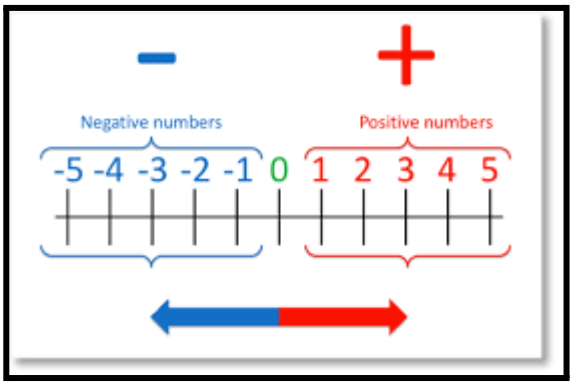
4zrIntegers

WALT understand using integers and applying the integers rule when adding, subtracting, multiplying and dividing

Success Criteria I know how to multiply and divide negative and positive, negative and negative add and subtract negative and negative, positive and positive



Few things to consider

- Owing the bank \$5 would be represented as $^{-}5$, whereas having a deposit of \$5 would be represented as $^{+}5$ or just 5.
- A temperature of 21°C above zero would be 21, whereas 3°C below zero would be $^{-}3$.
- A height of 16 m below sea level would be $^{-}16$, whereas 8848 m above sea level would be 8848.

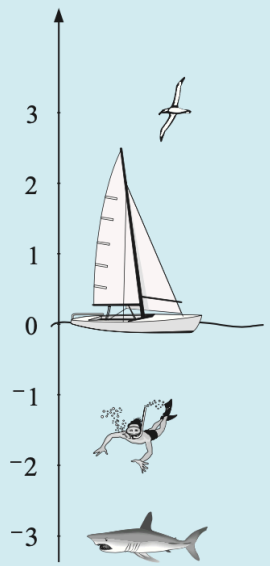
Some common uses of positive and negative signs are listed in the given table:

Positive (+)	*Negative (−)*
above	below
increase	decrease
profit	loss
right	left
fast	slow
win	loss
North	South

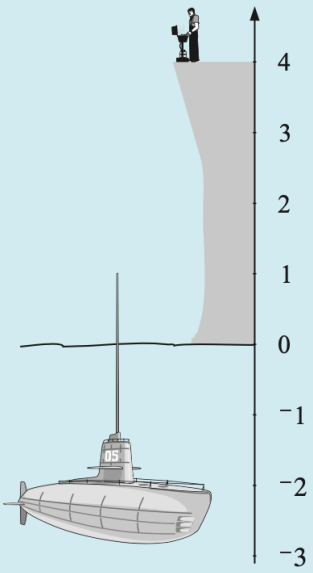
Example 1

Write the number (positive, negative or zero) which describes the position of each object. The reference position is the water level.

a



b



a Positions *above* the water level are marked off with *positive numbers*, so the bird is at 3.

The boat is level with the water, so it is at 0.

Positions *below* the water level are marked off with *negative numbers*. The diver is at $^{-}1.5$ and the shark is at $^{-}3$.

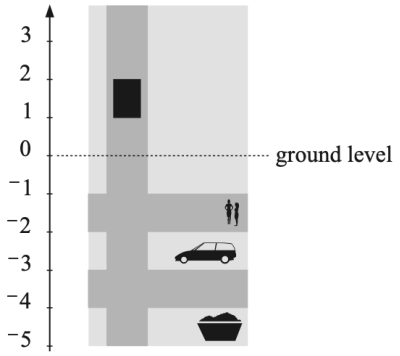
b The clifftop is at 4, the periscope is at 1, the water is at 0, the submarine is at $^{-}2$.

1 Copy and complete the following table:

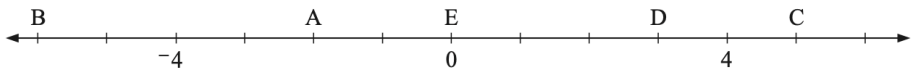
	Statement	*Number*	*Opposite to statement*	*Number*
a	20 m above sea level	20	20 m below sea level	$^{-}20$
b	45 km south of the city			
c	a loss of 2 kg in weight			
d	a clock is 2 min fast			
e	she arrives 5 min early			
f	a profit of $4000			
g	2 floors above ground level			
h	10°C below zero			
i	an increase of $400			
j	winning by 34 points			

2 Write positive or negative numbers for the position of the lift, the car, the parking attendant and the rubbish skip.

(Use the bottom of each object.)



3 If right is positive and left is negative, write the numbers for the positions of A, B, C, D and E using zero as the reference position.



4 Write these temperatures as positive or negative numbers. Zero degrees is the reference point.

a 11^o above zero **b** 6^o below zero **c** 8^o below zero
d 29^o above zero **e** 14^o below zero

5 Write these gains or losses as positive or negative numbers:

a \$30 loss **b** \$200 gain **c** \$431 loss
d \$751 loss **e** \$809 gain

6 If north is the positive direction, write these directions as positive or negative numbers:

a 7 metres north **b** 15 metres south **c** 115 metres south
d 362 metres north **e** 19.6 metres south

7 If the ground floor (street level) is regarded as zero, write a positive or negative number for the following positions:

a 6 floors above ground level **b** 3 floors below ground level
c 29 floors above ground level **d** 7 floors below ground level
e 4 floors below ground level

8 If right is positive, write a number for the position from zero which is:

a 7 units left **b** 5 units right **c** 12 units left
d 9 units right **e** 23 units left

9 State the combined effect of the following:

a a withdrawal of \$7 followed by a deposit of \$10
b a \$7 withdrawal followed by a \$6 withdrawal
c a rise in temperature of 13^oC followed by a fall of 8^oC
d a fall of 12^oC followed by a rise of 7^oC
e a 4 km trip east followed by a 3 km trip west
f a 7 km trip south followed by a 7 km trip north
g going up 5 floors in a lift and then coming down 6 floors
h a loss in mass of 4 kg followed by a gain in mass of 2 kg.

10 A baby boy weighed 3409 grams at birth. The record of his weight for the first five days showed the following:

Day 1: 28 g loss Day 2: 15 g loss Day 3: 13 g loss
Day 4: 17 g gain Day 5: 29 g gain

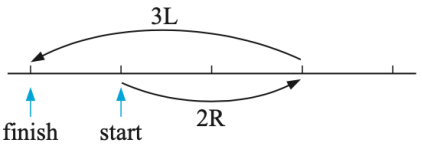
a Write each days gain or loss as a positive or negative number.
b What was the baby's weight at the end of the five days?

11 Luigi starts exploring at a point which is 7 km east of the campsite. What will be his position after he has travelled 3 km west then 5 km east then 6 km west?

12 Helene had \$155 in the bank. How much will be in her account after the following transactions?

Week 1	Deposit \$18	Week 2	Withdraw \$17	Week 3	Withdraw \$38
Week 4	Withdraw \$23	Week 5	Deposit \$29		

13 Suppose $2R$ means a trip to the right 2 units and $3L$ means a trip to the left 3 units, then $2R + 3L = 1L$ is clearly the combined trip. Find the combined trip for the following:



a $2R + 4L$ **b** $5L + 1R$ **c** $3R + 2R$ **d** $5L + 4L$

14 Find the combined trip for the following:

a $4R + 2L$ **b** $1R + 3R + 5L$ **c** $7L + 8L + 4R$

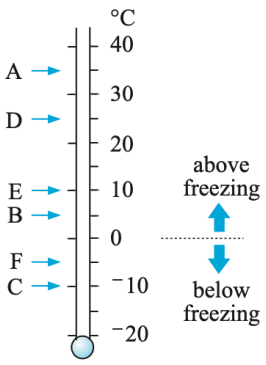
15 If $4\uparrow$ means go upwards 4 units and $2\downarrow$ means go downwards 2 units, then clearly $4\uparrow + 2\downarrow = 2\uparrow$ is the combined effect. Find the combined effect of:

a $2\uparrow + 5\downarrow$ **b** $3\downarrow + 4\uparrow$ **c** $7\downarrow + 6\downarrow$

16 Find the combined effect of the following:

a $1\uparrow + 2\uparrow + 6\downarrow$ **b** $9\uparrow + 3\downarrow + 6\downarrow$ **c** $3\downarrow + 4\downarrow + 5\uparrow$

17



The temperatures of cities A, B, C, D, E and F were recorded at 12 noon on a certain day last year.

a What was the temperature of each of the cities?

b How many °C is city D warmer than city:

i E **ii** B **iii** F **iv** C?

c How many °C is city C cooler than city:

i A **ii** E **iii** F **iv** B?

d What is the difference in temperature between:

i A and B **ii** D and E

iii E and C **iv** F and C

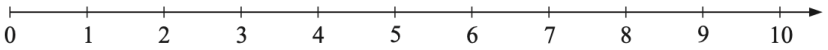
v B and F **vi** D and F?

Integers on a number line

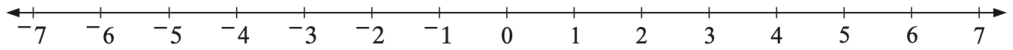
All negative whole numbers, zero and all positive whole numbers form the set of all **integers**.

Integers have both size and direction, and they can be illustrated on a number line.

You have used number lines before to place numbers in order. By convention (agreement), zero is on the left and numbers are marked off in equal intervals to the right. We can also show number lines vertically.



Imagine taking a number line and making a mirror image of the numbers on the right of zero so that the number line stretches in both directions. The numbers to the **right of zero** are shown with a positive sign + and the numbers to the **left of zero** shown with a negative sign, so the number line looks like this:



Every number on the *right* of 0 has a 'partner' on the *left* (except for 0 itself which lies at the centre of the number line).

The pairs of numbers like 7 and $^{-}7$, $^{-}5$ and 5, and so on are exactly the same distance from 0 but on opposite sides, so they are called **opposites**.

Example 2

What is the opposite of **a** 4 **b** $^{-}9$?

a The opposite of 4 is $^{-}4$ because they are the same distance from zero on opposite sides.

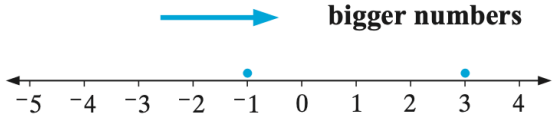
b The opposite of $^{-}9$ is 9.

COMPARING AND ORDERING NUMBERS

Thinking of the position of numbers on a number line makes it easy to compare their size and arrange numbers in order.

As you move along the number line from *left* to *right* the numbers increase in size so that the number furthest to the right is the largest.

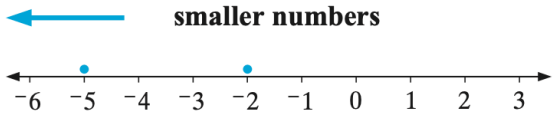
To increase, move to the right



3 is *bigger* than $^{-}1$ because it is further to the right on the number line

As you move along the number line from *right* to *left* the numbers decrease in size so that the number furthest to the left is the smallest.

To decrease, move to the left



$^{-}5$ is *smaller* than $^{-}2$ because it is further to the left on the number line

Remember that the symbol $>$ stands for '**is greater than**' and the symbol $<$ stands for '**is less than**',

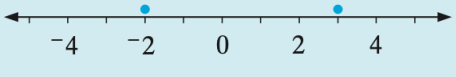
so, we could write these two statements as $3 > {}^{-}1$ and $^{-}5 < {}^{-}2$.

Example 3

a Show 3 and $^{-}2$ on a number line and write a sentence comparing their size.

b Write the statement $^{-}7 > {}^{-}4$ in words, then state whether it is True or False.

a



Since 3 is further to the right, we can say that 3 is greater than $^{-}2$. We could also say $^{-}2$ is less than 3.

b The statement reads 'negative 7 is greater than negative 4'.
This is false because $^{-}7$ is on the **left** of $^{-}4$, so it is smaller than $^{-}4$.

Summary:

- Positive numbers are to the right of zero; negative numbers are to the left of zero.
- 5 and $^{-}5$ are opposites as they are both 5 units from zero but in opposite directions.
- 0 is the only number which is neither positive nor negative.
- 4 is to the right of $^{-}1$ and $4 > {}^{-}1$, $^{-}2$ is to the right of $^{-}5$ and $^{-}2 > {}^{-}5$.
- The further to the **right** a number is on the number line, the **greater** its value.
- The further to the **left** a number is on the number line, the **smaller** its value.

Draw a number line to help you with these answers.

1 Write the opposite of these numbers:

a 8	**b** $^{-}5$	**c** 0	**d** 11	**e** $^{-}2$
f 6.4	**g** $^{-}3\frac{1}{2}$	**h** 56	**i** $^{-}23$	**j** $^{-}23.6$

2 Use a number line to:

a increase 2 by 3	**b** increase $^{-}1$ by 3	**c** decrease 5 by 2
d decrease $^{-}1$ by 3	**e** increase $^{-}4$ by 3	**f** increase $^{-}2$ by 1
g decrease 3 by 6	**h** decrease $^{-}2$ by 2	**i** increase $^{-}3$ by 5

3 Which is larger?

a 5 or 10	**b** 6 or $^{-}3$	**c** $^{-}4$ or 4
d 7 or $^{-}1$	**e** $^{-}6$ or $^{-}2$	**f** $^{-}5$ or $^{-}12$

4 Which is smaller?

a 15 or 12	**b** 8 or $^{-}2$	**c** $^{-}3$ or 3
d $^{-}7$ or $^{-}9$	**e** $^{-}2$ or 2	**f** $^{-}6$ or $^{-}6.5$

5 Write *true* or *false* for the following:

a $6 < {}^{-}3$	**b** $13 > {}^{-}5$	**c** $0 > {}^{-}4$
d $7 < {}^{-}2$	**e** $11 > {}^{-}5$	**f** $^{-}8 > {}^{-}1$
g $^{-}7 > {}^{-}3$	**h** $^{-}17 < 1$	**i** $^{-}5 > {}^{-}12$

6 Add $<$ or $>$ in the square to make each statement true:

a $4 \square {}^{-}1$	**b** $^{-}4 \square {}^{-}11$	**c** $8 \square {}^{-}8$
d $^{-}1 \square {}^{-}11$	**e** $^{-}6 \square {}^{-}8$	**f** $^{-}9 \square {}^{-}13$
g $0 \square {}^{-}8$	**h** $^{-}6 \square 0$	**i** $^{-}7 \square {}^{-}5.5$

Example 4

On a number line plot the numbers: $\{5, 3, 0, {}^{-}1, {}^{-}4\}$

7 On a number line plot these numbers. Use a different number line for each set.

a $\{^{-}2, 0, 3\}$ **b** $\{4, 3, 2, 0, ^{-}1, ^{-}5\}$

c $\{^{-}5, 3, ^{-}2, 0, 4, 1\}$ **d** $\{6, ^{-}3, 4, ^{-}1, 0, ^{-}6\}$

8 **a** Arrange in *ascending* order: $\{^{-}3, 0, ^{-}4, ^{-}1, 4\}$

b Arrange in *descending* order: $\{^{-}2, 2, 5, 0, ^{-}1\}$

9 Four friends have the following bank balances: Monica $^{-}\$592$, Joey $\$311$, Rachel $\$852$ and Ross $^{-}\$312$. Place them in order of richest to poorest.

10 The temperatures of five cities were: Sydney 12^oC, New York $^{-}3^oC$, Mexico City 15^oC, Moscow $^{-}7^oC$ and London 0^oC. Place them in order of coldest to hottest.

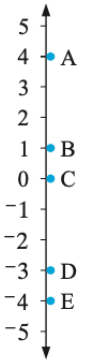
11 Arrange these numbers from smallest to largest:

a $^{-}5, 8, ^{-}2$ **b** $4, ^{-}3, ^{-}4, 0$

c $2.5, ^{-}1.2, 4, ^{-}3.1$ **d** $^{-}9.5, ^{-}8.9, ^{-}10, ^{-}9.7$

e $3\frac{1}{2}, ^{-}2\frac{1}{4}, 1, ^{-}1\frac{1}{5}$ **f** $-\frac{1}{8}, -\frac{7}{8}, \frac{5}{8}, -\frac{3}{8}, -\frac{5}{8}$

12 This number line is vertical. As you go up the number line, the numbers increase, and as you go down, the numbers decrease. Write the directed number for each of the points marked on the number line, and write True or False for the following statements.



a B is higher than D **b** $A < E$

c D is lower than A **d** $B < C$

e $C > E$ **f** $C < B$

g B and D are opposites **h** A and E are opposites

13 **a** Which number is furthest from 7?

i 3 or 15 **ii** 10 or $^{-}1$ **iii** $^{-}20$ or 28

b Which number is furthest from $^{-}3$?

i 5 or $^{-}8$ **ii** $^{-}10$ or 6 **iii** 32 or $^{-}28$

Example 5

Write down the next two members of the number sequence:

a $\{24, 17, 10, 3, ...\}$ **b** $\{^{-}3, ^{-}7, ^{-}11, ...\}$

a The members of the sequence are decreasing by 7
$\therefore$ the next two members of the sequence are $^{-}4$ and $^{-}11$.

b The members of the sequence are decreasing by 4
$\therefore$ the next two members are $^{-}15$ and $^{-}19$.

14 Write down the next two members of the number sequence:

a $\{3, 7, 11, 15, ...\}$ **b** $\{2, 1, 0, ^{-}1, ...\}$

c $\{8, 5, 2, ...\}$ **d** $\{^{-}11, ^{-}7, ^{-}3, ...\}$

15 What number is halfway between the following?

a 0 and 12 **b** 0 and 20 **c** 6 and 10 **d** 1 and 11

e 0 and $^{-}4$ **f** $^{-}2$ and 2 **g** $^{-}6$ and $^{-}2$ **h** $^{-}4$ and 2

Answers

EXERCISE 3A

1

	Statement	*Directed number*	*Opposite to statement*	*Directed number*
a	20 m above sea level	20	20 m below sea level	-20

b	45 km south of the city	-45	45 km north of the city	45
c	a loss of 2 kg in weight	-2	a gain of 2 kg in weight	2
d	a clock is 2 minutes fast	2	a clock is 2 minutes slow	-2
e	she arrives 5 minutes early	-5	she arrives 5 minutes late	5
f	a profit of \$4000	4000	a loss of \$4000	-4000
g	2 floors above ground level	2	2 floors below ground level	-2
h	10^oC below zero	-10	10^oC above zero	10
i	an increase of \$400	400	a decrease of \$400	-400
j	winning by 34 points	34	losing by 34 points	-34

2 lift 1, car -3, parking attendant -2, rubbish skip -5

3 A -2, B -6, C 5, D 3, E 0

4 **a** 11 **b** -6 **c** -8 **d** 29 **e** -14

5 **a** -30 **b** 200 **c** -431 **d** -751 **e** 809

6 **a** 7 **b** -15 **c** -115 **d** 362 **e** -19.6

7 **a** 6 **b** -3 **c** 29 **d** -7 **e** -4

8 **a** -7 **b** 5 **c** -12 **d** 9 **e** -23

9 **a** deposit of \$3 **b** \$13 withdrawal **c** 5^oC rise
d 5^oC fall **e** 1 km east
f remain in same position **g** 1 floor down
h 2 kg loss

10 **a** Day 1: -28 g Day 2: -15 g Day 3: -13 g
Day 4: 17 g Day 5: 29 g **b** 3399 g

11 3 km east **12** \$124

13 **a** 2L **b** 4L **c** 5R **d** 9L

14 **a** 2R **b** 1L **c** 11L

15 **a** 3 ↓ **b** 1 ↑ **c** 13 ↓

16 **a** 3 ↓ **b** 0 **c** 2 ↓

17 **a** A 35^oC, B 5^oC, C -10^oC, D 25^oC, E 10^oC, F -5^oC
b **i** 15^oC **ii** 20^oC **iii** 30^oC **iv** 35^oC
c **i** 45^oC **ii** 20^oC **iii** 5^oC **iv** 15^oC
d **i** 30^oC **ii** 15^oC **iii** 20^oC **iv** 5^oC
v 10^oC **vi** 30^oC

EXERCISE 3B

1 **a** $^{-}8$ **b** 5 **c** 0 **d** $^{-}11$ **e** 2 **f** $^{-}6.4$
g $3\frac{1}{2}$ **h** $^{-}56$ **i** 23 **j** 23.6

2 **a** 5 **b** 2 **c** 3 **d** $^{-}4$ **e** $^{-}1$ **f** $^{-}1$
g $^{-}3$ **h** $^{-}4$ **i** 2

3 **a** 10 **b** 6 **c** 4 **d** 7 **e** $^{-}2$ **f** $^{-}5$

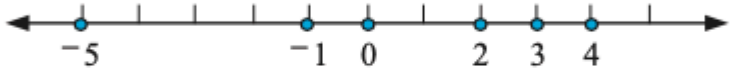
4 **a** 12 **b** $^{-}2$ **c** $^{-}3$ **d** $^{-}9$ **e** $^{-}2$ **f** $^{-}6.5$

5 **a** false **b** true **c** true **d** false **e** true
f false **g** false **h** true **i** true

6 **a** $4 > {}^{-}1$ **b** $^{-}4 > {}^{-}11$ **c** $8 > {}^{-}8$
d $^{-}1 > {}^{-}11$ **e** $^{-}6 > {}^{-}8$ **f** $^{-}9 > {}^{-}13$
g $0 > {}^{-}8$ **h** $^{-}6 < 0$ **i** $^{-}7 < {}^{-}5.5$

7 **a**

b



c

d

8 **a** $\{^{-}4, {}^{-}3, {}^{-}1, 0, 4\}$ **b** $\{5, 2, 0, {}^{-}1, {}^{-}2\}$

9 Rachel \$852, Joey \$311, Ross $^{-}$\$312, Monica $^{-}$\$592

10 Moscow $^{-}7^{o}$C, New York $^{-}3^{o}$C, London 0^{o}C, Sydney 12^{o}C, Mexico City 15^{o}C

11 **a** $^{-}5, {}^{-}2, 8$ **b** $^{-}4, {}^{-}3, 0, 4$
c $^{-}3.1, {}^{-}1.2, 2.5, 4$
d $^{-}10, {}^{-}9.7, {}^{-}9.5, {}^{-}8.9$
e $^{-}2\frac{1}{4}, {}^{-}1\frac{1}{5}, 1, 3\frac{1}{2}$ **f** $-\frac{7}{8}, -\frac{5}{8}, -\frac{3}{8}, -\frac{1}{8}, \frac{5}{8}$

12 A 4, B 1, C 0, D $^{-}3$, E $^{-}4$ **a** true **b** false
c true **d** false **e** true **f** true **g** false
h true

13 **a** **i** 15 **ii** $^{-}1$ **iii** $^{-}20$
b **i** 5 **ii** 6 **iii** 32

14 **a** 19, 23 **b** $^{-}2, {}^{-}3$ **c** $^{-}1, {}^{-}4$ **d** 1, 5

15 **a** 6 **b** 10 **c** 8 **d** 6 **e** $^{-}2$ **f** 0
g $^{-}4$ **h** $^{-}1$