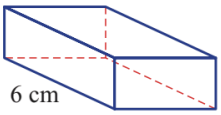
**f**



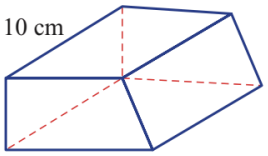
**2** Draw the cross-sectional shape for these prisms and state the given 'height' (perpendicular to the cross-section).

'perpendicular' means 'at a right angle (90°)'.

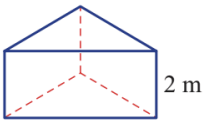
**a**



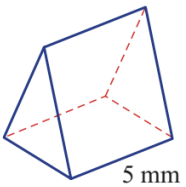

**b**



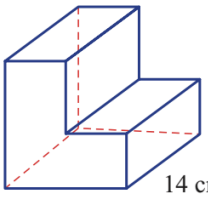

**c**




**d**




**e**




**f**


**3** Write the missing number.

**a** The number of mm in 1 cm is _____.

**b** The number of $mm^2$ in 1 $cm^2$ is _____.

**c** The number of $mm^3$ in 1 $cm^3$ is _____.

**d** There are _____ $cm^3$ in 1 $m^3$.

**e** There are _____ $m^3$ in 1 $km^3$.

**f** There are _____ mL in 1 L.

**g** There are _____ L in 1 kL.

**h** There are _____ $cm^3$ in 1 mL.

## Example 18 Converting units of volume

Convert the following volume measurements into the units given in the brackets.

**a** 2.5 $m^3$ ($cm^3$) **b** 458 $mm^3$ ($cm^3$)

| Solution | Explanation |
|---|---|
| **a** 2.5 $m^3 = 2.5 \times 100^3$ $cm^3$<br>$= 2\,500\,000$ $cm^3$ | $\times 100^3 = 1\,000\,000$<br>$m^3$ → $cm^3$<br>2.500000 |
| **b** 458 $mm^3 = 458 \div 10^3$ $cm^3$<br>$= 0.458$ $cm^3$ | $cm^3$ ← $mm^3$<br>$\div 10^3 = 1000$<br>458. |

**4** Convert the following volume measurements into the units given in brackets.

**a** 3 $cm^3$ ($mm^3$) **b** 0.3 $cm^3$ ($mm^3$) **c** 2000 $mm^3$ ($cm^3$)
**d** 0.001 $m^3$ ($cm^3$) **e** 8.7 $m^3$ ($cm^3$) **f** 5900 $cm^3$ ($m^3$)
**g** 0.00001 $km^3$ ($m^3$) **h** 21 700 $m^3$ ($km^3$) **i** 430 000 $cm^3$ ($m^3$)

1 $km^3 = 1000^3$ $m^3$
1 $m^3 = 100^3$ $cm^3$
1 $cm^3 = 10^3$ $mm^3$

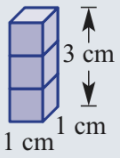
**5** Convert these units of capacity to the units given in brackets.

**a** 3 L (mL) **b** 0.2 kL (L) **c** 3500 mL (L)
**d** 0.021 L (mL) **e** 37 000 L (kL) **f** 42 900 kL (ML)
**g** 2 $cm^3$ (mL) **h** 2 L ($cm^3$) **i** 1 $m^3$ (L)

1 ML = 1000 kL
1 kL = 1000 L
1 L = 1000 mL

## Example 19 Finding the volume of a rectangular prism

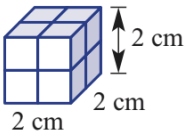
Find the volume of this rectangular prism.



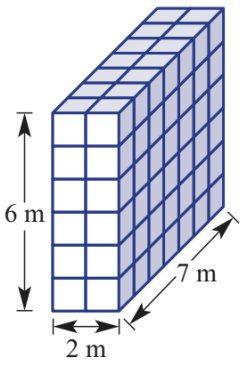

| Solution | Explanation |
|---|---|
| Volume $= l \times w \times h$<br>$= 1 \times 1 \times 3$<br>$= 3$ $cm^3$ | The solid is a rectangular prism.<br>Length = 1 cm, width = 1 cm and height = 3 cm |

**6** Find the volume of these three-dimensional rectangular prisms.

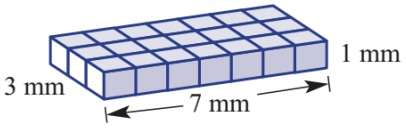
**a**




**b**

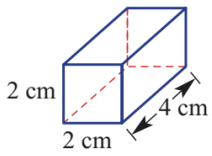



**c**

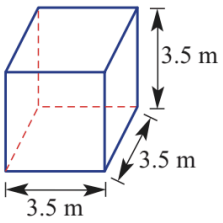


**7** Find the volume of each of these rectangular prisms (cuboids).

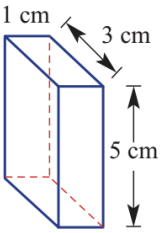
**a**




**b**



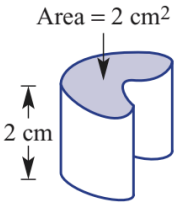

**c**



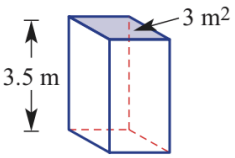

**8** Find the volume of each of these three-dimensional objects. The cross-sectional area has been given.

Simply use $V = A \times h$, since the area of the cross-section is given.

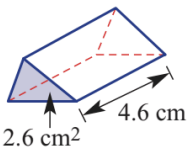
**a**



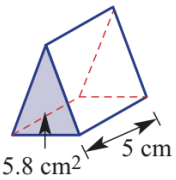

**b**



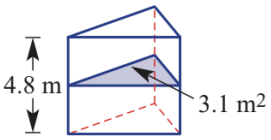

**c**



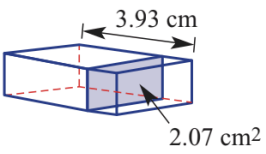

**d**




**e**

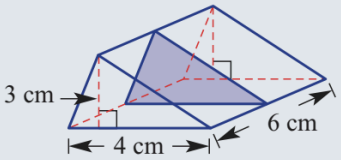



**f**




## Example 20 Finding the volume of a triangular prism

Find the volume of this triangular prism.




**Solution**

Area of cross-section $= \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times 4 \times 3$

$= 6 \text{ cm}^2$

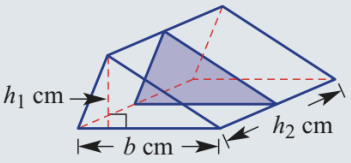
Volume = area of cross-section × height

$= 6 \times 6$

$= 36 \text{ cm}^3$

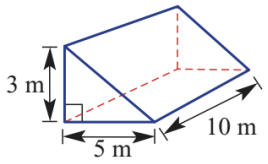
**Explanation**

The cross-section is a triangle.

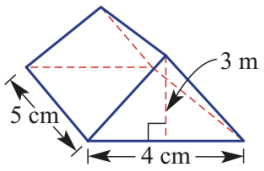


**9** Find the volume of these prisms.

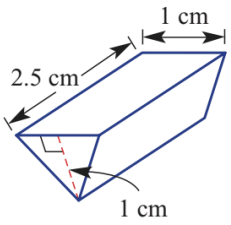
**a**




**b**




**c**




First find the area of the triangular cross-section.

## Problem-solving and Reasoning

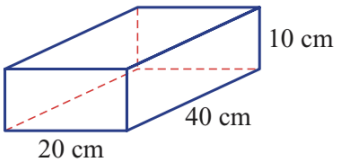
**10** A brick is 10 cm wide, 20 cm long and 8 cm high. How much space would five of these bricks occupy?

**11** 25 L of water is poured into a rectangular fish tank which is 50 cm long, 20 cm wide and 20 cm high. Will it overflow?

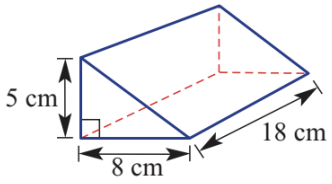
There are 1000 $cm^3$ in 1 L.

**12** Find the volume of these solids, converting your answer to litres.

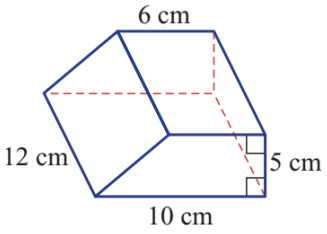
**a**




**b**




**c**

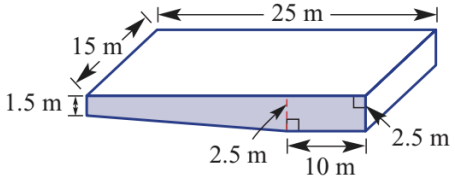



Area of a trapezium:
$A = \frac{1}{2}(a + b)h$

**13** This diagram is a sketch of a new 25 m swimming pool to be installed in a school sports complex.

**a** Find the area of one side of the pool (shaded).

**b** Find the volume of the pool in litres.
Use 1 $m^3$ = 1000 L.



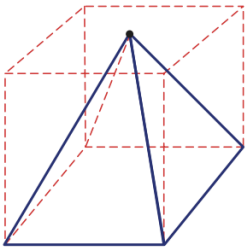

## Volume of a pyramid

**14** Someone tells you that the volume of a pyramid is half of the volume of a rectangular prism with the same base. Do you think this is true?

**a** Make an educated guess as to what fraction of the prism's volume is the pyramid's volume.

**b** Use the internet to find the actual answer to part **a**.

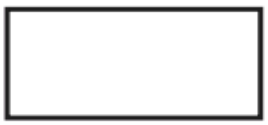
**c** Draw some pyramids and find their volume using the results from part **b**.

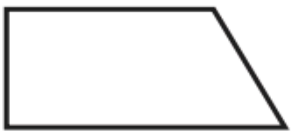


Check your answers

## Exercise 6H

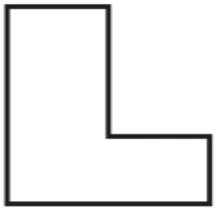
**1** **a** square **b** triangle **c** rectangle
**d** trapezium **e** hexagon **f** triangle

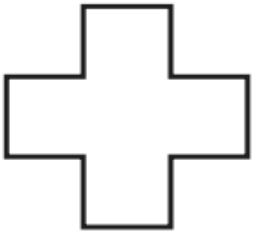
**2** **a**  , 6 cm

**b**  , 10 cm

**c**  , 2 m

**d**  , 5 mm

**e**  , 14 cm

**f**  , 10 m

**3** **a** 10 **b** 100 **c** 1000 **d** 1 000 000
**e** 1 000 000 000 **f** 1000 **g** 1000 **h** 1

**4** **a** 3000 $mm^3$ **b** 300 $mm^3$ **c** 2 $cm^3$
**d** 1000 $cm^3$ **e** 8 700 000 $cm^3$ **f** 0.0059 $m^3$
**g** 10 000 $m^3$ **h** 0.000 021 7 $km^3$ **i** 0.43 $m^3$

**5** **a** 3000 mL **b** 200 L **c** 3.5 L
**d** 21 mL **e** 37 kL **f** 42.9 ML
**g** 2 mL **h** 2000 $cm^3$ **i** 1000 L

**6** **a** 8 $cm^3$ **b** 84 $m^3$ **c** 21 $mm^3$

**7** **a** 16 $cm^3$ **b** 42.875 $m^3$ **c** 15 $cm^3$

**8** **a** 4 $cm^3$ **b** 10.5 $m^3$ **c** 11.96 $cm^3$
**d** 29 $cm^3$ **e** 14.88 $m^3$ **f** 8.1351 $cm^3$

**9** **a** 75 $m^3$ **b** 30 $cm^3$ **c** 1.25 $cm^3$

**10** 8000 $cm^3$

**11** Yes, the tank only holds 20 L

**12** **a** 8 L **b** 0.36 L **c** 0.48 L

**13** **a** 55 $m^2$ **b** 825 000 L

**14** **b** $\frac{1}{3}$