

WALT understand probability and the chance

Success Criteria I know ...

- A probability is a number between 0 and 1. 0 means impossible and 1 means certain
- If the outcomes are equally likely, we find the probability of an event by counting the ways it can happen and dividing it by the total number of outcomes
- [Video on probability](#)

## Let's start

### ▶ Let's start: Estimating probabilities

Try to estimate the probability of the following events, giving a number between 0 and 1. Compare your answers with other students in the class and discuss any differences.

- 1 Flipping a 'tail' on a 50-cent coin.
- 2 An albino whale is born.
- 3 Rolling three 6s in a row on a fair die.
- 4 Correctly guessing a number between 1 and 10.
- 5 Tomorrow being a rainy day.
- 6 Seeing a wombat in the Australian bush.

Are there some events for which there is more than one correct answer?

## Important notes

**Trial** - One run of an event

**Outcome**- One of the possibilities from an event

**Event**- Collection of possible outcomes

**Probability** - A measure of the likelihood that an event will occur

**Sample Space** -The list of all the possible outcomes of an event

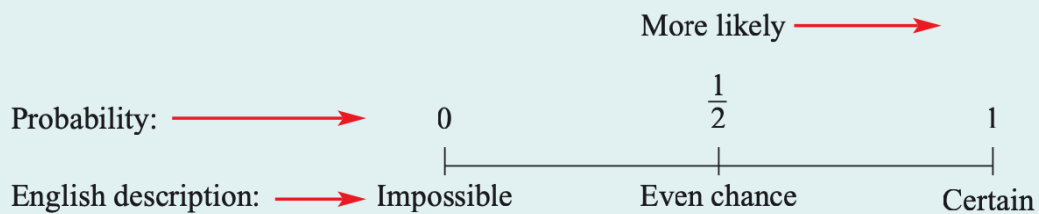
**Complement** - Set containing the elements that are not in a given set

## Teacher discussion

- A **trial** could be flipping a coin, rolling a die or spinning a spinner.
- There are multiple **outcomes** that could occur for any trial; for example, 'rolling a 3' or 'flipping tails on the coin'.
- An **event** is a collection of outcomes.
- The **probability** of an event is a number between 0 and 1 that represents the chance that the event occurs. If all the outcomes are equally likely:

$$\Pr(\text{Event}) = \frac{\text{number of outcomes where the event occurs}}{\text{total number of outcomes}}$$

- Probabilities are often written as fractions, but can also be written as decimals or percentages.



- The **sample space** is the set of possible outcomes of a trial or event. For example, the sample space for the roll of a die is 1, 2, 3, 4, 5, 6.
- The **complement** of some event  $E$  is written  $E'$  (or not  $E$ ).  $E'$  is the event that  $E$  does not occur. For example, the complement of 'rolling the number 3' is 'rolling a number other than 3'.

- 1 Write the missing word from each statement.
- a An example of a \_\_\_\_\_ is flipping a coin.
  - b After rolling a die the possible \_\_\_\_\_ are 1, 2, 3, 4, 5 and 6.
  - c The set of all possible outcomes from a trial is called the \_\_\_\_\_.
  - d The \_\_\_\_\_ of an event is the opposite of that event.
  - e If an event is called A then the complement is written as \_\_\_\_\_.
- 2 Match each experiment with the set of possible outcomes.
- a Flipping a coin
  - b Choosing a number between 1 and 5
  - c Choosing a letter of the word MATHS
  - d Rolling a die
- A** 1, 2, 3, 4, 5, 6      **B** Heads, Tails      **C** 1, 2, 3, 4, 5      **D** M, A, T, H, S
- 3 The following events are shown with their probabilities.  
 Event A: 0    Event B: 0.9    Event C: 1    Event D: 0.5
- a Which of the four events is most likely to occur?
  - b Which of the four events is sure not to occur?
  - c Which is more likely – event B or event D?
  - d Which event is sure to occur?

Impossible events are sure not to occur.

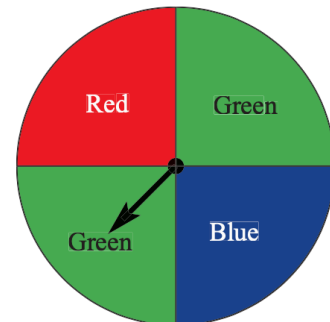


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- 4 The spinner is spun and could land with the pointer on any of the four sections.  
 Answer true or false:
- a Red and blue are equally likely outcomes.
  - b Green is less likely to occur than blue.
  - c The probability of it landing orange is 0.
  - d Red is less likely to occur than green.



### Working with probabilities

The letters of the word PRINCE are written onto 6 equally-sized cards and one is chosen at random.

- a State the sample space.
- b Find  $\Pr(\text{the letter N is chosen})$ .
- c What is the sample space of the event  $V = \text{choosing a vowel}$ ?
- d Find  $\Pr(V)$ .
- e State the sample space of the complement of choosing a vowel, written  $V'$ .
- f Hence find  $\Pr(V')$ .

## Solution

## Explanation

**a** P, R, I, N, C, E

The sample space is all the possible outcomes when a single card is chosen. In this case each of the letters in the word.

**b**  $\Pr(N) = \frac{1}{6}$

There are 6 equally likely cards and 1 of them has the letter N.

**c** I, E

The sample space  $V$  includes all the vowels in the word PRINCE.

**d**  $\Pr(V) = \frac{2}{6}$   
 $= \frac{1}{3}$

There are 2 cards with vowels, so probability =  $2 \div 6$ .

**e**  $V'$  includes P, R, N, C

The complement of  $V$  ( $V'$ ) is all the outcomes that are not in  $V$ , i.e. all the letters that are not vowels.

**f**  $\Pr(V') = \frac{4}{6}$   
 $= \frac{2}{3}$

There are 4 cards that do not have vowels, so  $\Pr(V') = 4 \div 6$ .

**5** The letters of the word PIANO are written on 5 cards and then one card is drawn from a hat at random.

**a** List the sample space.

**b** Find  $\Pr$ (the letter A is chosen).

**c** Find  $\Pr$ (a vowel is chosen).

**d** Find  $\Pr$ (a consonant is drawn).

**e** Find  $\Pr$ (the letter chosen is not an N).

**f** State the sample space of the complement of choosing a vowel, written  $V'$ .

**g** Hence find ( $\Pr(V')$ )

Pr means probability.



Write probability answers as fractions.



**6** A fair die is rolled.

**a** List the sample space.

**b** Find  $\Pr(5)$ . That is, find the probability that a 5 is rolled.

**c** Find  $\Pr$ (even number).

**d** State the sample space of the complement of 'rolling a 5'.

**e** State the probability that a 5 is not rolled.

**f** What is the probability of rolling a 14?



- 7 There are five red marbles, two green marbles and three black marbles. The 10 marbles are placed into a hat and one is picked out.



- What is  $\Pr(\text{red})$ ? That is, what is the probability that the picked marble is red?
  - Find  $\Pr(\text{green})$ .
  - Find  $\Pr(\text{black})$ .
  - Find  $\Pr(\text{a black or a red marble is drawn})$ .
  - Find  $\Pr(\text{red}')$ , that is find the probability of the complement of choosing a red marble.
  - Find  $\Pr(\text{black}')$ .
  - Give an example of an event that has a probability of 0.
- 8 The numbers 1 to 10 are written on cards. A card is chosen at random.
- List the sample space.
  - Find the probability of choosing a 5.
  - Find  $\Pr(7 \text{ or } 9)$ .
  - Find  $\Pr(\text{a multiple of } 3 \text{ is chosen})$ .
  - Find  $\Pr(\text{prime number})$ .
  - Find  $\Pr(\text{a factor of } 24)$ .

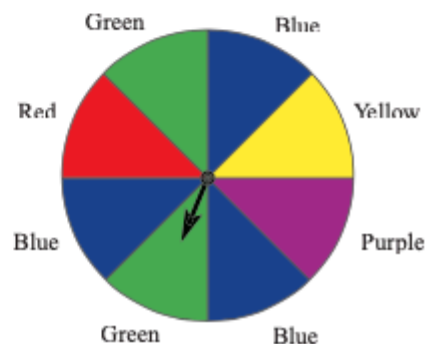
A factor of 24 divides into 24 with no remainder. A prime has 2 factors. 1 is not prime.



Problem-solving and Reasoning

- 9 A spinner has the arrangement of colours as shown.
- List the sample space when this spinner is spun.
  - Find  $\Pr(\text{red})$ .
  - State  $\Pr(\text{green})$ .
  - Find  $\Pr(\text{blue})$ .
  - List the sample space of the complement of 'spinner landing on blue'.
  - What is  $\Pr(\text{not blue})$ ?
  - Find  $\Pr(\text{red or green or blue})$ .
  - What is an event that is equally likely to 'spinning red'?
  - Give an example of an event that has a probability of 0.

List the colour as many times as it is on the spinner.



## Extension

10 On a game show, a wheel is spun for a prize with the options as shown.

- Joan wants to go on a \$10 000 holiday so she is happy with the cash or the holiday. What is the probability she will get what she wants?
- What is the probability of getting a prize that is not the cash?
- What is  $\Pr(\text{car or motorbike})$ ?
- What is the probability of winning a prize?



11 Jamie has a collection of marbles in his pocket. Four of them are blue, three are green and three are white. He chooses one at random.

- What is the probability that a green marble is chosen?
- What is the probability that he does not choose a white marble?
- He adds two more marbles and now  $\Pr(\text{blue}) = \frac{1}{2}$ . What colour were the marbles he added?
- If instead of adding the two marbles he removed two, is it possible for  $\Pr(\text{blue})$  to become  $\frac{1}{2}$ ? Explain your answer.

12 Six counters coloured red, purple or orange are placed in a pocket. You are told that

$$\Pr(\text{red or orange}) = \frac{1}{2} \text{ and } \Pr(\text{red or purple}) = \frac{2}{3}.$$

- How many counters of each colour are there?
  - State  $\Pr(\text{red})$ .
  - Find  $\Pr(\text{purple})$ .
  - Find  $\Pr(\text{orange})$ .
- 13 Draw a spinner that has  $\Pr(\text{red}) = \frac{1}{8}$ ,  $\Pr(\text{blue}) = \frac{5}{8}$  and  $\Pr(\text{green}) = \frac{1}{4}$ .

Change the probabilities to have a common denominator.

First divide a circle into 8 equal sectors.



### Changing probabilities

14 In a large bucket there are 2 red balls and 8 blue balls.

- State  $\Pr(\text{red})$ .
- One of each colour is added. What is the new  $\Pr(\text{red})$ ?
- The procedure of adding a red ball and a blue ball is repeated several times. How many balls are in the bucket when  $\Pr(\text{red}) = \frac{1}{3}$ ?
- Imagine the procedure is repeated many times. What value does  $\Pr(\text{red})$  eventually approach as more balls are added? It might be helpful to imagine 1000 balls of each colour are added and use decimals.

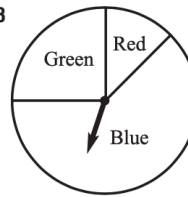
Make a table.



## Check your answers

- 1 a trial      b outcomes      c sample space  
 d complement      e  $A'$
- 2 a B      b C      c D      d A
- 3 a Event C      b Event A      c Event B      d Event C
- 4 a true      b false      c true      d true
- 5 a P, I, A, N, O      b  $\frac{1}{5}$       c  $\frac{3}{5}$   
 d  $\frac{2}{5}$       e  $\frac{4}{5}$       f  $V' = P, N$       g  $\Pr(V') = \frac{2}{5}$
- 6 a 1, 2, 3, 4, 5, 6      b  $\frac{1}{6}$       c  $\frac{1}{2}$   
 d 1, 2, 3, 4, 6      e  $\frac{5}{6}$       f 0
- 7 a  $\frac{1}{2}$       b  $\frac{1}{5}$       c  $\frac{3}{10}$       d  $\frac{4}{5}$   
 e  $\frac{1}{2}$       f  $\frac{7}{10}$       g choosing a purple marble
- 8 a 1, 2, 3, 4, 5, 6, 7, 8, 9, 10      b  $\frac{1}{10}$       c  $\frac{1}{5}$   
 d  $\frac{3}{10}$       e  $\frac{2}{5}$       f  $\frac{3}{5}$
- 9 a red, green, blue, yellow, purple, blue, green, blue  
 b  $\frac{1}{8}$       c  $\frac{1}{4}$       d  $\frac{3}{8}$   
 e green, green, red, yellow, purple      f  $\frac{5}{8}$   
 g  $\frac{3}{4}$       h spinning purple (or spinning yellow)  
 i spinning orange
- 10 a  $\frac{1}{3}$       b  $\frac{5}{6}$       c  $\frac{1}{3}$       d  $\frac{5}{6}$
- 11 a  $\frac{3}{10}$       b  $\frac{7}{10}$       c Both were blue.  
 d Yes, for instance if he removed two green marbles.
- 12 a 1 red, 2 orange, 3 purple      b  $\frac{1}{6}$   
 c  $\frac{1}{2}$       d  $\frac{2}{3}$

13



- 14 a  $\frac{1}{5}$       b  $\frac{1}{4}$       c 18  
 d It approaches  $\frac{1}{2}$  or 0.5.