Probability for multistage events

WALT to create tree diagrams for a multistage event,

Success Criteria

I know that A **multistage event** is one that is made up of simple events such as tossing a coin and rolling a die.

I can find probabilities in these situations which are made easier if we use a table, tree diagram or lattice diagram to show all the possible outcomes.

I need to read the question carefully and decide which format is most appropriate. List the sample space to show all possible outcomes and use this to find the probabilities.

Watch video 1

Watch a video 2

EXAMPLE 1

a A coin is tossed and a die is thrown. List the sample space.
b What is the probability of a tail and a 3?

	Solve	Think	Apply
a	Sample space using a table: **Coin** | **Die**: H | 1 | 2 | 3 | 4 | 5 | 6; T | 1 | 2 | 3 | 4 | 5 | 6 Sample space using a tree diagram: H: 1 H1, 2 H2, 3 H3, 4 H4, 5 H5, 6 H6 T: 1 T1, 2 T2, 3 T3, 4 T4, 5 T5, 6 T6	Tossing a coin has two possible outcomes, throwing a die has six possible outcomes. Combine these in an organised way in a table or tree diagram to show the sample space.	Use an organised method to combine the outcomes of each simple event.
b	$P(\text{T3}) = \frac{1}{12}$	Number of outcomes in the sample space = 12 Number of favourable outcomes = 1	Use: Probability $= \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}}$

EXAMPLE 2

a A bag contains 2 blue, 3 red and 2 white counters. A coin is tossed and a counter is chosen at random from the bag. List the sample space.
b Find the probability of getting:
i a head and a red counter **ii** a tail and blue counter.

	Solve	Think	Apply
a	Sample space using a tree diagram: H: B HB, B HB, R HR, R HR, R HR, W HW, W HW T: B TB, B TB, R TR, R TR, R TR, W TW, W TW	Tossing a coin has two possible outcomes, selecting a counter has seven possible outcomes. Combine these in an organised way in a tree diagram.	Use an organised method to combine the outcomes of each simple event.
b i	$P(\text{HR}) = \frac{3}{14}$	Total number of equally likely outcomes = 14 Number of favourable outcomes = 3	Use: Probability $= \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}}$
ii	$P(\text{TB}) = \frac{2}{14} = \frac{1}{7}$	Number of favourable outcomes = 2	

Exercise 7C

1 **a** A coin is tossed and a die is thrown. List the sample space.

b Find the probability of getting:

i a tail and a 4 **ii** a tail and an odd number

iii a head and a 6 **iv** a head and an even number.

2 **a** A jar contains 2 red discs and 1 blue disc. A coin is tossed and a disc is selected at random from the jar. List the sample space.

b Find the probability that the result is:

i a head and a red disc **ii** a head and a blue disc **iii** a tail and a red disc.

3 **a** A spinner has the numbers 1 to 4, each with an equal chance of occurring. A coin is tossed and the spinner is spun. List all the equally likely outcomes of this experiment.

b Find the probability of obtaining:

i a head and a 2 **ii** a tail and a 3 **iii** a head and an even number.

4 **a** A bag contains 2 green discs and 1 black disc. A die is rolled and a disc is chosen at random from the bag. List the sample space.

b What is the probability of getting:

i a 3 and a green disc? **ii** a 5 and a black disc?

iii an odd number and a green disc? **iv** an even number and a black disc?

5 **a** Marie and Peter are planning a family with 2 children. Assume the chance of having a boy or a girl are equally likely. List all the possible outcomes.

b Find the probability of having:

i 2 boys **ii** a boy and a girl, in that order

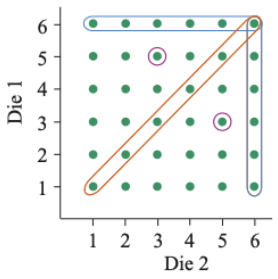
iii a boy and a girl, in any order **iv** at least 1 girl.

EXAMPLE 3

a Two dice are thrown. List the sample space.

b What is the probability of obtaining:

i a 3 and a 5? **ii** a double? **iii** at least one 6?

	Solve	Think	Apply
a		Show the sample space using a lattice diagram. Each point on the lattice represents the numbers on each die.	Use an organised method to combine the outcomes of each simple event.
b i	$P(3 \text{ and } 5) = \frac{2}{36} = \frac{1}{18}$	Total number of equally likely possible outcomes = 36 Number of favourable outcomes = 2	Use: Probability $= \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}}$

EXAMPLE 3 CONTINUED

	Solve	Think
ii	$P(\text{a double}) = \frac{6}{36} = \frac{1}{6}$	Number of favourable outcomes = 6
iii	$P(\text{at least one 6}) = \frac{11}{36}$	Number of favourable outcomes = 11

6 Two dice are thrown. Find the probability of throwing:

a a 4 and a 6 **b** a 3 and a 2 **c** double 6

d at least one 2 **e** a 5 and an even number.

7 Two spinners each have the numbers 1 to 5, with equal probabilities of each number occurring.

a Draw a lattice diagram to illustrate all the possible outcomes when the two spinners are spun.

b Find the probability of getting:

i a 3 and a 4 **ii** a 4 and an even number

iii a double **iv** a 2 and any other number.

8 A spinner has the numbers 1 to 5, with equal probabilities of each number occurring. A bag contains discs numbered 1 to 6. The spinner is spun and a disc is selected from the bag.

a List the sample space.

b Find the probability of obtaining

i a 4 and a 6 **ii** a double

iii a 2 and an even number **iv** at least one 3.

EXAMPLE 4

a Three coins are tossed. List the sample space.

b What is the probability of getting:

i 3 heads? **ii** 2 heads and a tail, in any order?

a

1st coin	2nd coin	3rd coin	Sample space
H	H	H	HHH
		T	HHT
	T	H	HTH
		T	HTT
T	H	H	THH
		T	THT
	T	H	TTH
		T	TTT

Use a tree diagram.

	Solve	Think	Apply
b i	$P(\text{3 heads}) = \frac{1}{8}$	There are 8 equally likely outcomes. Number of favourable outcomes = 1	Use: Probability $= \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}}$
ii	$P(\text{2 heads and a tail, in any order}) = \frac{3}{8}$	Probability of 2 heads and a tail, in any order = P(HHT or HTH or THH) Number of favourable outcomes = 3	

9 Three coins are tossed. What is the probability of getting:

a 3 tails? **b** 2 tails and a head, in any order? **c** no tails?

d at least one tail? **e** at most one tail?

10 **a** A family has 3 children. Assuming the chance of a boy or a girl is equally likely, list the sample space.

b Calculate the probability of having:

i 3 boys **ii** 2 girls and a boy, in that order

iii 2 girls and a boy, in any order **iv** no girls **v** at least 1 girl.

11 **a** A spinner has the numbers 1 to 4, each with an equal chance of occurring. A bag contains a blue, a red and a green disc. A coin is tossed, the spinner is spun and a disc is selected from the bag. List all the equally likely outcomes of this experiment.

b Find the probability of obtaining:

i a head, a 4 and a blue disc **ii** a tail, an even number and a red disc

iii a head, a 1 and either a blue or a green disc **iv** a head, a number <4 and a green disc.

Check your answers

1 **a** H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6

b **i** $\frac{1}{12}$ **ii** $\frac{1}{4}$ **iii** $\frac{1}{12}$ **iv** $\frac{1}{4}$

2 **a**

H	R	R	B
T	R	R	B

b **i** $\frac{1}{3}$ **ii** $\frac{1}{6}$ **iii** $\frac{1}{3}$

3 **a**

H	1	2	3	4
T	1	2	3	4

b **i** $\frac{1}{8}$ **ii** $\frac{1}{8}$ **iii** $\frac{1}{4}$

4 **a**

G	1	2	3	4	5	6
G	1	2	3	4	5	6
B	1	2	3	4	5	6

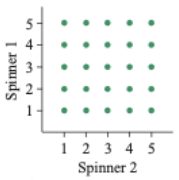
b **i** $\frac{1}{9}$ **ii** $\frac{1}{18}$ **iii** $\frac{1}{3}$ **iv** $\frac{1}{6}$

5 **a** BB, BG, GB, GG

b **i** $\frac{1}{4}$ **ii** $\frac{1}{4}$ **iii** $\frac{1}{2}$ **iv** $\frac{3}{4}$

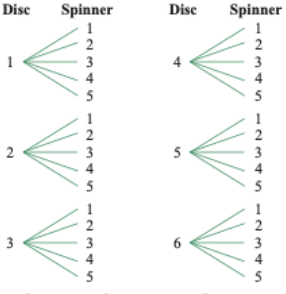
6 **a** $\frac{1}{18}$ **b** $\frac{1}{18}$ **c** $\frac{1}{36}$ **d** $\frac{11}{36}$ **e** $\frac{1}{6}$

7 **a**



b **i** $\frac{2}{25}$ **ii** $\frac{3}{25}$ **iii** $\frac{1}{5}$ **iv** $\frac{8}{25}$

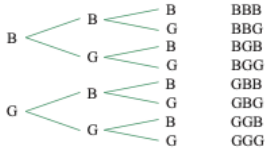
8 **a**



b **i** $\frac{1}{15}$ **ii** $\frac{1}{6}$ **iii** $\frac{2}{15}$ **iv** $\frac{1}{3}$

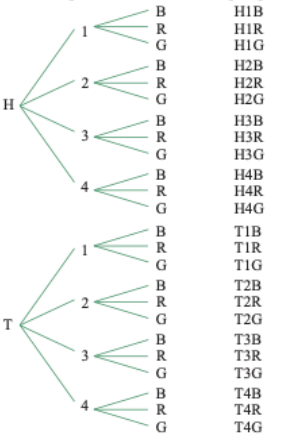
9 **a** $\frac{1}{8}$ **b** $\frac{3}{8}$ **c** $\frac{1}{8}$ **d** $\frac{7}{8}$ **e** $\frac{1}{2}$

10 **a**



b **i** $\frac{1}{8}$ **ii** $\frac{1}{8}$ **iii** $\frac{3}{8}$ **iv** $\frac{1}{8}$ **v** $\frac{7}{8}$

11 **a**



b **i** $\frac{1}{24}$ **ii** $\frac{1}{12}$ **iii** $\frac{1}{12}$ **iv** $\frac{1}{8}$