

Walt Factorise quadratic expression

Success criteria I can use my knowledge of multiplication and addition facts to see the two numbers that factorise.

[Watch the video](#) __--



- 1 The **coefficient** (number in front) of x is the sum of the numbers a and b in the brackets.
- 2 The **constant term** (number at the end by itself) is the product of the numbers a and b in the brackets.

Therefore, working in reverse, to find the numbers a and b we want two numbers that:

- 1 add to the coefficient of x
- 2 multiply to the constant term.

Example

Factorise $x^2 + 8x + 12$.

Answer

To write $x^2 + 8x + 12$ in the form $(x \quad)(x \quad)$ we have to find a pair of numbers that add to 8 and multiply to 12.

Some pairs of numbers that add to 8	Some pairs of numbers that multiply to 12
-1 9	1 12
0 8	2 6
1 7	3 4
2 6	-1 -12
3 5	-2 -6
4 4	-3 -4

The pair we want is the one that is in *both* columns, i.e. 2 6.

Therefore, $x^2 + 8x + 12$ factorises to $(x + 2)(x + 6)$.



A competent Maths student explores the possibilities in their head rather than writing them all down. This takes some practice at first, but you will gradually speed up!

Example

Factorise $x^2 - x - 6$.

Answer

Here we want two numbers that add to -1 and multiply to -6 . If they multiply to a negative number, the two numbers must have different signs – one positive and one negative.

-3 and 2 are the only pair of numbers that have this property.

$$x^2 - x - 6 = (x - 3)(x + 2)$$

Example

Factorise $x^2 - 26x + 48$.

Answer

Here we want two numbers that add to -26 and multiply to 48. The two numbers must both be negative.

-24 and -2 are the only pair of numbers that have this property.

$$x^2 - 26x + 48 = (x - 24)(x - 2)$$



It does not matter which bracket comes first, e.g.

$$(x - 3)(x + 2) \text{ is the same as } (x + 2)(x - 3).$$



Expanding is the opposite of factorising.

$$\begin{array}{c}
 \xrightarrow{\text{Factorising}} \\
 x^2 + 3x - 10 \Leftrightarrow (x + 5)(x - 2) \\
 \xleftarrow{\text{Expanding}}
 \end{array}$$

EXERCISE 8.07

1–20 Factorise these quadratic expressions.

1 $x^2 + 8x + 15$

2 $x^2 + 10x + 24$

3 $x^2 + 4x + 3$

4 $x^2 - 2x - 15$

5 $x^2 - x - 12$

6 $x^2 + x - 56$

7 $x^2 - 7x + 12$

8 $x^2 - 13x + 42$

9 $x^2 - x - 2$

10 $x^2 - 8x - 20$

11 $x^2 - 15x + 56$

12 $x^2 + 16x + 39$

13 $x^2 + 11x - 60$

14 $x^2 - 6x - 40$

15 $x^2 - 22x - 23$

16 $x^2 - 13x - 48$

17 $x^2 + 15x + 50$

18 $x^2 - 3x - 70$

19 $x^2 - 17x + 72$

20 $x^2 + 5x - 36$

21–28 It is not always possible to factorise a quadratic expression. Four of the following factorise; the other four do not. Either factorise correctly or write “Cannot be factorised.”

21 $x^2 - 2x + 8$

22 $x^2 + 12x - 13$

23 $x^2 - 5x - 6$

24 $x^2 + 2x + 3$

25 $x^2 - 4x + 18$

26 $x^2 + x - 7$

27 $x^2 + x - 6$

28 $x^2 + 13x - 14$

29 There are nine quadratic expressions in the left column. Eight of them have a corresponding factorisation placed in the right column, but they have been muddled up.

Use factorising or expanding to match up each expression (a–i) with its correct factorisation. For one of them write “Cannot be factorised.”

Expansion	Factorisation
a $x^2 - 3x - 4$	A $(x + 3)(x + 4)$
b $x^2 - 5x + 4$	B $(x + 4)(x - 1)$
c $x^2 - x - 12$	C $(x + 4)(x - 3)$
d $x^2 - 7x + 12$	D $(x - 1)(x - 4)$
e $x^2 + 7x + 12$	E $(x + 4)(x + 1)$
f $x^2 + 5x + 4$	F $(x - 3)(x - 4)$
g $x^2 - 2x - 12$	G $(x - 4)(x + 3)$
h $x^2 + x - 12$	H $(x + 1)(x - 4)$
i $x^2 + 3x - 4$	

Two special cases

- 1 A **perfect square** is a quadratic expression where both brackets are the same. To write it as simply as possible we use squaring rather than repeating the same set of brackets.

Example

Factorise $x^2 - 6x + 9$.

Answer

The pair of numbers that multiply to 9 and add to -6 are -3 and -3 .
$$x^2 - 6x + 9 = (x - 3)(x - 3)$$
$$= (x - 3)^2$$

- 2 The **difference of two squares** factorises to two sets of brackets which are identical except for the signs; one $+$ and the other $-$. It is called the difference of two squares because in its expanded form it is:

$$x^2 - (\text{number})^2$$

and there is no x term. If an x term was written in it would be $0x$.

Example

Factorise $x^2 - 64$.

Answer

$$x^2 - 64 = x^2 + 0x - 64$$

The two numbers that add to 0 and multiply to -64 are 8 and -8 .
$$x^2 - 64 = (x + 8)(x - 8)$$

EXERCISE 8.08

Factorise these quadratic expressions. Each one is either a perfect square or a difference of two squares.

1 $x^2 - 16$

7 $x^2 - 18x + 81$

13 $x^2 - 36x + 324$

2 $x^2 + 4x + 4$

8 $x^2 - 24x + 144$

14 $x^2 - \frac{1}{9}$

3 $x^2 - 10x + 25$

9 $x^2 - 64$

15 $x^2 + \frac{1}{5}x + \frac{1}{100}$

4 $x^2 - 1$

10 $x^2 - 121$

5 $x^2 + 6x + 9$

11 $x^2 + 40x + 400$

6 $x^2 - 36$

12 $x^2 - 169$

Quadratics with a common factor

When the coefficient (number in front) of x^2 is not 1 the whole quadratic expression may have a common factor. Deal with this first, and then factorise the resulting quadratic.

Example

Factorise $4x^2 - 20x + 24$.

Answer

On inspection each term has a common factor of 4.

$$4x^2 - 20x + 24 = 4(x^2 - 5x + 6)$$

Then factorise the quadratic in brackets, *leaving* the 4 in place:

$$= 4(x - 3)(x - 2)$$

Factorise these quadratic expressions *completely*.

1 $3x^2 + 15x + 18$

5 $3x^2 + 6x + 3$

9 $5x^2 - 10x - 15$

13 $4x^2 - 4x - 24$

2 $2x^2 - 24x + 72$

6 $2x^2 + 2x - 4$

10 $2x^2 + 12x + 10$

14 $5x^2 - 40x + 35$

3 $5x^2 - 125$

7 $4x^2 - 400$

11 $3x^2 + 18x + 27$

15 $3x^2 + 9x + 6$

4 $3x^2 - 12$

8 $2x^2 - 8x + 8$

12 $6x^2 - 6$