Walt calculate volume to describe the amount of space inside a three dimensional object
Success Criteria: I know we use Cubic kilometres for the volume of concrete poured at a building
Units for Capacity for liquids and gases - Millilitres, litres, kilolitres, and Megalitres

6.8 Volume

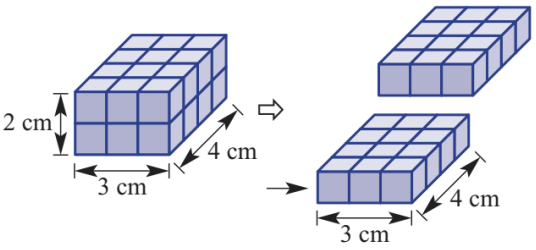
We use volume to describe the amount of space inside a three-dimensional object. We use metric units, such as:

- cubic kilometres for the volume of water in the sea
- cubic metres for the volume of concrete poured at a building site
- cubic centimetres for the volume of space occupied by this book
- cubic millimetres for the volume of metal in a pin.

Units for capacity (millilitres, litres, kilolitres and megalitres) are used for liquids and gases.

Let's start: Why length × width × height?

For most people, the first thing that comes to mind when dealing with volume is length × width × height. But this rule only applies to finding the volume of rectangular prisms.

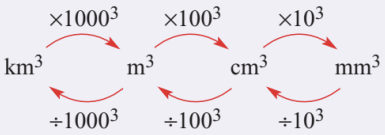


Let's look at a rectangular prism split into two layers.

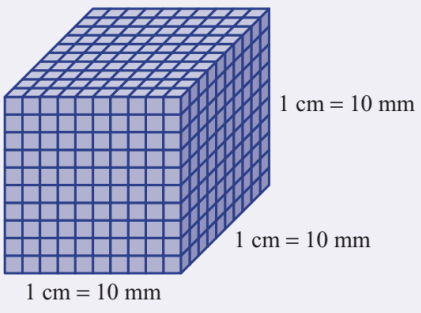
- How many cubes sit on one layer?
- What is the area of the base? What do you notice?
- What is the height and how many layers are there?
- Why is the volume rule given by $V = lwh$ in this case?

Key ideas

- Common metric units for **volume** include cubic kilometres (km^3), cubic metres (m^3), cubic centimetres (cm^3) and cubic millimetres (mm^3).



$1000^3 = 1\,000\,000\,000$
$100^3 = 1\,000\,000$
$10^3 = 1000$

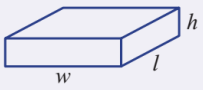


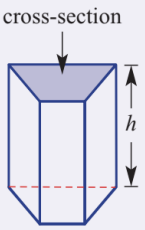
$1\text{ cm}^3 = 10 \times 10 \times 10$
$= 10^3\text{ mm}^3$

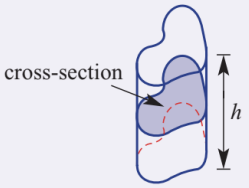
Volume The amount of three-dimensional space inside an object

- For **capacity**, common units include:
  - Megalitres (ML) 1 ML = 1000 kL
  - Kilolitres (kL) 1 kL = 1000 L
  - Litres (L) 1 L = 1000 mL
  - Millilitres (mL)

  Also: 1 cm^3 = 1 mL so 1 L = 1000 cm^3 and 1 m^3 = 1000 L
- Volume of solids with a uniform **cross-section** is equal to area of cross-section (A) × height (h).
  $V = A \times h$
- Volume of a rectangular prism:
  $V = l \times w \times h$







- The 'height' is the length of the edge that runs perpendicular to the cross-section in any solid.

Capacity The amount of liquid a container can hold

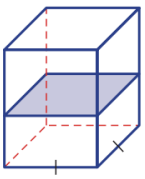
Cross-section The plane figure formed when you slice a solid figure parallel to one of its surfaces

Exercise 6H

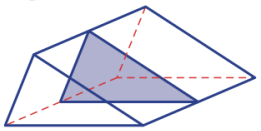
Understanding

1 What is the name given to the shape of the shaded cross-section of each of the following solids?

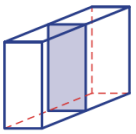
a



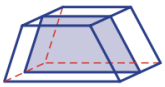
b



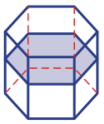
c



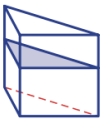
d



e



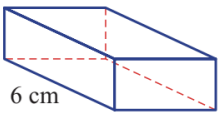
f



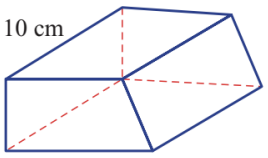
2 Draw the cross-sectional shape for these prisms and state the given 'height' (perpendicular to the cross-section).

'perpendicular' means 'at a right angle (90°)'.

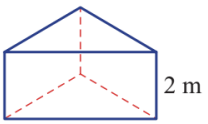
a



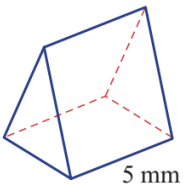
b



c



d



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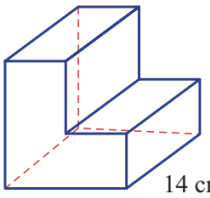

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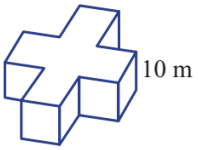


3 Write the missing number.

a The number of mm in 1 cm is _____.
b The number of mm^2 in 1 cm^2 is _____.
c The number of mm^3 in 1 cm^3 is _____.
d There are _____ cm^3 in 1 m^3.
e There are _____ m^3 in 1 km^3.
f There are _____ mL in 1 L.
g There are _____ L in 1 kL.
h There are _____ cm^3 in 1 mL.

Fluency

Example 18 Converting units of volume

Convert the following volume measurements into the units given in the brackets.

a 2.5 m^3 (cm^3) **b** 458 mm^3 (cm^3)

Solution	Explanation
a $2.5\ m^3 = 2.5 \times 100^3\ cm^3$ $= 2\,500\,000\ cm^3$	$\times 100^3 = 1\,000\,000$ m^3 → cm^3 2.500000
b $458\ mm^3 = 458 \div 10^3\ cm^3$ $= 0.458\ cm^3$	cm^3 ← mm^3 $\div 10^3 = 1000$ 458.

4 Convert the following volume measurements into the units given in brackets.

a 3 cm^3 (mm^3) **b** 0.3 cm^3 (mm^3) **c** 2000 mm^3 (cm^3)
d 0.001 m^3 (cm^3) **e** 8.7 m^3 (cm^3) **f** 5900 cm^3 (m^3)
g 0.00001 km^3 (m^3) **h** 21 700 m^3 (km^3) **i** 430 000 cm^3 (m^3)

1 $km^3 = 1000^3\ m^3$
1 $m^3 = 100^3\ cm^3$
1 $cm^3 = 10^3\ mm^3$

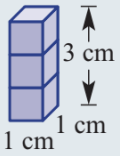
5 Convert these units of capacity to the units given in brackets.

a 3 L (mL) **b** 0.2 kL (L) **c** 3500 mL (L)
d 0.021 L (mL) **e** 37 000 L (kL) **f** 42 900 kL (ML)
g 2 cm^3 (mL) **h** 2 L (cm^3) **i** 1 m^3 (L)

1 ML = 1000 kL
1 kL = 1000 L
1 L = 1000 mL

Example 19 Finding the volume of a rectangular prism

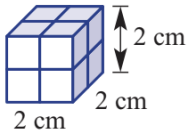
Find the volume of this rectangular prism.



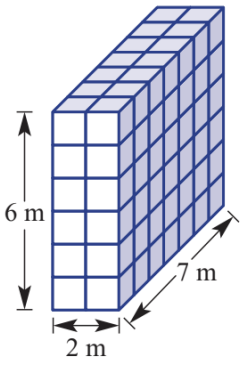
Solution	Explanation
Volume $= l \times w \times h$ $= 1 \times 1 \times 3$ $= 3\ \text{cm}^3$	The solid is a rectangular prism. Length = 1 cm, width = 1 cm and height = 3 cm

6 Find the volume of these three-dimensional rectangular prisms.

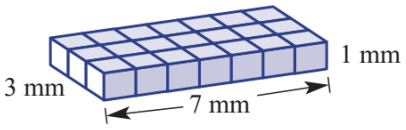
a



b

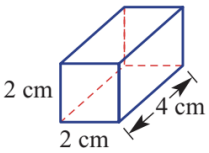


c

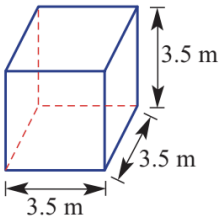


7 Find the volume of each of these rectangular prisms (cuboids).

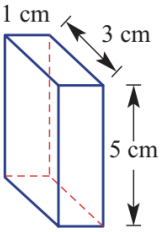
a



b

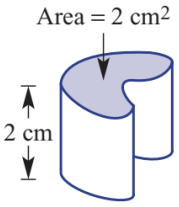


c

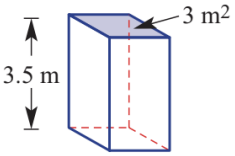


8 Find the volume of each of these three-dimensional objects. The cross-sectional area has been given.

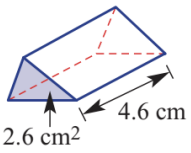
a



b

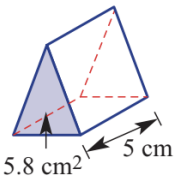


c

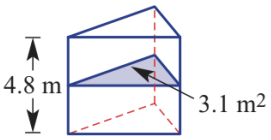


Simply use $V = A \times h$, since the area of the cross-section is given.

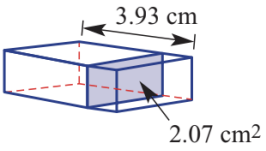
d



e

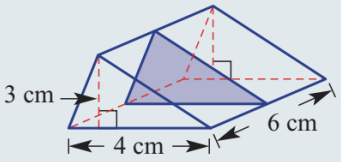


f



Example 20 Finding the volume of a triangular prism

Find the volume of this triangular prism.



Solution

Area of cross-section $= \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times 4 \times 3$

$= 6 \text{ cm}^2$

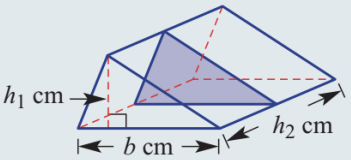
Volume = area of cross-section × height

$= 6 \times 6$

$= 36 \text{ cm}^3$

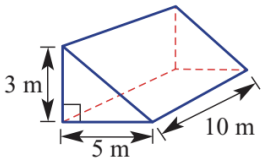
Explanation

The cross-section is a triangle.

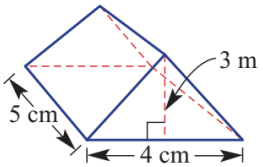


9 Find the volume of these prisms.

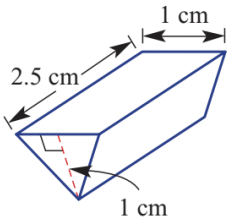
a



b



c



First find the area of the triangular cross-section.

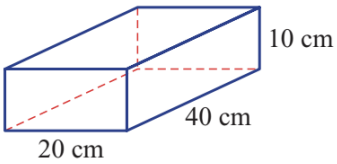
10 A brick is 10 cm wide, 20 cm long and 8 cm high. How much space would five of these bricks occupy?

11 25 L of water is poured into a rectangular fish tank which is 50 cm long, 20 cm wide and 20 cm high. Will it overflow?

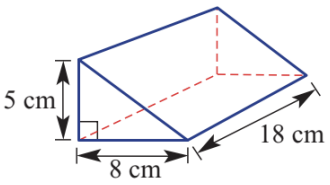
There are 1000 cm^3 in 1 L.

12 Find the volume of these solids, converting your answer to litres.

a

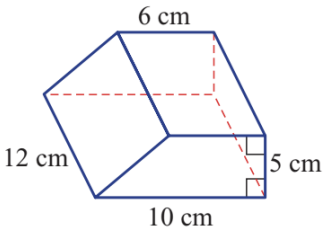


b



Area of a trapezium:
$A = \frac{1}{2}(a + b)h$

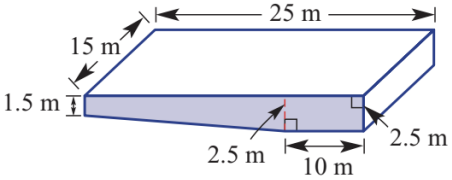
c



13 This diagram is a sketch of a new 25 m swimming pool to be installed in a school sports complex.

a Find the area of one side of the pool (shaded).

b Find the volume of the pool in litres.
Use 1 m^3 = 1000 L.



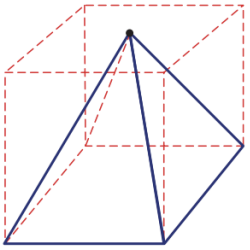
Volume of a pyramid

14 Someone tells you that the volume of a pyramid is half of the volume of a rectangular prism with the same base. Do you think this is true?

a Make an educated guess as to what fraction of the prism's volume is the pyramid's volume.

b Use the internet to find the actual answer to part **a**.

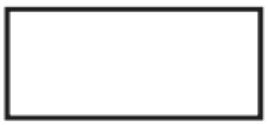
c Draw some pyramids and find their volume using the results from part **b**.



Check your answers

Exercise 6H

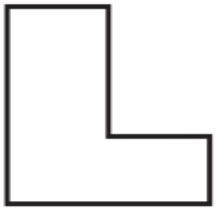
1 **a** square **b** triangle **c** rectangle
d trapezium **e** hexagon **f** triangle

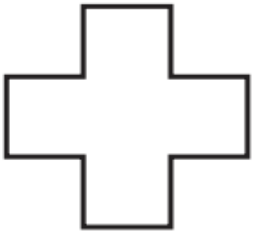
2 **a**  , 6 cm

b  , 10 cm

c  , 2 m

d  , 5 mm

e  , 14 cm

f  , 10 m

3 **a** 10 **b** 100 **c** 1000 **d** 1 000 000
e 1 000 000 000 **f** 1000 **g** 1000 **h** 1

4 **a** 3000 mm^3 **b** 300 mm^3 **c** 2 cm^3
d 1000 cm^3 **e** 8 700 000 cm^3 **f** 0.0059 m^3
g 10 000 m^3 **h** 0.000 021 7 km^3 **i** 0.43 m^3

5 **a** 3000 mL **b** 200 L **c** 3.5 L
d 21 mL **e** 37 kL **f** 42.9 ML
g 2 mL **h** 2000 cm^3 **i** 1000 L

6 **a** 8 cm^3 **b** 84 m^3 **c** 21 mm^3

7 **a** 16 cm^3 **b** 42.875 m^3 **c** 15 cm^3

8 **a** 4 cm^3 **b** 10.5 m^3 **c** 11.96 cm^3
d 29 cm^3 **e** 14.88 m^3 **f** 8.1351 cm^3

9 **a** 75 m^3 **b** 30 cm^3 **c** 1.25 cm^3

10 8000 cm^3

11 Yes, the tank only holds 20 L

12 **a** 8 L **b** 0.36 L **c** 0.48 L

13 **a** 55 m^2 **b** 825 000 L

14 **b** $\frac{1}{3}$