## Do now

Expand and simplify:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| **a** | $(x+1)^2$ | **b** | $(x+3)^2$ | **c** | $(x-2)^2$ | **d** | $(x-5)^2$ |
| **e** | $(2+x)^2$ | **f** | $(2-x)^2$ | **g** | $(2x+1)^2$ | **h** | $(2x-1)^2$ |
| **i** | $(3x+2)^2$ | **j** | $(3x-2)^2$ | **k** | $(x+y)^2$ | **l** | $(x-y)^2$ |

Expand and simplify:

| | | | | | |
|---|---|---|---|---|---|
| **a** | $(x-1)(x+1)$ | **b** | $(x+4)(x-4)$ | **c** | $(x+5)(x-5)$ |
| **d** | $(2x+1)(2x-1)$ | **e** | $(4-x)(4+x)$ | **f** | $(3-2x)(3+2x)$ |

Why did the $x$-terms disappear in the expansions of question **6**?

**WALT: Factorise algebraic expressions**

**Success Criteria: I know in expansions we have to remove brackets whereas in factorisation we have to insert brackets**

[Video Dr Frost]

**1** Copy and complete:

| | | | |
|---|---|---|---|
| **a** | $3x+6=3(x+\ldots)$ | **b** | $4a-12=4(a-\ldots)$ |
| **c** | $20-5p=5(\ldots-p)$ | **d** | $16x+12=4(\ldots+3)$ |
| **e** | $3x^2-9x=3x(x-\ldots)$ | **f** | $2m+8m^2=2m(\ldots+4m)$ |

**2** Copy and complete:

| | | | |
|---|---|---|---|
| **a** | $4x+12=4(\ldots+\ldots)$ | **b** | $9+3d=3(\ldots+\ldots)$ |
| **c** | $3c-3=3(\ldots-\ldots)$ | **d** | $cd+de=d(\ldots\quad\ldots)$ |
| **e** | $6a+8ab=\ldots(3+4b)$ | **f** | $4x-2x^2=\ldots(2-x)$ |
| **g** | $4ab-4a=\ldots(b-1)$ | **h** | $4ab-6bc=\ldots(2a-3c)$ |

**3** Fully factorise:

| | | | | | |
|---|---|---|---|---|---|
| **a** | $5a+5b$ | **b** | $2x-4$ | **c** | $7d+14$ |
| **d** | $21-14x$ | **e** | $6x-12$ | **f** | $12+3x$ |
| **g** | $ac+bc$ | **h** | $12y-6a$ | **i** | $2a+ab$ |
| **j** | $bc-3cd$ | **k** | $2x-xy$ | **l** | $xy+y$ |
| **m** | $a+ab$ | **n** | $ab-bc$ | **o** | $2an+ab$ |
| **p** | $ab-a$ | **q** | $ab+bc$ | **r** | $2x+xy-4$ |

# Factorising Quadratics - Challenge

**Example 17**

Factorise: $x^2 + 11x + 24$

We need to find two numbers which have sum $= 11$, product $= 24$.
Pairs of factors of 24:

| *Factor product* | $1 \times 24$ | $2 \times 12$ | $3 \times 8$ | $4 \times 6$ |
|---|---|---|---|---|
| *Factor sum* | 25 | 14 | 11 | 10 |

↑ this one

The numbers we want are 3 and 8.

So, $x^2 + 11x + 24$

$= (x + 3)(x + 8)$

Most of the time we can find these two numbers mentally.

**Note:** Only the last two lines of this example need to be shown in your working.

**2** Factorise:

| | | | | | |
|---|---|---|---|---|---|
| **a** | $x^2 + 4x + 3$ | **b** | $x^2 + 11x + 24$ | **c** | $x^2 + 10x + 21$ |
| **d** | $x^2 + 15x + 54$ | **e** | $x^2 + 9x + 20$ | **f** | $x^2 + 8x + 15$ |
| **g** | $x^2 + 10x + 24$ | **h** | $x^2 + 9x + 14$ | **i** | $x^2 + 6x + 8$ |
| **j** | $x^2 + 11x + 18$ | **k** | $x^2 + 9x + 18$ | **l** | $x^2 + 13x + 42$ |
| **m** | $x^2 + 11x + 24$ | **n** | $x^2 + 15x + 26$ | **o** | $x^2 + 29x + 100$ |

**Example 15**

Fully factorise: $^{-}2a + 6ab$

$$
\begin{aligned}
& {}^{-}2a + 6ab \\
&= 6ab - 2a && \{\text{Rewrite with } 6ab \text{ first. Why?}\} \\
&= 2 \times 3 \times a \times b \; - \; 2 \times a \\
&= 2a(3b - 1) && \{\text{as } 2a \text{ is the HCF}\}
\end{aligned}
$$

**5** Fully factorise:

| | | | | | |
|---|---|---|---|---|---|
| **a** | $^{-}2a + 2b$ | **b** | $^{-}3 + 6b$ | **c** | $^{-}4a + 8b$ |
| **d** | $^{-}3c + cd$ | **e** | $^{-}a + ab$ | **f** | $^{-}7x^2 + 14x$ |
| **g** | $^{-}6x + 12x^2$ | **h** | $^{-}4b^2 + 2ab$ | **i** | $^{-}a + a^2$ |

**Example 16**

Fully factorise: $^{-}2x^2 - 4x$

$$
\begin{aligned}
& {}^{-}2x^2 - 4x \\
&= {}^{-}2 \times x \times x + {}^{-}2 \times 2 \times x \\
&= {}^{-}2x(x + 2) && \{\text{as HCF is } {}^{-}2x\}
\end{aligned}
$$

**6** Fully factorise:

| | | | | | |
|---|---|---|---|---|---|
| **a** | $^{-}3a - 3b$ | **b** | $^{-}4 - 8x$ | **c** | $^{-}3y - 6b$ |
| **d** | $^{-}5c - cd$ | **e** | $^{-}x - xy$ | **f** | $^{-}5x^2 - 10x$ |
| **g** | $^{-}4y - 12y^2$ | **h** | $^{-}6a^2 - 3ab$ | **i** | $^{-}8x^2 - 24x$ |

**Example 18**

Factorise: $x^2 - 7x + 12$

sum $= {}^-7$ and product $= 12$

$\therefore$ numbers are ${}^-3$ and ${}^-4$

So, $x^2 - 7x + 12$

$= (x - 3)(x - 4)$


**3** Factorise:

| | | | | | |
|---|---|---|---|---|---|
| **a** | $x^2 - 3x + 2$ | **b** | $x^2 - 4x + 3$ | **c** | $x^2 - 5x + 6$ |
| **d** | $x^2 - 14x + 33$ | **e** | $x^2 - 16x + 39$ | **f** | $x^2 - 19x + 48$ |
| **g** | $x^2 - 11x + 28$ | **h** | $x^2 - 14x + 24$ | **i** | $x^2 - 20x + 36$ |
| **j** | $x^2 - 7x + 12$ | **k** | $x^2 - 17x + 30$ | **l** | $x^2 - 11x + 30$ |
| **m** | $x^2 - 13x + 36$ | **n** | $x^2 - 13x + 42$ | **o** | $x^2 - 17x + 60$ |

**Example 19**

Factorise: **a** $x^2 - 2x - 15$ **b** $x^2 + x - 6$

**a** sum $= {}^-2$ and product $= {}^-15$

$\therefore$ numbers are ${}^-5$ and $3$

So, $x^2 - 2x - 15$

$= (x - 5)(x + 3)$

**b** sum $= 1$ and product $= {}^-6$

$\therefore$ numbers are $3$ and ${}^-2$

So, $x^2 + x - 6$

$= (x + 3)(x - 2)$


**4** Factorise:

| | | | | | |
|---|---|---|---|---|---|
| **a** | $x^2 - 7x - 8$ | **b** | $x^2 + 4x - 21$ | **c** | $x^2 - x - 2$ |
| **d** | $x^2 - 2x - 8$ | **e** | $x^2 + 5x - 24$ | **f** | $x^2 - 3x - 10$ |
| **g** | $x^2 + 3x - 54$ | **h** | $x^2 + x - 72$ | **i** | $x^2 - 4x - 21$ |
| **j** | $x^2 - x - 6$ | **k** | $x^2 - 7x - 60$ | **l** | $x^2 + 7x - 60$ |

**Example 20**

Fully factorise by first removing a common factor: $3x^2 + 6x - 72$

$$\begin{aligned} & 3x^2 + 6x - 72 && \{\text{first look for a } \textbf{common factor}\} \\ &= 3(x^2 + 2x - 24) && \{\text{sum} = 2, \quad \text{product} = {}^-24 \quad \text{i.e., } 6 \text{ and } {}^-4\} \\ &= 3(x + 6)(x - 4) \end{aligned}$$

**5** Fully factorise by first removing a common factor:

| | | | | | |
|---|---|---|---|---|---|
| **a** | $2x^2 + 10x + 8$ | **b** | $3x^2 - 21x + 18$ | **c** | $2x^2 + 14x + 24$ |
| **d** | $2x^2 - 44x + 240$ | **e** | $4x^2 - 8x - 12$ | **f** | $3x^2 - 42x + 99$ |
| **g** | $2x^2 - 2x - 180$ | **h** | $3x^2 - 6x - 24$ | **i** | $2x^2 + 18x + 40$ |
| **j** | $x^3 - 7x^2 - 8x$ | **k** | $x^3 - 3x^2 - 28x$ | **l** | $x^4 + 2x^3 + x^2$ |

**"the difference of two squares".**

**Example 21**

Fully factorise:

**a** $x^2 - 4$ **b** $1 - 25y^2$

**a**
$$\begin{aligned} & x^2 - 4 \\ &= x^2 - 2^2 \\ &= (x + 2)(x - 2) \end{aligned}$$

**b**
$$\begin{aligned} & 1 - 25y^2 \\ &= 1^2 - (5y)^2 \\ &= (1 + 5y)(1 - 5y) \end{aligned}$$


## EXERCISE 11E

**1** Fully factorise:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| **a** | $c^2 - d^2$ | **b** | $m^2 - n^2$ | **c** | $n^2 - m^2$ | **d** | $m^2 - x^2$ |
| **e** | $x^2 - 16$ | **f** | $x^2 - 81$ | **g** | $a^2 - 9$ | **h** | $4x^2 - 1$ |
| **i** | $4x^2 - 9$ | **j** | $9y^2 - 25$ | **k** | $64 - x^2$ | **l** | $16 - 9a^2$ |
| **m** | $9x^2 - 1$ | **n** | $4a^2 - 9b^2$ | **o** | $16a^2 - x^2$ | **p** | $9x^2 - 16b^2$ |

# F PERFECT SQUARE FACTORISATION (EXTENSION)

Recall that

$$\begin{aligned} & (a + b)^2 \\ &= (a + b)(a + b) \\ &= a^2 + ab + ab + b^2 \\ &= a^2 + 2ab + b^2 \end{aligned}$$

and

$$\begin{aligned} & (a - b)^2 \\ &= (a - b)(a - b) \\ &= a^2 - ab - ab + b^2 \\ &= a^2 - 2ab + b^2 \end{aligned}$$