Slope (gradient)

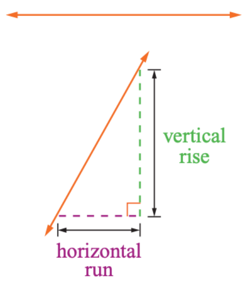
WALT Calculate slope of a line
Success Criteria I can count the way lines go/move up/down (rise) and go/move across (run)

We use the word **slope** or **gradient** when talking about the degree of steepness of a line or a line segment. Horizontal lines have zero slope.

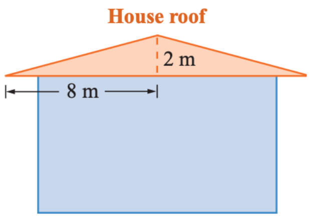
This line is very steep. It must therefore have a large slope.

To compare the slopes of different lines, we use the ratio of **vertical rise** to **horizontal run**. For a horizontal line, the vertical rise is 0, so the slope is 0.

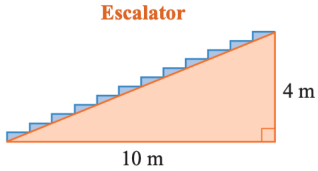
$$\textbf{Slope} = \frac{\textbf{vertical rise}}{\textbf{horizontal run}}$$



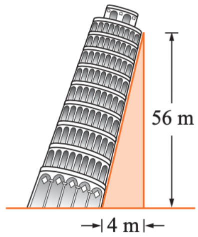
The following illustrations indicate slopes of varying amounts.



slope $= \frac{2}{8} = \frac{1}{4}$



slope $= \frac{4}{10} = \frac{2}{5}$

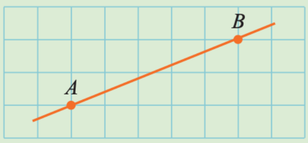


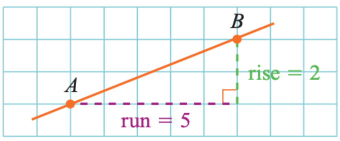
slope $= \frac{56}{4} = 14$

When line segments are drawn on graph paper, we can determine the slope of the line segments by drawing horizontal and vertical lines to complete a right-angled triangle.

EXAMPLE 1

Find the slope of line *AB*.

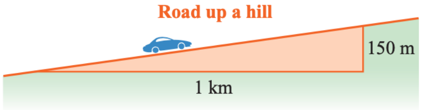


Solve	Think	Apply
 Slope of $AB = \frac{\text{vertical rise}}{\text{horizontal run}} = \frac{2}{5}$	Complete a right-angled triangle using *AB* as the hypotenuse. Then vertical rise = 2 and horizontal run = 5, so slope = $\frac{2}{5}$.	Complete a right-angled triangle. Determine the vertical rise and horizontal run to calculate the gradient. Gradient = $\frac{\text{rise}}{\text{run}}$

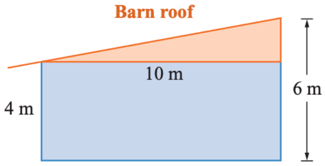
Its time to try

1 Find these slopes.

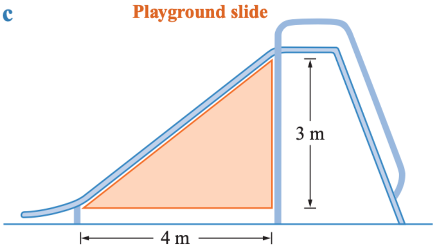
a



b



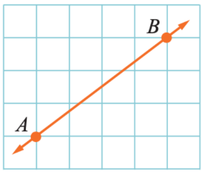
c



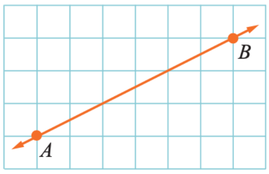
2 In each diagram, draw a right-angled triangle and find the gradient using:

$$\text{Gradient} = \frac{\text{vertical rise}}{\text{horizontal run}}$$

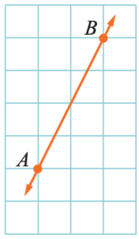
a



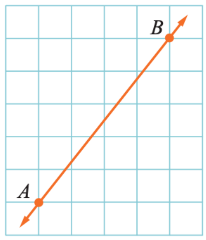
b



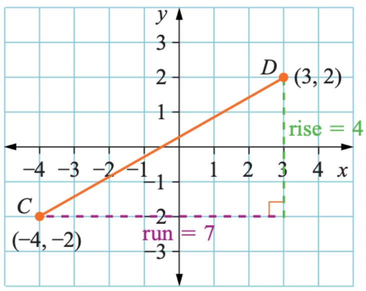
c



d



Find the gradient of the line passing through points $C(-4, -2)$ and $D(3, 2)$.

Solve	Think	Apply
 Gradient $= \frac{\text{rise}}{\text{run}} = \frac{4}{7}$	Complete a right-angled triangle using CD as the hypotenuse. Then vertical rise = 4 and horizontal run = 7, so gradient $= \frac{4}{7}$.	Complete a right-angled triangle using CD as the hypotenuse. Determine the vertical rise and horizontal run from C to D and calculate the gradient. Gradient $= \frac{\text{rise}}{\text{run}}$

3 Find the gradient of the line passing through each pair of points.

a $C(-5, -2)$ and $D(4, 5)$

b $A(-3, -1)$ and $B(5, 2)$

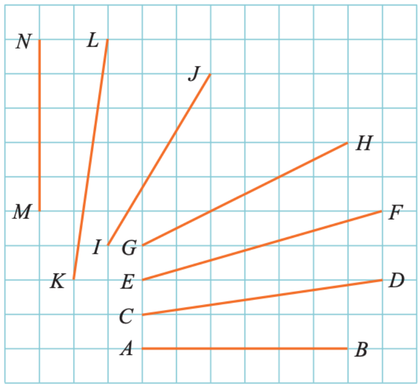
c $C(-5, 3)$ and $P(7, 7)$

d $M(1, -5)$ and $N(2, 6)$

Investigation 2 Varying the slope

1 Complete the table.

Line segment	*x*-run	*y*-rise	Slope
AB			
CD			
EF			
GH			
IJ			
KL			
MN			



2 Complete the following.

a The slope of a horizontal line is ______.

b The slope of a vertical line is ______.

c As the line segments become steeper, their slopes ______.

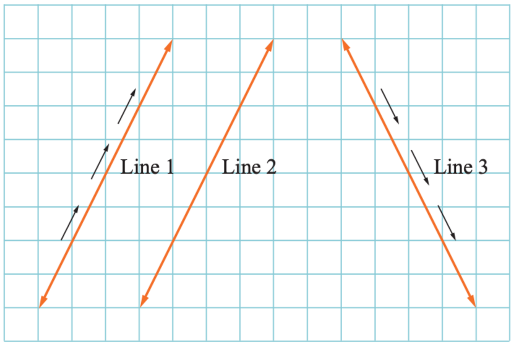
Positive and negative gradients

In the diagram, lines 1 and 2 are parallel, and have the same slope of 2.

Line 3 is not parallel to lines 1 and 2, yet it has the same degree of steepness.

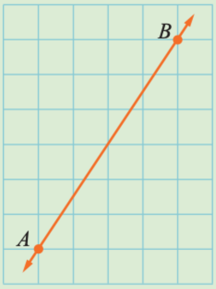
We say that lines 1 and 2 are **forwards sloping**, whereas line 3 is **backwards sloping**.

As we go from *left to right*, on line 1 we are going *uphill* and the slope (gradient) is **positive**, whereas on line 3 we are going *downhill* and the slope (gradient) is **negative**.

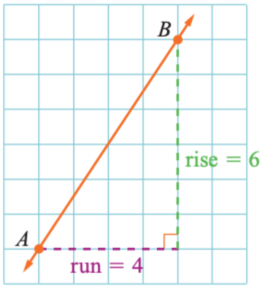
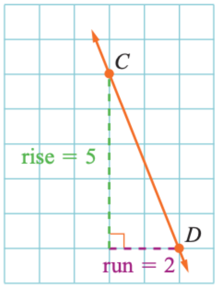


Find the slope of each line.

a

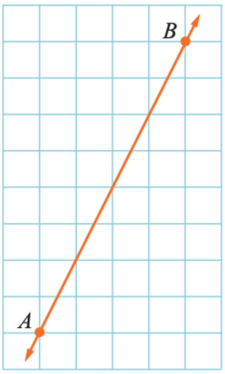


b

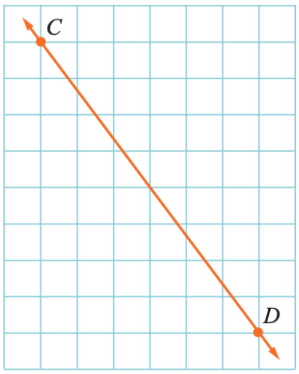
	Solve	Think		Apply
a	The slope of AB is positive (uphill). Slope $AB = \frac{\text{rise}}{\text{run}}$ $= +\frac{6}{4}$ $= +1\frac{1}{2}$	Draw in a right-angled triangle and find the rise and run.		First determine whether the slope is positive or negative. For downhill slopes, the 'rise' is a 'drop', so the slope is a negative value.
b	The slope of CD is negative (downhill). Slope $CD = \frac{\text{rise}}{\text{run}}$ $= -\frac{5}{2}$ $= -2\frac{1}{2}$	Draw in a right-angled triangle and find the rise and run.		

1 Determine whether the slope is positive or negative and then find the gradient.

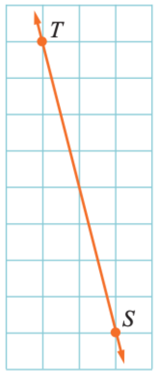
a



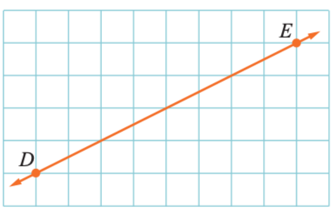
b



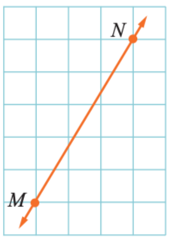
c



d



e



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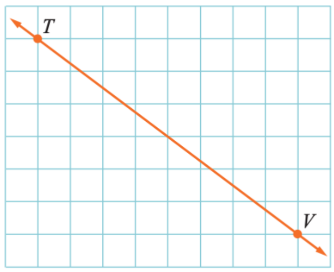
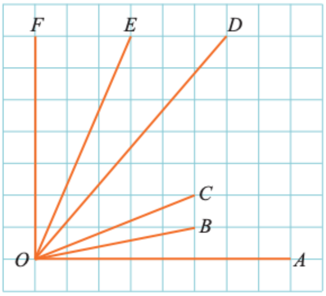


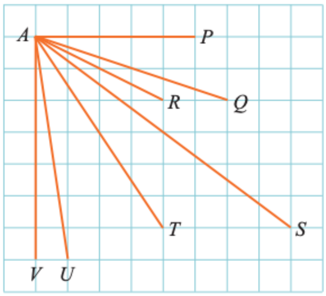
2 Find the gradient of each line.

a OA **b** OB
c OC **d** OD
e OE **f** OF

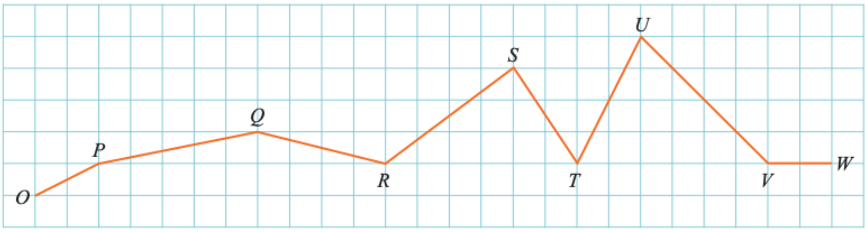


3 Find the gradient of each line.

a AP **b** AQ **c** AR
d AS **e** AT **f** AU
g AV

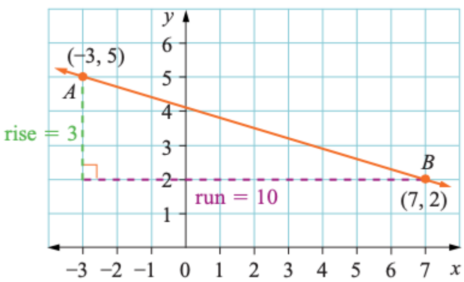


4 Imagine you are walking across the countryside from O to W (from left to right).



a When are you going uphill?
b When are you going downhill?
c Where is the steepest positive slope?
d Where is the steepest negative slope?
e Where is the slope 0?
f Where is the slope not zero but least?

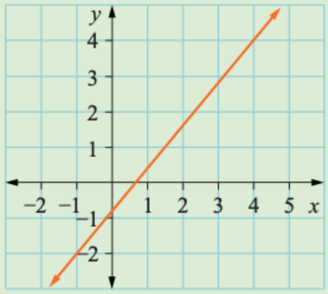
Plot points $A(-3, 5)$ and $B(7, 2)$ and find the gradient of the line passing through them.

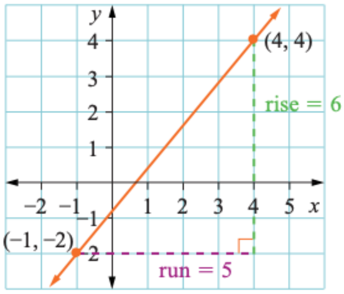
Solve	Think	Apply
 Gradient $= \frac{\text{rise}}{\text{run}}$ $= -\frac{3}{10}$	From the right-angled triangle, the slope is downhill, so the rise is -3 and the run is 10.	Plot the points, and draw a right-angled triangle. Find the rise and run. The gradient is negative (downhill).

5 Plot each pair of points and find the gradient of the line passing through them.

a $A(-4, 6)$ and $B(7, 2)$ **b** $C(-4, -1)$ and $D(5, 3)$ **c** $P(1, 3)$ and $Q(-4, -1)$
d $R(0, 0)$ and $S(5, 3)$ **e** $M(5, 3)$ and $N(-5, 2)$ **f** $S(-3, -2)$ and $T(4, -6)$

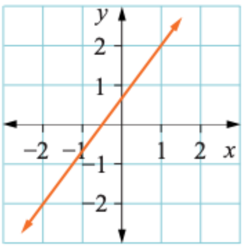
Find the gradient of this line.



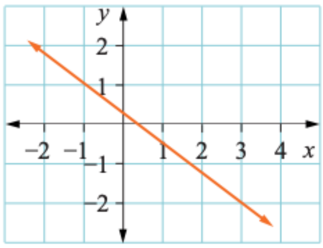
Solve	Think	Apply
 Gradient $= \frac{\text{rise}}{\text{run}}$ $= +\frac{6}{5}$	Choose any two points on the line, say $(-1, -2)$ and $(4, 4)$. Draw in a right-angled triangle. The gradient is positive (uphill). The rise is 6 and the run is 5.	A straight line has the same gradient for its entire length. Choose any two points to calculate the gradient.

6 By choosing two points on each line, find the gradients.

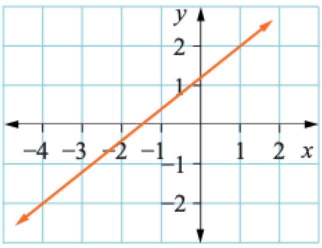
a



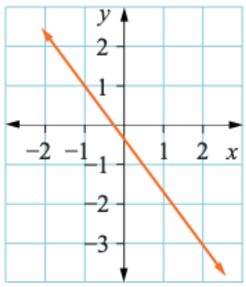
b



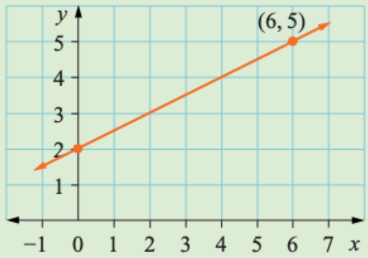
c

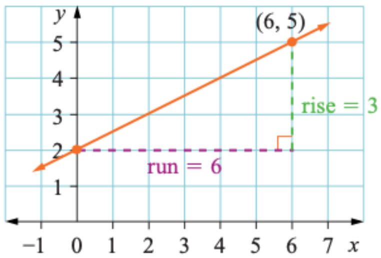


d



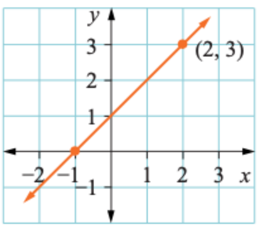
Find the gradient of the given line.



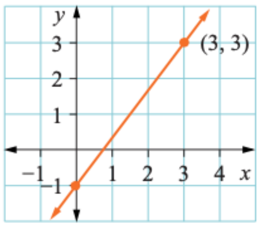
Solve	Think	Apply
 Gradient $= \frac{\text{rise}}{\text{run}}$ $= +\frac{3}{6} = +\frac{1}{2}$	The gradient is positive (uphill). The rise is 3 and the run is 6.	Draw in a right-angled triangle. Find the rise and run.

7 Find the gradients of these lines.

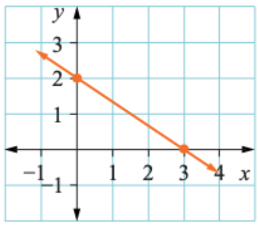
a



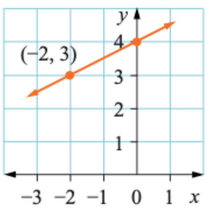
b



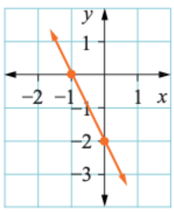
c



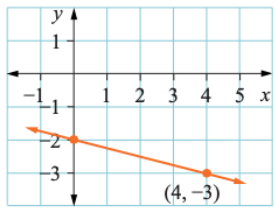
d



e



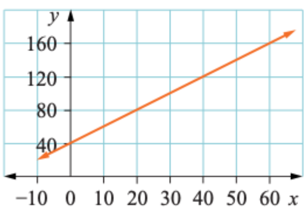
f



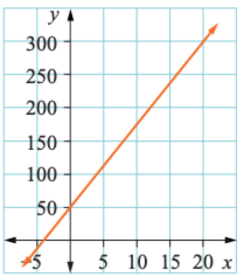
Be careful as the scales are not the same.

8 Find the gradient of each line.

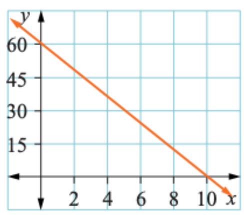
a



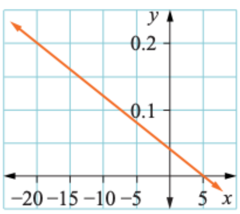
b



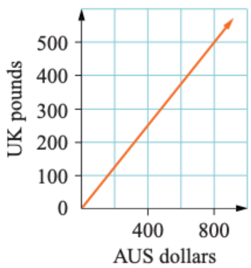
c



d



e



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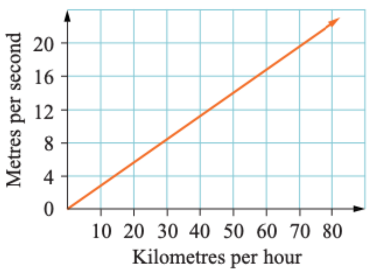


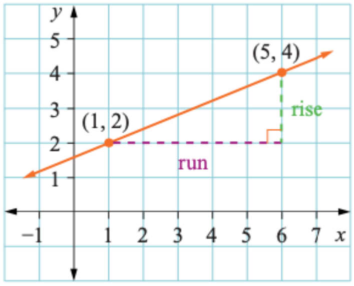
Investigation 3 Formula for gradient

The gradient has been found by drawing a right-angled triangle and finding the vertical rise and horizontal run.

Gradient $= \frac{\text{rise}}{\text{run}}$

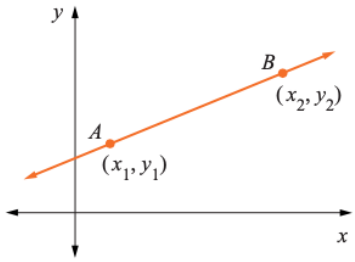
1 **a** Find values for the vertical rise and horizontal run as shown in the triangle on this graph.

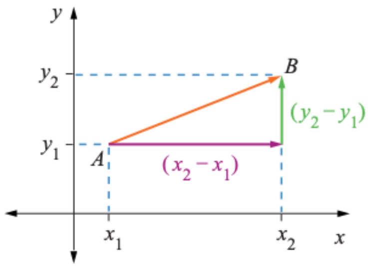
b Calculate the gradient.



2 **a** Copy this diagram.

b Draw in the triangle as shown on the right-hand diagram.





c If A is (x_1, y_1) and B is (x_2, y_2) then from the diagram:

- rise $= y_2 - y_1$
- run $= x_2 - x_1$
- the vertical rise from A to B is $y_2 - y_1$ (the difference between the y-coordinates)
- the horizontal run from A to B is $x_2 - x_1$ (the difference between the x-coordinates).

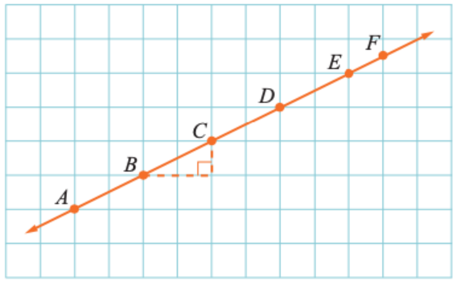
d The symbol for gradient is m. Complete the following.

$$m = \frac{y_2 - \square}{\square - \square}$$

Investigation 4 The slope of a line

1 Complete the table.

Line segment	x-run	y-rise	$\frac{y\text{-rise}}{x\text{-run}}$
BC	2	1	$\frac{1}{2}$
DE			
AC			
BE			
AE			
AF			



2 State, in sentence form, any conclusions you can draw from the graph and table.

Investigation 5 Relating gradient and the tangent ratio

1 Plot points $A(1, 2)$ and $B(5, 9)$.

2 Draw a right-angled triangle and write the lengths of the horizontal and vertical sides.

3 Find the gradient of AB.

4 Label the angle at A as θ.

5 With respect to θ, label the sides as opposite, adjacent and hypotenuse.

6 Write an expression for $\tan \theta$.

7 Compare $\tan \theta$ and the gradient.

8 Explain the result from question **7**.

9 Calculate the size of the angle that the line makes with the x-axis.

10 Calculate the angles for the gradients of the line joining the points in Exercise 10D question **5**.

11 Copy and complete the following.
The gradient of a line is equal to _____ θ, where θ is the angle made by the line and the _____ axis.