WAIt identify and apply the properties of similar figures.

Success Criteria:-

- I can correctly identify corresponding sides and angles in similar figures.write similarity statements using appropriate notation.
- I can solve problems involving similar figures.by matching corresponding sides and angles:
- Writing similarity statements: Students are given a pair of similar figures and asked to write a similarity statement in the form of "AA",(angle /angle) "SSS", (Side side side) or "SAS".(Side angle side)

Video We also need to learn about congruent triangles

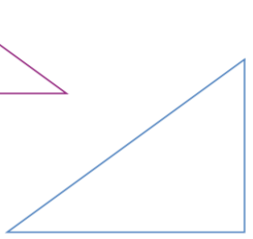
Video

By measuring the angles and the lengths of the sides, determine whether or not the following pairs of figures are similar. If they are similar, state the enlargement factor.

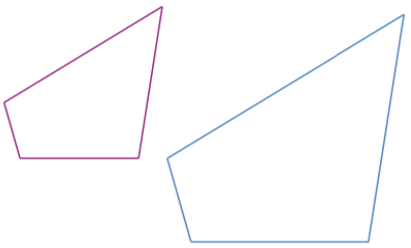
Exercise 9B

1 By measuring the angles and the lengths of the sides, determine whether or not the following pairs of figures are similar. If they are similar, state the enlargement factor.

a



b



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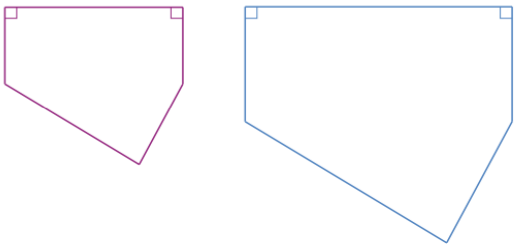
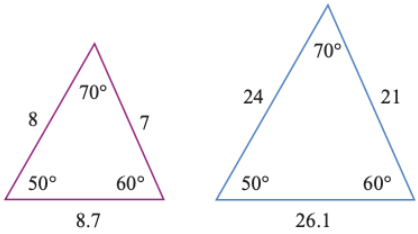


2 Use the information given to determine whether or not the following pairs of figures are similar. If they are similar, state the enlargement factor. (Diagrams are not drawn to scale. All lengths are in centimetres.)

a



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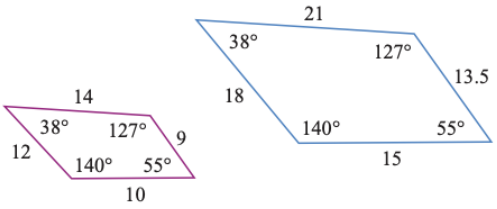

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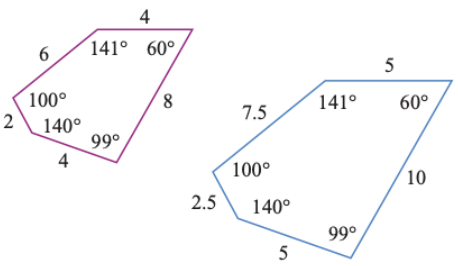
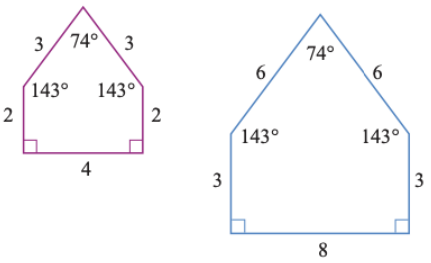
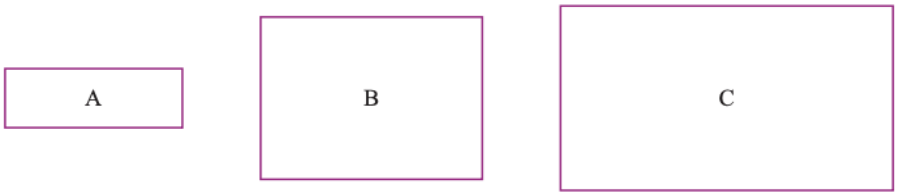


d



3 **a** Here are three rectangles. Are all rectangles equiangular? Give reasons for your answer.



b Complete the table below for the rectangles on the previous page.

	Rectangle A	Rectangle B	Rectangle C
Length			
Width			

c Complete the following.

$\frac{\text{length A}}{\text{length B}} = \_\_$ $\frac{\text{length B}}{\text{length C}} = \_\_$ $\frac{\text{length A}}{\text{length C}} = \_\_$

$\frac{\text{width A}}{\text{width B}} = \_\_$ $\frac{\text{width B}}{\text{width C}} = \_\_$ $\frac{\text{width A}}{\text{width C}} = \_\_$

d Are any of these rectangles similar? Explain.

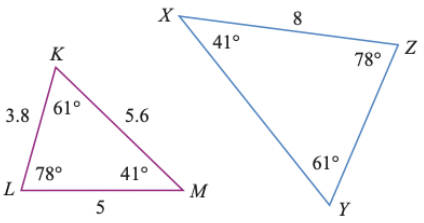
e Comment on the statement 'All rectangles are similar'.

Exercise 9C

1 **i** Name the matching sides in the following pairs of similar triangles. (Diagrams are not drawn to scale. All lengths are in centimetres.)

ii Hence find the enlargement factor.

a



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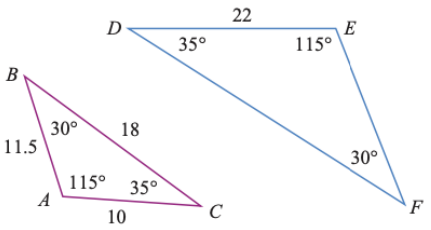

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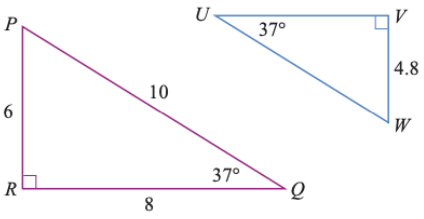

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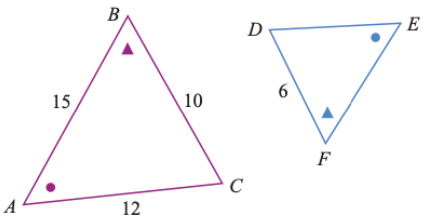

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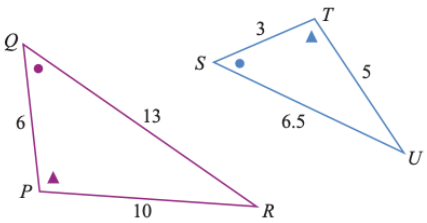

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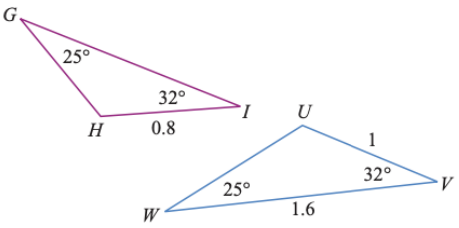
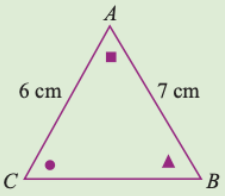
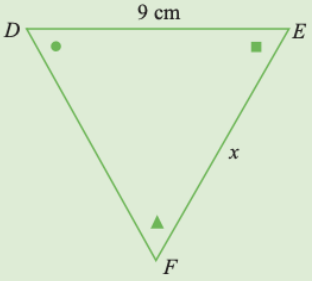


EXAMPLE 2

Find the length of the unknown side in these similar triangles.

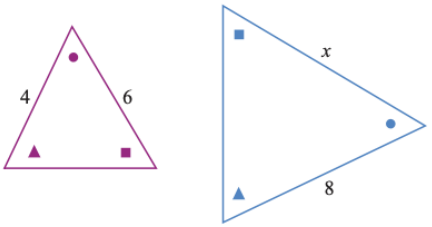




Solve	Think	Apply
$k = \frac{9}{6} = 1.5$ Hence: $x = 1.5 \times 7$ $= 10.5$ cm	As DE and AC are matching sides, $k = \frac{DE}{AC}$. As FE and BA are matching sides, $FE = k \times BA$.	Use a pair of matching sides of known length to find the scale factor, k. Each side of the second triangle is k times its matching side in the first triangle.

2 Find the length of the unknown side in the following pairs of similar triangles. (Diagrams are not drawn to scale. All lengths are in centimetres.)

a



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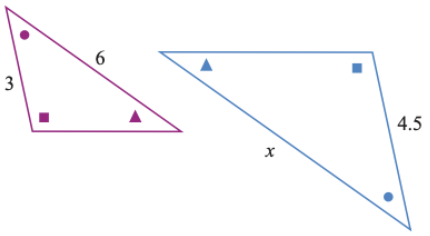
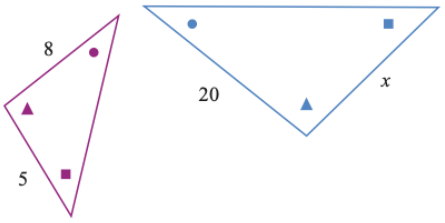


c



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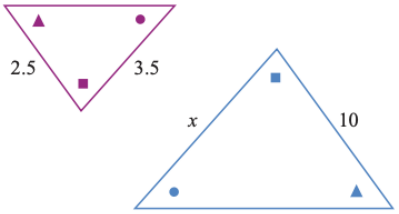
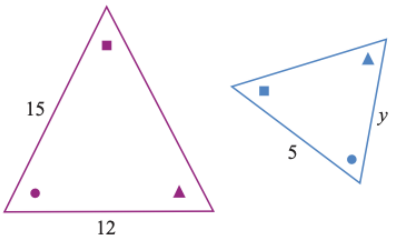


e



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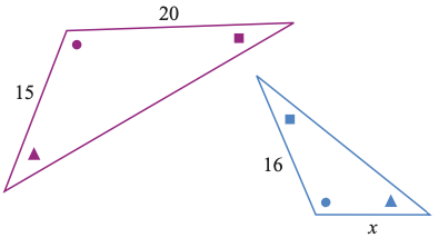
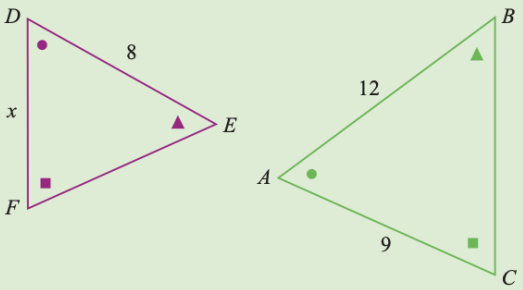


EXAMPLE 3

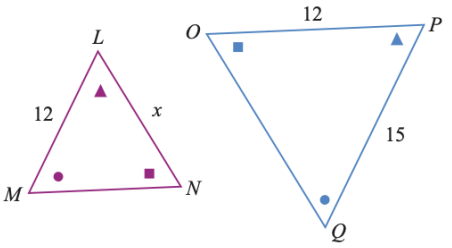
Find the length of the unknown side in the similar triangles shown. (All lengths are in centimetres.)



Solve	Think	Apply
$k = \frac{8}{12} = \frac{2}{3}$ Hence: $x = \frac{2}{3} \times 9$ $= 6$ cm	As we are to find a side in the first triangle, we need the enlargement factor from $\triangle ABC$ to $\triangle DEF$. DE and AB are matching sides hence $k = \frac{DE}{AB}$. As DF and AC are matching sides, $DF = k \times AC$.	Use a pair of matching sides of known length to find the scale factor, k. Each side of the first triangle is then k times its matching side in the second triangle.

3 Find the length of the unknown side in the similar triangles shown. (All lengths are in centimetres.)

a



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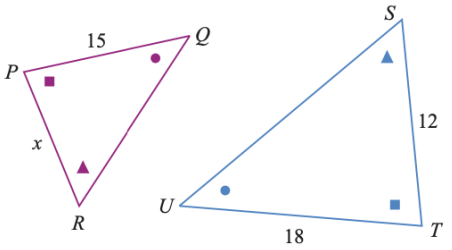

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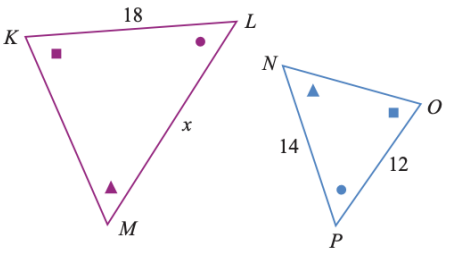

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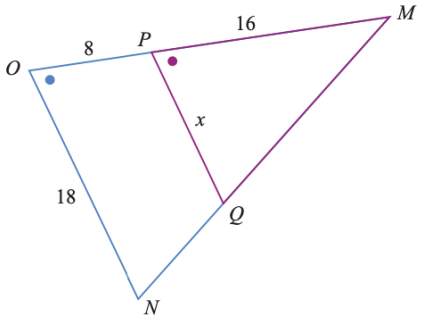
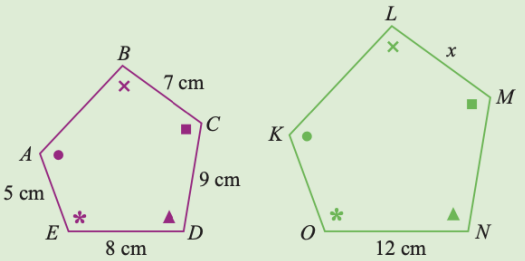


EXAMPLE 4

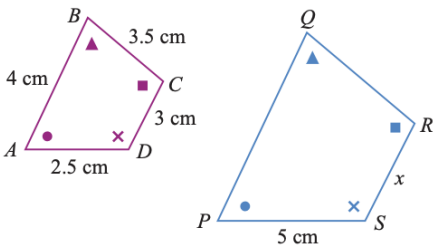
Find the length of the unknown side, given that the figures are similar. (Figures are not drawn to scale.)



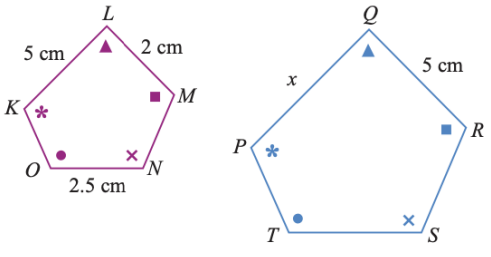
Solve	Think	Apply
$k = \frac{12}{8} = 1.5$ Hence: $x = 1.5 \times 7$ $= 10.5$ cm	ON and ED are matching sides, hence $k = \frac{ON}{ED}$. As LM and BC are matching sides, $LM = k \times BC$.	Use a pair of matching sides of known length to find the scale factor, k. Each side of the second figure is then k times its matching side in the first figure.

4 Find the length of the unknown side in the following pairs of similar figures.

a



b



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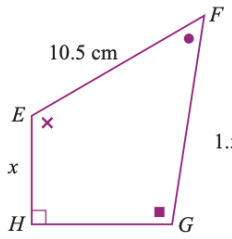

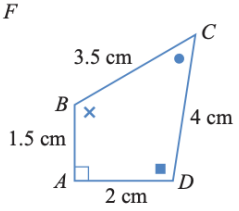

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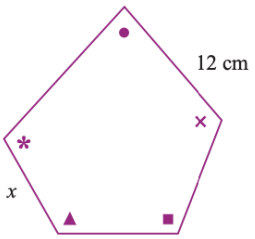

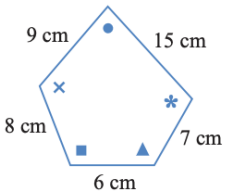

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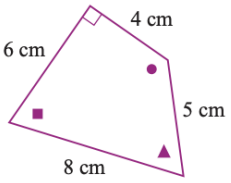
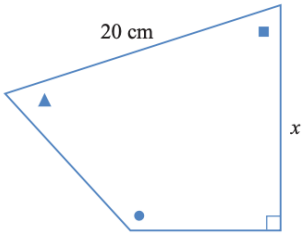
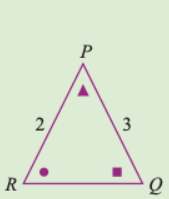
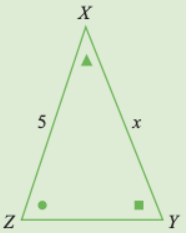




EXAMPLE 5

Given that the triangles are similar, find the length of the unknown side. (All lengths are in centimetres.)



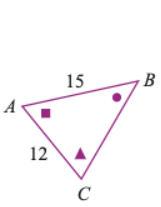


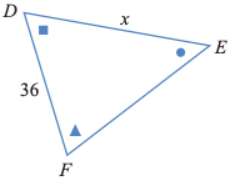
This question can be answered by finding and using the enlargement factor as in the questions above. Here is an alternative method for solving these types of questions.

Solve	Think	Apply
$\frac{x}{3} = \frac{5}{2}$ $x = 3 \times \frac{5}{2} = 7.5$	$\frac{XY}{PQ} = \frac{XZ}{PR} \left(= \frac{ZY}{RQ} = k\right)$	As the triangles are similar, the matching side lengths are in the same ratio.

5 Use the method of Example 5 to find the length of the unknown side in the following pairs of similar triangles. (Diagrams are not drawn to scale. All lengths are in centimetres.)

a





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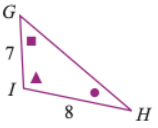

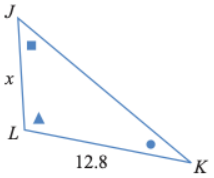

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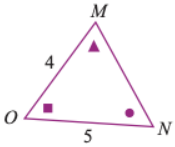

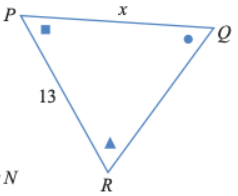

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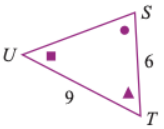
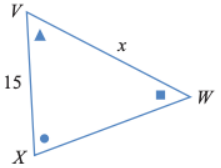
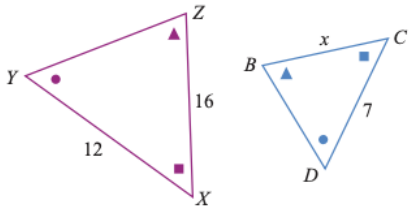




e



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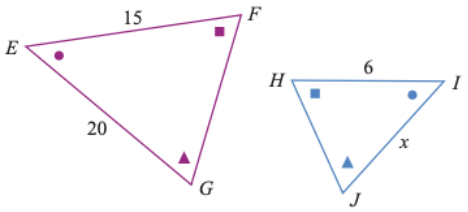

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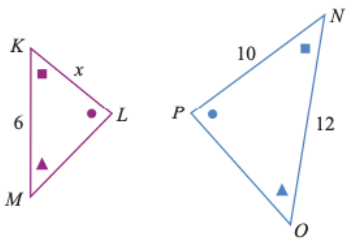

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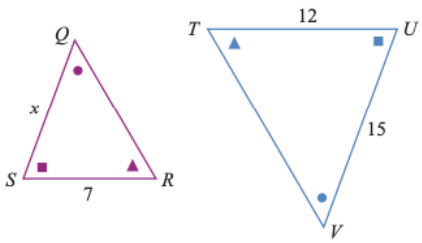
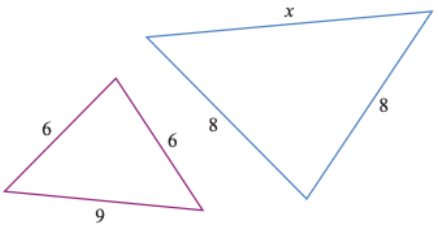


6 These triangles are similar. Find the value of the pronumerals. (All lengths are in centimetres.)

a



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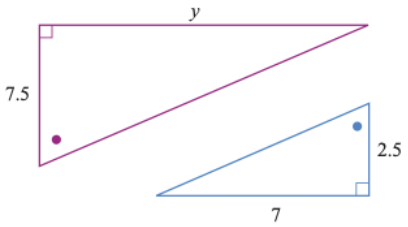
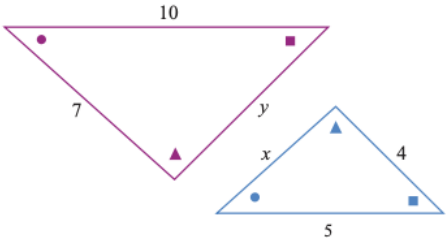
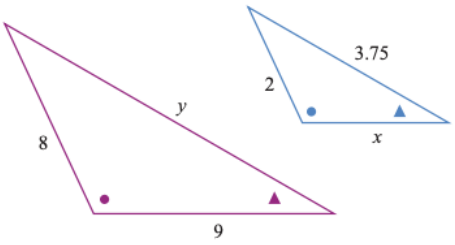


c



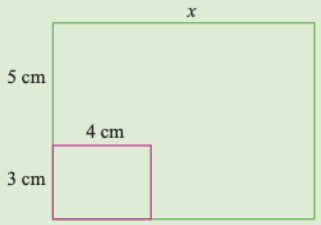
d



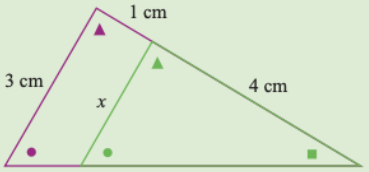
EXAMPLE 6

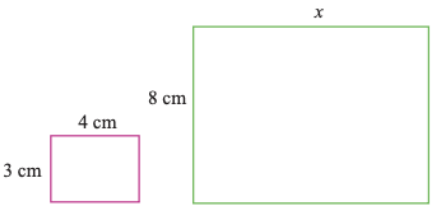
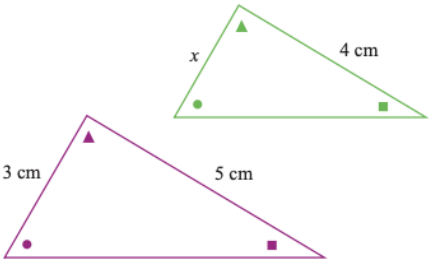
Find the lengths of the unknown sides in these pairs of similar figures.

a



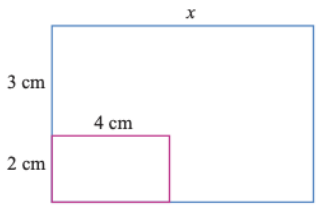
b



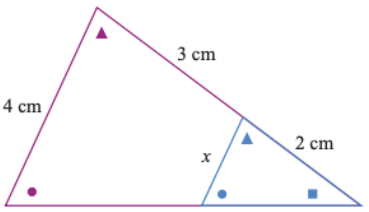
	Solve	Think	Apply
a	$k = \frac{8}{3}$ or $\frac{x}{4} = \frac{8}{3}$ $x = \frac{8}{3} \times 4$ $\quad x = 4 \times \frac{8}{3}$ $= 10\frac{2}{3}$ $\quad = 10\frac{2}{3}$ $x = 10\frac{2}{3}$ cm	Separate the similar rectangles. 	Separate into similar figures, find k and use it to find the unknown side.
b	$k = \frac{4}{5}$ or $\frac{x}{3} = \frac{4}{5}$ $x = \frac{4}{5} \times 3$ $\quad x = 3 \times \frac{4}{5}$ $= 2.4$ $\quad = 2.4$ $x = 2.4$	Separate the similar triangles. 	

7 Find the lengths of the unknown sides in the following pairs of similar figures.

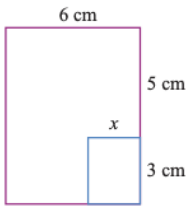
a



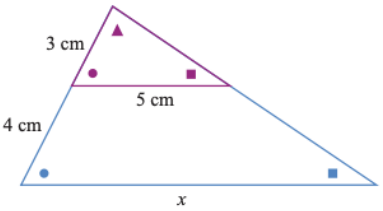
b



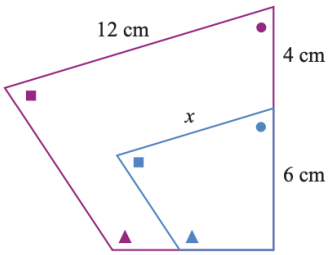
c



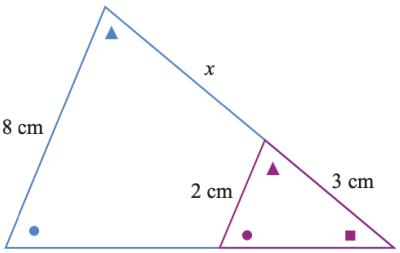
d



e



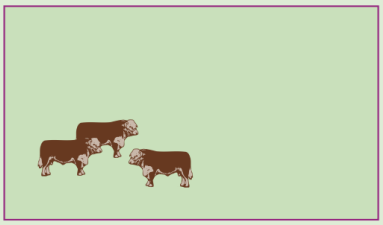
f

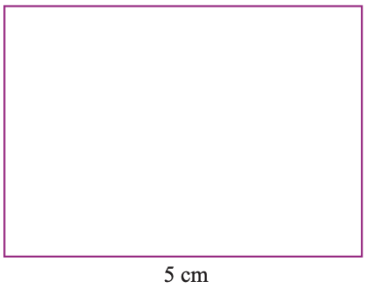


Scale drawings

EXAMPLE 1

The measurements of a rectangular cattle yard are shown on the diagram. Make a scale drawing of the paddock using a scale of 1 : 1000.



Solve	Think	Apply
Scaled length $= \frac{1}{1000} \times 50$ m $= 5$ cm Scaled breadth $= \frac{1}{1000} \times 35$ m $= 3.5$ cm  	Scale $= \frac{\text{length on drawing}}{\text{real length}}$ $= \frac{1}{1000}$ $\frac{\text{length on drawing}}{50 \text{ m}} = \frac{1}{1000}$ Length on drawing $= \frac{1}{1000} \times 50$ m $= \frac{1}{1000} \times 50 \times 100$ cm $\frac{\text{breadth on drawing}}{35 \text{ m}} = \frac{1}{1000}$ Breadth on drawing $= \frac{1}{1000} \times 35$ m $= \frac{1}{1000} \times 35 \times 100$ cm	If the scale is $1 : n$, the length on the drawing or scaled length $= \frac{1}{n} \times$ real length.

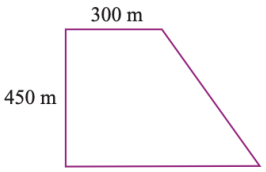
Exercise 9D

1 Make scale drawings of the following shapes using the scale given.

a

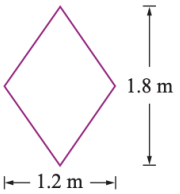
Scale 1 : 1000

b



Scale 1 : 10 000

c



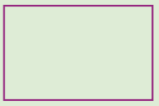
Scale 1 : 50

2 The length of a boat is 11.2 m. What would be its length on a scale drawing with a scale of 1 : 200?

3 The distance from Sydney to Melbourne by air is 710 km. What would this distance be on a map of Australia with a scale of 1: 10 000 000?

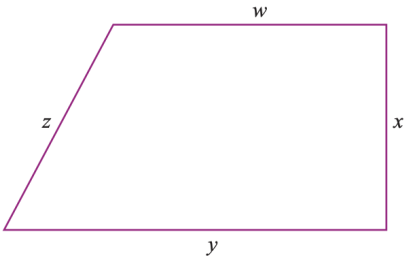
EXAMPLE 2

This scale drawing of a paddock was made by a surveyor using a scale of 1 : 10 000. What are the real dimensions of the paddock?



Solve	Think	Apply
By measurement: scaled length = 2.2 cm scaled breadth = 1.4 cm Real length = 10 000 × 2.2 cm = 22 000 cm = 220 m Real breadth = 10 000 × 1.4 cm = 14 000 cm = 140 m	$\frac{\text{length on drawing}}{\text{real length}} = \frac{1}{10\,000}$ $\frac{2.2\text{ cm}}{\text{real length}} = \frac{1}{10\,000}$ Take the reciprocal of both sides. $\frac{\text{real length}}{2.2\text{ cm}} = \frac{10\,000}{1}$ real length = 10 000 × 2.2 cm $\frac{\text{breadth on drawing}}{\text{real breadth}} = \frac{1}{10\,000}$ $\frac{1.4\text{ cm}}{\text{real breadth}} = \frac{1}{10\,000}$ Take the reciprocal of both sides. $\frac{\text{real breadth}}{1.4\text{ cm}} = \frac{10\,000}{1}$ real breadth = 10 000 × 1.4 cm	If the scale factor is 1: n, real length = n × scaled length.

4 The scale drawing of a field was made by a surveyor using a scale of 1 : 10 000. Find the real dimensions w, x, y, z of this field.



Exercise 9B

1 **a** Similar, $k = 2$ **b** Similar, $k = 1.5$

c Not similar (matching angles are equal but not all matching sides are in proportion)

2 **a** Similar, $k = 3$ **b** Similar, $k = 1.5$

c Similar $k = 1.25$ **d** Not similar

3 **a, e** All rectangles are equiangular, but do not necessarily have their matching sides in proportion and so not all rectangles are similar.

4 **a** True **b** True **c** False

d True **e** False

5 They are congruent when $k = 1$.

6 **a** $k = 3$ **b** $k = 1.25$

Exercise 9C

1 **a** **i** KL and YZ, LM and ZX, KM and YX

ii 1.6

b **i** AB and EF, AC and ED, BC and FD

ii 2.2

c **i** PQ and WU, QR and UV, PR and WV

ii 0.8

d **i** AB and EF, BC and FD, AC and ED

ii 0.6

e **i** QP and ST, QR and SU, PR and TU

ii $\frac{1}{2}$

f **i** GH and WU, HI and UV, GI and WV

ii 1.2

2 **a** $x = 12$ cm **b** $x = 9$ cm **c** $x = 12.5$

d $x = 14$ cm **e** $y = 4$ cm **f** $x = 12$ cm

3 **a** $x = 9.6$ cm **b** $x = 10$ cm **c** $x = 21$ cm

d $x = 12$ cm

4 **a** $x = 6$ cm **b** $x = 12.5$ cm **c** $x = 4.5$ cm

d $x = 9\frac{1}{3}$c m **e** $x = 15$ cm

5 **a** $x = 45$ cm **b** $x = 11.2$ cm **c** $x = 16.25$ cm

d $x = 22.5$ cm **e** $x = 9.33$ cm **f** $x = 8$ cm

g $x = 5$ cm **h** $x = 8.75$ cm

6 **a** $x = 12$ cm **b** $y = 21$ cm

c $x = 3.5$ cm, $y = 8$ cm **d** $x = 2.25$ cm, $y = 15$ cm

7 **a** $x = 10$ cm **b** $x = 1.6$ cm **c** $x = 2.25$ cm

d $x = 11\frac{2}{3}$c m **e** $x = 7.2$ cm **f** $x = 9$ cm

Exercise 9D

2 56 mm **3** 71 mm

4 $w = 400$ m, $x = 300$ m, $y = 560$ m, $z = 340$ m

5 **a** 1 : 400 **b** 12 m

6 **a** 1 : 4000 **b** 136 m

7 **a** 1 : 2000 **b** 67 m

8 8 m **9** 10.8 m

10 1 m **11** 100 m

12 25 paces