Rational Numbers

WALT Understand rational numbers and fractions

Success Criteria: I know what are rational numbers and be able to work on different operations of rational numbers such as fractions and decimals

Rational numbers can be written as a **ratio** of two integers in the form $\frac{a}{b}$.

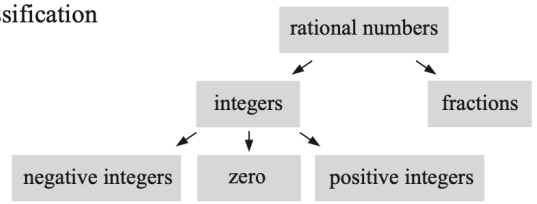
Rational numbers appear in many forms.

For example, 4, $^{-}2$, 0, 10%, $^{-}\frac{4}{7}$, 1.3, $0.\overline{6}$ are all rational numbers,

i.e., they can be written in the form $\frac{a}{b}$ as: $\frac{4}{1}$, $\frac{^{-}2}{1}$, $\frac{0}{1}$, $\frac{1}{10}$, $\frac{^{-}4}{7}$, $\frac{13}{10}$, $\frac{2}{3}$.

Note that 4, $^{-}2$ and 0 are *integers*, and 10%, $^{-}\frac{4}{7}$, 1.3 and $0.\overline{6}$ are *fractions*.

We can extend our classification of numbers as shown:



A **common fraction** consists of two whole numbers, a **numerator** and a **denominator**, separated by a bar symbol.

For example,

$\frac{4}{5}$ ← numerator
— ← bar (which also means *divide*)
5 ← denominator

TYPES OF FRACTIONS

$\frac{4}{5}$ is a **proper fraction**	{as the numerator is less than the denominator}
$\frac{7}{6}$ is an **improper fraction**	{as the numerator is greater than the denominator}
$2\frac{3}{4}$ is a **mixed number**	{as it is really $2+\frac{3}{4}$}
$\frac{1}{2}$, $\frac{3}{6}$ are **equivalent fractions**	{as both fractions represent equivalent portions}

Example 12

Simplify: **a** $\frac{9}{12}$ **b** $\frac{16}{20}$ **c** $\frac{9}{4}$ **d** $\frac{12}{5}$

a $\frac{9}{12} = \frac{9 \div 3}{12 \div 3}$
$= \frac{3}{4}$

b $\frac{16}{20} = \frac{16 \div 4}{20 \div 4}$
$= \frac{4}{5}$

c $\frac{9}{4} = \frac{8+1}{4}$
$= \frac{8}{4} + \frac{1}{4}$
$= 2\frac{1}{4}$

d $\frac{12}{5} = \frac{10+2}{5}$
$= \frac{10}{5} + \frac{2}{5}$
$= 2\frac{2}{5}$

EXERCISE 1B.1

1 Simplify:

a $\frac{6}{8}$ **b** $\frac{6}{10}$ **c** $\frac{14}{20}$ **d** $\frac{7}{21}$

e $\frac{24}{30}$ **f** $\frac{40}{60}$ **g** $\frac{32}{48}$ **h** $\frac{75}{125}$

2 Simplify:

a $\frac{7}{4}$ **b** $\frac{10}{7}$ **c** $\frac{16}{5}$ **d** $\frac{19}{8}$

Addition and Subtraction of fractions

Example 13

Find: $\frac{3}{4} + \frac{5}{6}$

$\frac{3}{4} + \frac{5}{6}$

$= \frac{3\times3}{4\times3} + \frac{5\times2}{6\times2}$ {Lowest Common Denominator, LCD = 12}
{to achieve a common denominator of 12}

$= \frac{9}{12} + \frac{10}{12}$

$= \frac{19}{12}$

$= 1\frac{7}{12}$

1 Find:

a $\frac{1}{2} + \frac{1}{2}$ **b** $\frac{3}{4} + \frac{1}{4}$ **c** $1\frac{2}{3} + \frac{1}{3}$ **d** $\frac{1}{6} + \frac{1}{6}$

e $\frac{3}{14} + \frac{6}{14}$ **f** $\frac{7}{16} + \frac{2}{16}$ **g** $\frac{2}{11} + \frac{5}{11}$ **h** $\frac{11}{15} + \frac{2}{15}$

2 Find:

a $\frac{1}{2} + \frac{1}{4}$ **b** $\frac{2}{3} + \frac{1}{2}$ **c** $\frac{1}{5} + \frac{1}{2}$ **d** $\frac{2}{3} + \frac{1}{5}$

e $\frac{3}{4} + \frac{1}{3}$ **f** $\frac{1}{6} + \frac{2}{3}$ **g** $\frac{5}{8} + \frac{3}{4}$ **h** $\frac{2}{5} + \frac{1}{6}$

Example 14

Find: $1\frac{2}{3} + 3\frac{5}{8}$

$$
\begin{aligned}
& 1\tfrac{2}{3} + 3\tfrac{5}{8} \\
&= 4 + \tfrac{2}{3} + \tfrac{5}{8} && \{\text{adding the whole numbers first}\} \\
&= 4 + \tfrac{2\times 8}{3\times 8} + \tfrac{5\times 3}{8\times 3} && \{\text{to get a common denominator of 24}\} \\
&= 4 + \tfrac{16}{24} + \tfrac{15}{24} && \{\text{simplifying}\} \\
&= 4 + \tfrac{31}{24} && \{\text{adding the fractions}\} \\
&= 4 + 1\tfrac{7}{24} \\
&= 5\tfrac{7}{24}
\end{aligned}
$$

3 Find:

a $1\frac{1}{2} + \frac{1}{2}$	**b** $1\frac{1}{2} + \frac{1}{3}$	**c** $1\frac{1}{2} + 1\frac{1}{3}$	**d** $2\frac{3}{4} + 1\frac{1}{4}$
e $3\frac{5}{8} + 2\frac{1}{3}$	**f** $3\frac{3}{4} + 1\frac{5}{6}$	**g** $2\frac{1}{3} + 3\frac{1}{4}$	**h** $5\frac{1}{8} + 2\frac{2}{3}$
i $2\frac{3}{7} + 1\frac{1}{3}$	**j** $2\frac{2}{3} + 1\frac{5}{6}$	**k** $2\frac{3}{7} + 1\frac{2}{5}$	**l** $1\frac{1}{2} + 2\frac{1}{3} + 3\frac{1}{4}$

Subtraction

Example 15

Find: **a** $\frac{3}{4} - \frac{1}{5}$ **b** $3\frac{2}{3} - 1\frac{4}{5}$

a

$$
\begin{aligned}
& \tfrac{3}{4} - \tfrac{1}{5} \\
&= \tfrac{3\times 5}{4\times 5} - \tfrac{1\times 4}{5\times 4} && \{\text{to get a common denominator of 20}\} \\
&= \tfrac{15}{20} - \tfrac{4}{20} \\
&= \tfrac{11}{20}
\end{aligned}
$$

b

$$
\begin{aligned}
& 3\tfrac{2}{3} - 1\tfrac{4}{5} \\
&= 2 + \tfrac{2}{3} - \tfrac{4}{5} && \{\text{subtracting the whole numbers first}\} \\
&= 2 + \tfrac{2\times 5}{3\times 5} - \tfrac{4\times 3}{5\times 3} && \{\text{to get a common denominator of 15}\} \\
&= 2 + \tfrac{10}{15} - \tfrac{12}{15} \\
&= 1 + \tfrac{15}{15} + \tfrac{10}{15} - \tfrac{12}{15} \\
&= 1 + \tfrac{15+10-12}{15} \\
&= 1\tfrac{13}{15}
\end{aligned}
$$

4 Find:

a $\frac{7}{9} - \frac{2}{9}$ **b** $\frac{4}{5} - \frac{3}{5}$ **c** $\frac{2}{3} - \frac{1}{3}$ **d** $\frac{3}{4} - \frac{3}{4}$

e $\frac{5}{8} - \frac{3}{5}$ **f** $\frac{7}{8} - \frac{1}{4}$ **g** $\frac{6}{11} - \frac{1}{3}$ **h** $\frac{7}{9} - \frac{2}{3}$

5 Find:

a $1 - \frac{1}{8}$ **b** $2 - \frac{3}{8}$ **c** $5 - 3\frac{1}{8}$ **d** $3 - 2\frac{1}{2}$

e $1\frac{1}{2} - \frac{1}{3}$ **f** $2\frac{2}{3} - \frac{1}{4}$ **g** $1\frac{1}{2} - \frac{1}{2}$ **h** $3\frac{3}{4} - \frac{1}{6}$

6 Find:

a $2\frac{1}{3} - 1\frac{1}{4}$ **b** $3\frac{5}{8} - 2\frac{1}{3}$ **c** $1\frac{3}{4} - 1\frac{1}{3}$ **d** $5\frac{3}{8} - 2\frac{1}{4}$

e $1\frac{2}{3} - \frac{3}{4}$ **f** $2\frac{3}{5} - 1\frac{3}{4}$ **g** $3\frac{1}{4} - 1\frac{1}{2}$ **h** $2\frac{3}{4} - \frac{5}{6}$

i $3\frac{1}{3} - 2\frac{1}{2}$ **j** $2\frac{3}{5} - 1\frac{5}{6}$ **k** $3\frac{5}{6} - 2\frac{7}{8}$ **l** $3\frac{4}{5} - 1\frac{7}{8}$

Multiplication

To multiply you multiply the top numbers together and bottom numbers tiger first and then simplify the answer

Example 16

Find: **a** $\frac{2}{3} \times \frac{4}{5}$ **b** $1\frac{3}{4} \times 2\frac{1}{3}$

a
$$\frac{2}{3} \times \frac{4}{5}$$
$$= \frac{2 \times 4}{3 \times 5}$$
$$= \frac{8}{15}$$

b
$$1\frac{3}{4} \times 2\frac{1}{3}$$
$$= \frac{7}{4} \times \frac{7}{3} \quad \{\text{converting to improper fractions}\}$$
$$= \frac{7 \times 7}{4 \times 3}$$
$$= \frac{49}{12}$$
$$= 4\frac{1}{12}$$

7 Find:

a $\frac{1}{2} \times \frac{1}{2}$ **b** $\frac{1}{2} \times \frac{1}{3}$ **c** $\frac{1}{2} \times \frac{1}{4}$ **d** $\frac{1}{3} \times \frac{1}{4}$

e $1\frac{2}{5} \times \frac{1}{3}$ **f** $\frac{3}{5} \times \frac{3}{4}$ **g** $\frac{2}{3} \times \frac{1}{5}$ **h** $\frac{4}{5} \times \frac{2}{5}$

8 Find:

a $1\frac{1}{2} \times \frac{1}{3}$ **b** $\frac{2}{3} \times 1\frac{1}{4}$ **c** $1\frac{1}{2} \times 1\frac{1}{2}$ **d** $2\frac{1}{3} \times 1\frac{3}{4}$

e $1\frac{4}{5} \times 1\frac{1}{2}$ **f** $2\frac{1}{5} \times 1\frac{1}{3}$ **g** $1\frac{1}{3} \times 1\frac{1}{3}$ **h** $1\frac{1}{2} \times 1\frac{1}{3} \times 1\frac{1}{4}$

1 Multiply these fractions.

a $\frac{1}{2} \times \frac{2}{3}$ **b** $\frac{4}{5} \times \frac{2}{3}$

c $\frac{1}{4} \times \frac{2}{5}$ **d** $\frac{14}{15} \times \frac{12}{21}$

2 Evaluate these products.

a $\frac{1}{8} \times 5$ **b** $3 \times \frac{1}{4}$

c $5 \times \frac{2}{15}$ **d** $\frac{5}{12} \times 2$

e $\frac{3}{4} \times 2$

3 Multiply these fractions.

a $\frac{25}{42} \times \frac{7}{60}$

b $\frac{2}{3} \times \frac{4}{5} \times \frac{1}{3}$

c $\frac{2}{5} \times \frac{1}{2} \times \frac{3}{4}$