Expanding and Factorising

Curriculum Ready

Mathletics

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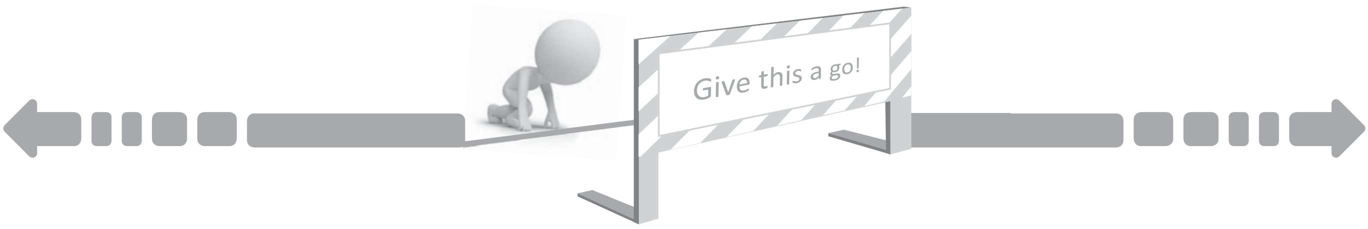
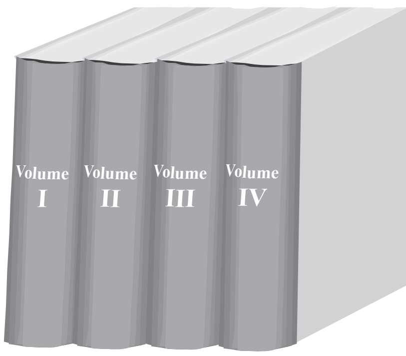
This booklet gives you the skills to simplify algebraic expressions by writing them in different ways.

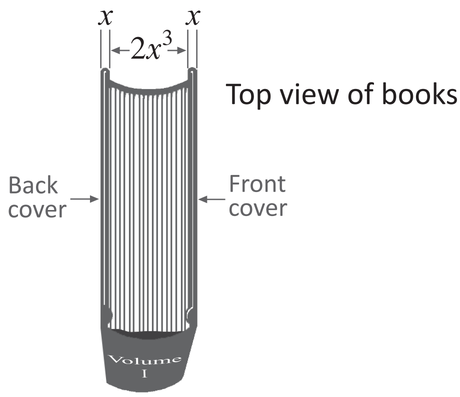
Investigate these terms and write a one sentence description of their meaning in Mathematics.

Expand	Simplify	Factorise



Q The picture below shows the first four volumes from a set of ten books in order on a shelf. All volumes are exactly the same size.





A small insect got caught inside the front cover of Volume I and ate through the books until it reached the last page of Volume IV. How many units did it travel while eating if $x = 1.5$?

Work through the book for a great way to solve this

Expanding

Grouping symbols such as [brackets], {braces} and (parentheses), can be removed without changing the value of the expression by expanding.

$$\begin{aligned} a(b+c) &= a \times b \text{ and } a \times +c \\ &= ab+ac \end{aligned}$$

Every term inside the parentheses is multiplied by the term in front of the parentheses.

Expand $2(3x+5)$

$$\begin{aligned} 2(3x+5) &= 2(3x+5) \quad \text{2 × every term inside the parentheses} \\ &= 2\times 3x \text{ and } 2\times +5 \\ &= 6x \text{ and } +10 \\ &= 6x+10 \end{aligned}$$

$$\begin{aligned} a(b+c) &= a \times b \text{ and } a \times -c \\ &= ab-ac \end{aligned}$$

Expand $3a(a-4)$

$$\begin{aligned} 3a(a-4) &= 3a(a-4) \quad \text{3a × every term inside the parentheses} \\ &= 3a\times a \text{ and } 3a\times -4 \\ &= 3a^2 \text{ and } -12a \\ &= 3a^2-12a \end{aligned}$$

Now that you have seen how it works, it's time to learn the mathematical name we give this is method:

The Distributive Law

- $a(b+c) = a \times b$ and $a \times +c$
  $= ab+ac$
- $a(b-c) = a \times b$ and $a \times -c$
  $= ab-ac$

Be careful multiplying positive and negative values when expanding.

$+ \times +$ or $- \times -$

Signs the same we get a **positive** answer

$3 \times 5 = 15$ $\quad -3 \times -5 = 15$

$- \times +$ or $+ \times -$

If the signs change we get a **negative** answer

$-3 \times 5 = -15$ $\quad 3 \times -5 = -15$

When expanding parenthesis multiplied by negative numbers, The Distributive Law becomes:

$$-a(b+c) = -a \times b \text{ and } -a \times +c$$
$$= -ab - ac$$

$$-a(b-c) = -a \times b \text{ and } -a \times -c$$
$$= -ab + ac$$

If the term in front of the parentheses is negative, all the terms inside change sign after expanding.

Expand $-p(4p+7)$

$$-p(4p+7) = -p(4p+7)$$ $-p \times$ every term inside the parentheses

$$= -p \times 4p \text{ and } -p \times 7$$ $-p(4p+7) = (-p \times 4p) + (-p \times 7)$

$$= -4p^2 \text{ and } -7p$$

$$= -4p^2 - 7p$$

Expand $-5(3y-1)$

$$-5(3y-1) = -5(3y-1)$$ $5 \times$ every term inside the parentheses

$$= -5 \times 3y \text{ and } -5 \times -1$$ $-5(3y-1) = (-5 \times 3y) + (-5 \times -1)$

$$= -15y \text{ and } +5$$

$$= -15y + 5$$

Expanding

1. Expand:

   a. $2(a+7)$

   b. $9(b-3)$

   c. $6c(3d+1)$

   d. $4d(3-c)$

   e. $3x(6+4y)$

   f. $3m(p-q)$

   g. $\frac{1}{2}(6m-14)$

   h. $2ab(3c+2d)$

   i. $4(-3-9x)$

   j. $-2p\left(2-\frac{q}{2}\right)$

Expanding

2 Expand:

a $-(a+11)$

Psst! Remember the 1 can be hidden: $-1(a+11)$

b $-2(b-5)$

c $-n(6+8m)$

d $-3(2-7d)$

e $-2x(y+4)$

f $-5mn(p-q)$

3 The same rules apply for expanding the following questions:

a $0.2a(25a+15)$

b $-2b(c-3.5b)$

More expanding

Why limit yourself to parentheses with only two terms? The Distributive Law works for parentheses with more.

Every term inside the parentheses is multiplied by the term in front.

Expand $4(2m+3n-2)$

$$\begin{aligned}4(2m+3n-2) &= 4(2m+3n-2) \\ &= 4\times 2m \text{ and } 4\times +3n \text{ and } 4\times -2 \\ &= 8m \text{ and } +12n \text{ and } -8 \\ &= 8m+12n-8\end{aligned}$$

$4\times$ every term inside the parentheses

Take care with the multiplications when there is a negative term out the front.

Expand $-a(a-b+3c+2)$

$$\begin{aligned}-a(a-b+3c+2) &= -a(a-b+3c+2) \\ &= -a\times a \text{ and } -a\times -b \text{ and } -a\times +3c \text{ and } -a\times +2 \\ &= -a^2 \text{ and } +ab \text{ and } -3ac \text{ and } -2a \\ &= -a^2+ab-3ac-2a\end{aligned}$$

$a\times$ every term inside the parentheses

The basic index laws are often used when expanding expressions.

Expand $p^2(p-3pq+5q)$

$$\begin{aligned}p^2(p-3pq+5q) &= p^2(p-3pq+5q) \\ &= p^2\times p \text{ and } p^2\times -3pq \text{ and } p^2\times +5q \\ &= p^{2+1} \text{ and } -3p^{2+1}q \text{ and } +5p^2q \\ &= p^3-3p^3q+5p^2q\end{aligned}$$

$p^2\times$ every term inside the parentheses

Remember:
$a^m \times a^n = a^{m+n}$

More expanding

1 Expand:

a $3(a+b+2)$

b $4(x-y-5)$

c $3p(2p+q+4)$

d $-d(e+2f+6)$

e $2x(4x+3y-3+z)$

f $-a(b-2c+d-5)$

2 Expand: (psst: remember the multiplication rule for indices)

a $n(n^2+3n)$

b $xy(x^2-y^3)$

c $-ab(ab^2+2a^2b)$

d $2p(2p^2-4pq+5)$

Expanding and simplifying

Always simplify the expression after expanding where possible.

Simplify by collecting like terms after the expansion of any parentheses.

Expand and simplify: $3(7m-6)-16m$

$$3(7m-6)-16m = 3(7m-6)-16m$$ 3 × every term inside the parentheses

$= 3\times 7m$ and 3×-6 and $-16m$

$= 21m-18-16m$ (Like terms: $21m$, $16m$)

$= 5m-18$ Combine the like terms

For expressions with multiple parentheses, expand each separately then look to simplify.

Expand and simplify: $5(2a+4)-4(a-3)$

$$5(2a+4)-4(a-3) = 5(2a+4)-4(a-3)$$ Expand each grouping separately

$5\times 2a$ and $5\times +4$ $\quad -4\times a$ and -4×-3

$= 10a+20-4a+12$ Identify the like terms (Like terms: $10a$, $4a$; Like terms: 20, 12)

$= 10a-4a+20+12$ Group the like terms

$= 6a+32$ Simplify

Be careful to apply the index laws correctly when expanding expressions with multiple variables.

Expand and simplify: $xy(5x+y)-2x^2y$

$$xy(5x+y)-2x^2y = xy(5x+y)-2x^2y$$ xy × every term inside the parentheses

$= xy\times 5x$ and $xy\times +y$ and $-2x^2y$

$= 5x^{1+1}y$ and xy^{1+1} and $-2x^2y$ Identify the like terms

$= 5x^2y+xy^2-2x^2y$ (Like terms: $5x^2y$, $2x^2y$)

$= 5x^2y-2x^2y+xy^2$ Group the like terms

$= 3x^2y+xy^2$ Simplify

Expanding and simplifying

1 Expand and simplify:

a $4(a+3)+2a$

b $-3(2-x)+1$

c $12p+5(p-2)$

d $5d-4(9-3d)$

e $-5b(4-b)+3b+5b^2$

f $9(x-2y)-x+4y$

2 Expand and simplify:

a $8(c-4)+3(c+2)$

b $9(d+2)-(5-3d)$

c $3(x-5)-2(4+x)$

d $a(a+8)-5(a+3)$

Expanding and simplifying

3 Expand and simplify:

a) $-(y+4x)-5(2x-y)$

Psst! Remember the 1 can be hidden: $-1(y+4x)$

b) $x(2+x-y)+3x-xy$

c) $2a(3+4b)+4(ab+2a)$

d) $-3b(2+b)-(6-b)$

e) $-(2-d)-2(d-2)$

f) $xy(40x+5)-3y(10x^2-x)$

g) $-mn(5m-2n^2)+mn^3+3m^2n$

h) $q(4p+3q^2-2)+2q(q+5p)$

Highest common factor

The highest common factor (HCF) is the **largest term** that divides **exactly** into all the given terms.

The HCF must divide exactly into **every** term.

Find the HCF for the terms $12a$ and 18

For $12a$ and 18 — Both numbers are divisible by $1, 2, 3, 4$ and 6

$\therefore 6$ is the HCF for the numbers

For $12a$ and 18 — There are no variables common to both terms

$\therefore$ The HCF for $12a$ and 18 is: $\mathbf{6}$

Take care to ensure all common factors have been found.

Find the HCF for the terms $15xy$, $30x$ and $25x^2$

For $15xy$, $30x$ and $25x^2$ — All three numbers are divisible by 1 and 5

$\therefore \mathbf{5}$ is the HCF for the numbers

For $15xy$, $30x$ and $25x^2$ — One x variable is common to all terms

$\therefore \boldsymbol{x}$ is the HCF for the variables

$\therefore$ The HCF for $15xy$, $30x$ and $25x^2$ is: $\boldsymbol{5x}$ — Multiply the HCFs together.

Some terms need to be simplified first.

Find the HCF for the terms $8m^2n$, $24mn^2$ and $(6mn)^2$

$(6mn)^2 = 36m^2n^2$ — Use the power rule to simplify $(6mn)^2$

For $8m^2n$, $24mn^2$ and $36m^2n^2$ — All the numbers are divisible by $1, 2$ and 4

$\therefore \mathbf{4}$ is the HCF for the numbers

For $8m^2n$, $24mn^2$ and $36m^2n^2$ — m and n are common to all terms

$\therefore \boldsymbol{mn}$ is the HCF for the variables

$\therefore$ The HCF for $8m^2n$, $24mn^2$ and $(6mn^2)$ is: $\boldsymbol{4mn}$ — Multiply the HCFs together

Highest common factor

1. Find the HCF for these groups of terms:

   a. 6 and 9

   b. 14 and $8b$

   c. $30m$ and $15m$

   d. $-10d$ and 15
   hint: there are positive and negative HCFs for this one

   e. $18h^2$ and $24h^2$

   f. $28w^2$ and $35w$

2. Find the HCF for these groups of terms:

   a. $6ab$, $12a$ and $8b$

   b. $14x^2y$, $21xy$ and $7y$

   c. $(4m)^2, 16m$ and $24m^2$

   d. $8pq^2$, $-p^2q$ and $-4pq$
   hint: there are positive and negative HCFs for this one

Factorising

This is the opposite of expanding. Put the highest common factor (HCF) out the front of a pair of parentheses and put the remaining parts inside.

Factorise $12a+4$

For $12a+4$, the HCF is: 4

$\therefore 12a+4 = 4(\quad ? \quad)$ — Put the HCF out the front of a pair of parentheses

HCF

$3a+1$ — Find what the HCF is multiplied by to get each term

Because $4 \times 3a = 12a$ $\quad 4 \times 1 = 4$

$\therefore 12a+4 = 4(3a+1)$

A good way to check your factorisation is by expanding your answer it to see if you get the original expression.

$\times$

$4(3a+1) = 4 \times 3a$ and $4 \times + 1$

$= 12a + 4$

Make sure you also look for variables common to all terms as possible factors.

Factorise $5xy - 2x$

For $5xy - 2x$, the HCF is: x

$\therefore 5xy - 2x = x(\quad ? \quad)$ — Put the HCF out the front of a pair of parentheses

HCF

$5y - 2$ — Find what the HCF is multiplied by to get each term

Because: $x \times 5y = 5xy$ $\quad x \times -2 = -2x$

$\therefore 5xy - 2x = x(5y-2)$

Be very careful when there are negative signs involved.

Factorise $-6mn - 8m$

For $-6mn - 8m$, the HCF is: $-2m$ — Put negative sign in HCF if both terms are negative

$\therefore -6mn - 8m = -2m(\quad ? \quad)$ — Put the HCF out the front of a pair of parentheses

HCF

$3n + 4$ — Find what the HCF is multiplied by to get each term

Because: $-2m \times 3n = -6mn$ $\quad -2m \times 4 = -8m$

$\therefore -6mn - 8m = -2m(3n+4)$

For more complex questions, be careful to ensure that you find **all** the common factors.

Factorise $2xy + 6x^2y - 4xy^2$

For $2xy + 6x^2y - 4xy^2$, the HCF is $2xy$

$\therefore 2xy + 6x^2y - 4xy^2 = 2xy(\quad ? \quad)$ — Put the HCF out the front of a pair of parentheses

HCF

$1 + 3x - 2y$ — Find what the HCF is multiplied by to get each term

Because: $2xy \times 1 = 2xy$ $\quad 2xy \times 3x = 6x^2y$ $\quad 2xy \times -2y = -4xy^2$

$\therefore 2xy + 6x^2y - 4xy^2 = 2xy(1 + 3x - 2y)$

Dividing each term by the HCF is another method to help find what goes inside the parentheses.

Factorise $2xy + 6x^2y - 4xy^2$

For $2xy + 6x^2y - 4xy^2$, the HCF is $2xy$

$\therefore 2xy + 6x^2y - 4xy^2 = 2xy(\quad ? \quad)$ — Put the HCF out the front of a pair of parentheses

HCF

$$\frac{2xy}{2xy} + \frac{6x^2y}{2xy} - \frac{4xy^2}{2xy}$$ — Divide every term by the HCF and simplify

$$1 + 3x - 2y$$

$\therefore 2xy + 6x^2y - 4xy^2 = 2xy(1 + 3x - 2y)$

Go back and try this last method with the other three factorisation examples to see which one you prefer.

Factorising

1 Factorise:

a $2a - 6$

b $8b + 16$

c $6c + 8$

d $9 - 3d$

e $-8 - 6e$

f $-24f + 36$

g $14m - 18n$

h $-48p - 28q$

2 Factorise:

a $8ab + 2a$

b $7m - 14mn$

c $22vw + 16w$

d $-15pq - 25p$

Factorising

e $3xyz + 9xy$

f $-32ab + 16abc$

3 Factorise:

a $20pqr + 14pq^2$

b $-12y^2z + 36yz^2$

c $-3a^2bc - 6b^2c$

d $6x + 3y + 12xy$

e $ab - 2a^2 + ac$

f $33j - 22jk + 11j^2k$

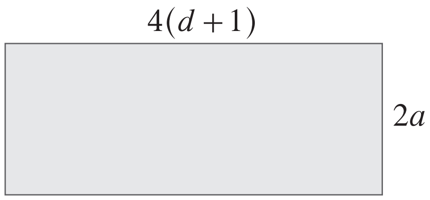
4 Factorise $39a^4b^3c^5 - 91ab^2c^3$

Algebraic calculations

Many calculations contain algebraic terms. These can be simplified using the techniques learned so far.

For the rectangle shown here:

(i) Write a simplified expression for the area of the rectangle.



$$\begin{aligned}
\text{Area} &= \text{length} \times \text{width units}^2 \\
&= 4(d+1) \times 2a \text{ units}^2 \\
&= 4 \times 2a \times (d+1) \text{ units}^2 && \text{Multiply the terms outside the parentheses} \\
&= 8a(d+1) \text{ units}^2 && \text{Factorised form} \\
&= 8a(d+1) \text{ units}^2 \\
&= 8ad + 8a \text{ units}^2 && \text{Expanded form}
\end{aligned}$$

Both are simplified expressions

(ii) Find the area of the rectangle when $a = 5$ and $d = 4.2$

Using expanded form or **Using factorised form**

Area $= 8ad + 8a$ units^2 | Area $= 8a(d+1)$ units^2

substitute in the variable values

$\therefore$ when $a = 5$ and $d = 4.2$

Area $= 8 \times 5 \times 4.2 + 8 \times 5$ units^2 | Area $= 8 \times 5 \times (4.2 + 1)$ units^2

use correct order of operations

(parentheses, then $\times$ and $\div$, then $+$ and $-$)

$= 168 + 40$ units^2 | $= 40 \times 5.2$ units^2

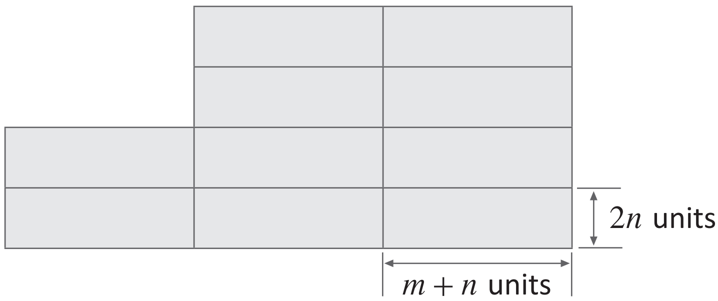
$= 208$ units^2 | $= 208$ units^2

You can simply substitute the variable values into the expressions for each side and calculate the area straight away if not asked to simplify first.

Looking for identical variable parts in a question will help with simplification.

For this shape made using 10 equal sized rectangles:

(i) Write a simplified expression for the perimeter.



Perimeter $=$ 6 lengths of $(m+n)$ plus 8 lengths of $2n$

$\therefore$ Perimeter $= 6(m+n)+8\times 2n$ units — Multiply equal sections by their total number

$= 6m+6n+16n$ units — Expand the parentheses

$= 6m+22n$ units — Simplify by collecting like terms

(ii) Calculate the perimeter of the shape when $m = 2$ and $n = 1.5$

Perimeter $= 6m+22n$ units

$\therefore$ when $m = 2$ and $n = 1.5$

Perimeter $= 6\times 2+22\times 1.5$ units — Substitute in the variable values

$= 45$ units — Calculate the perimeter

(iii) If one variable dimension is changed, a new expression needs to be found.

Write a simplified expression for the new perimeter if the length of each small rectangle is doubled $(\times 2)$.

New Perimeter $=$ 6 lengths of $2(m+n)$ plus 8 lengths of $2n$

$\therefore$ New Perimeter $= 6\times 2(m+n)+8\times 2n$ units — Multiply equal sections by their total number

$= 12(m+n)+16n$

$= 12m+12n+16n$ units — Expand the parentheses

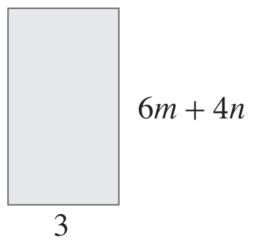
$= 12m+28n$ units — Simplify by collecting like terms

You can see that the answer to part (iii) is not simply double the original expression.

Algebraic calculations

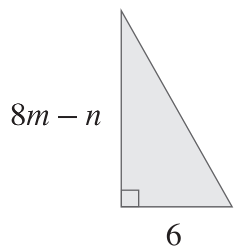
1 (i) Write simplified expressions for the **area** of each of these shapes
(ii) Calculate the area of each shape when $m = 2.5$ and $n = 3$

a



Area = length $\times$ width

b

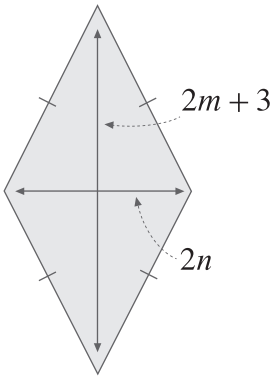


Area = (base $\times$ height) $\div$ 2

Algebraic calculations

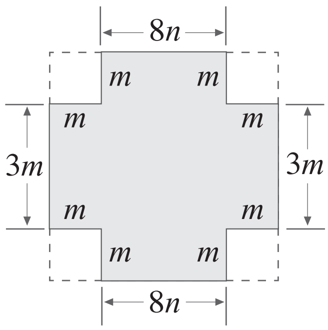
(i) Write simplified expressions for the **area** of each of these shapes
(ii) Calculate the area of each shape when $m = 2.5$ and $n = 3$

c



Area = (short diagonal $\times$ long diagonal) $\div 2$

d Earn an Awesome passport stamp with this one

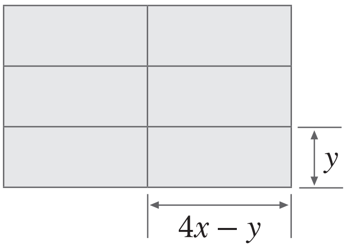


Help:
- Area of large rectangle = $(8n + 2m) \times (3m + 2m)$
- Area for each of the cut out squares = $m \times m$
- Shaded Area = Area of large rectangle – Area of 4 small squares

Algebraic calculations

2 (i) Write simplified expressions for the **perimeter** of these shapes made with equal sized rectangles.
(ii) Calculate the perimeter by substituting in the variable values given in [brackets].
(iii) Write simplified expressions for the new perimeter if the vertical side of each rectangle was tripled ($\times 3$).

a [$x = 5$ and $y = 3$]

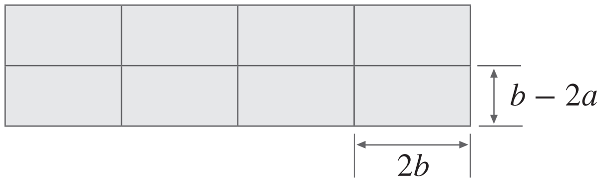


(i)

(ii)

(iii)

b [$a = -2$ and $b = 6$]



(i)

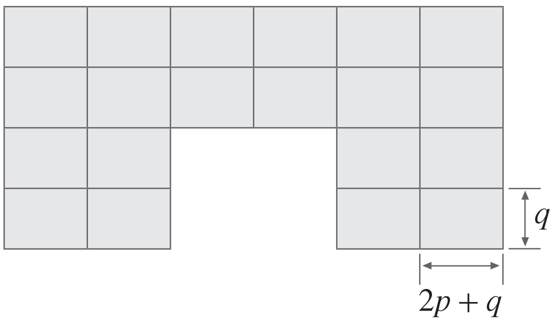
(ii)

(iii)

Algebraic calculations

2 (i) Write simplified expressions for the **perimeter** of these shapes made with equal sized rectangles.
(ii) Calculate the perimeter by substituting in the variable values given in [brackets].
(iii) Write simplified expressions for the new perimeter if the vertical side of each rectangle was tripled $(\times 3)$.

c $[p = 3$ and $q = 4]$

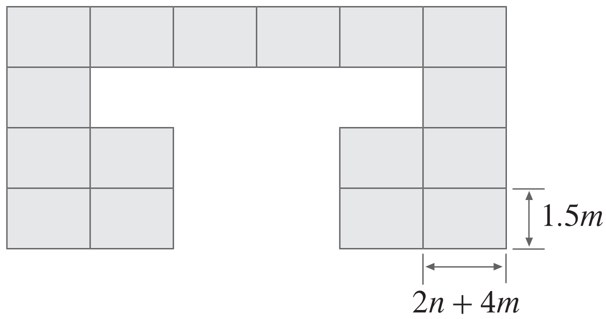


(i)

(ii)

(iii)

d $[m = 2$ and $n = -1]$



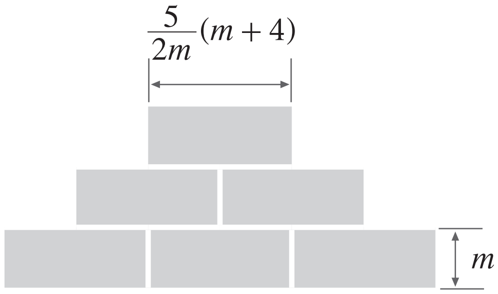
(i)

(ii)

(iii)

Algebraic calculations

3 Ignoring the space between the bricks, calculate the total area of the six, equal sized brick faces.



Follow these steps to solve the problem:

(i) Show by expanding and simplifying, that the **total area** of the six brick faces is given by:

Area of six brick faces $= 15m + 60$ units^2

Hint: multiply $\frac{5}{2m}$ by m first and simplify before expanding

(ii) Factorise the expression for the total area.

(iii) Use substitution to calculate the area of the six brick faces when $m = 3.6$ units.

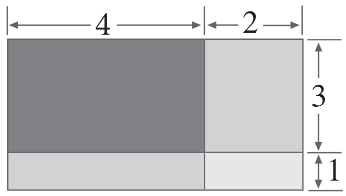
Product of two parentheses

This factorisation method is a key to unlock future Algebra work.

When finding the area of simple shapes, we usually find the length of each side separately first.

For this shape made up of four different rectangles:

(i) Calculate the length of each side of the large rectangle formed.



Length of the long side $= 4 + 2$
$= 6$

Length of the short side $= 3 + 1$
$= 4$

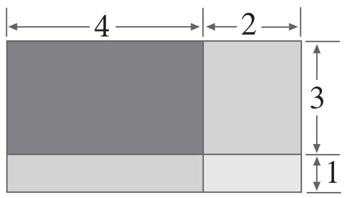
(ii) Calculate the area of the whole shape:

Area of Rectangle = long side × short side
$= 6 \times 4$
$= 24$ units^2

Looking at the same problem, this time we will write the answer in a different way.

For this shape made up of four different rectangles:

(i) Write down a sum expression for the length of each side.



Length of the long side $= 4 + 2$ Length of the short side $= 3 + 1$

(ii) Write an expression that will find the area of the whole shape and has two pairs of parentheses.

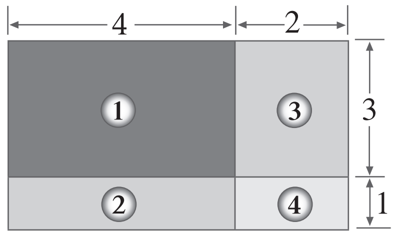
Area of Rectangle = long side × short side
$= (4 + 2) \times (3 + 1)$ units^2 Put in parentheses so the sums are done first
$= (4 + 2)(3 + 1)$ units^2 Simplify by writing without '×' sign

Let's take a look at the question one more time to see **another** way to write the area.

This time we will calculate the area of each small rectangle and add them together.

For this shape made up of four different rectangles:

(i) Calculate the area of each small rectangle.



① Area $= 4 \times 3$
$= 12$ units^2

② Area $= 4 \times 1$
$= 4$ units^2

③ Area $= 2 \times 3$
$= 6$ units^2

④ Area $= 2 \times 1$
$= 2$ units^2

(ii) Calculate the total area by adding the areas of the smaller rectangles together.

Total Area = Area ① + Area ② + Area ③ + Area ④
$= 12 + 4 + 6 + 2$ units^2
$= 24$ units^2

(iii) Combine the part (ii) answer with the previous example to write two expressions for the area.

Area of Rectangle	$= (4+2)(3+1)$ units^2	Factorised form with two pairs of parentheses
	$= 12 + 4 + 6 + 2$ units^2	Parentheses converted to expanded form
	$= 24$ units^2	Simplified form

Can you see the relationship between the factorised form and expanded form?

$$(4+2)(3+1) = 12 + 4 + 6 + 2$$
$$= 4 \times 3 + 4 \times 1 + 2 \times 3 + 2 \times 1$$
$$= (4+2)(3+1)$$

The name in Mathematics for expressions written like $(4+2)(3+1)$ is a:

Binomial product

The name for writing the binomial product like $12 + 4 + 6 + 2$ is a:

Binomial expansion

Product of two parentheses

For each of these questions:

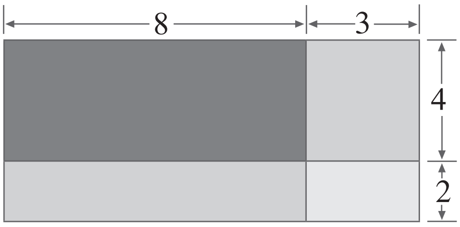
(i) Write down a sum expression for the length of each side.

(ii) Write an expression that will find the area of the whole shape **and** contains two parentheses.

(iii) Write an expression for the total area by adding together the area of each smaller rectangle.

(iv) Combine the answers to part (ii) and (iii) to write the area in factorised and expanded forms.

1



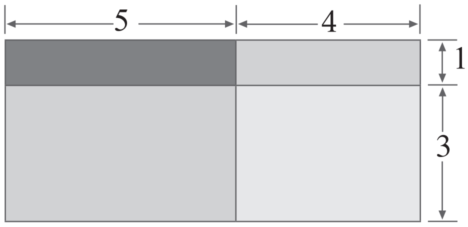
(i)

(ii)

(iii)

(iv)

2



(i)

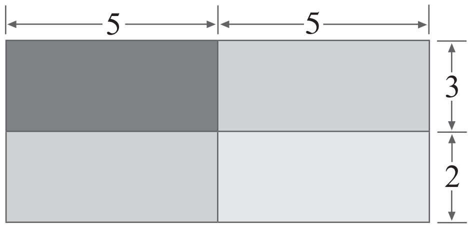
(ii)

(iii)

(iv)

Product of two parentheses

For each of these questions:
(i) Write down a sum expression for the length of each side.
(ii) Write an expression that will find the area of the whole shape **and** contains two parentheses.
(iii) Write an expression for the total area by adding together the area of each smaller rectangle.
(iv) Combine the answers to part (ii) and (iii) to write the area in factorised and expanded forms.

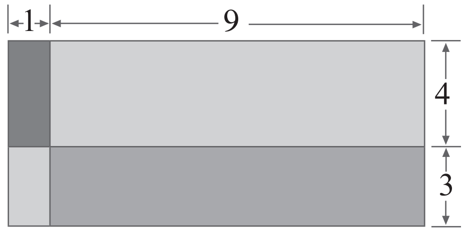
3 

(i)

(ii)

(iii)

(iv)

4 

(i)

(ii)

(iii)

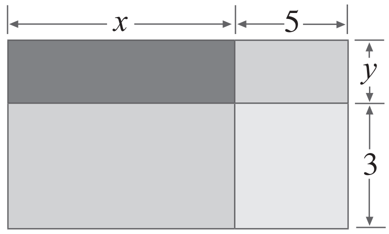
(iv)

Product of two parentheses with algebraic terms

Here is the exact same thing but with algebra in it.

For this rectangle made up of four smaller rectangles:

(i) Write down the **sum** for the length of each side.



Length of the long side $= x + 5$ Length of the short side $= y + 3$

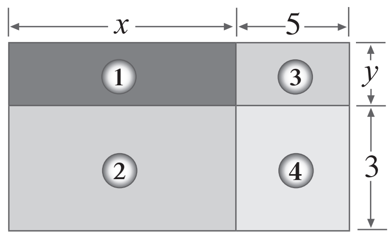
(ii) Write an expression for the area of the whole shape that contains two pairs of parentheses.

Area of Rectangle = long side × short side

$= (x + 5) \times (y + 3)$ units^2 Put in parentheses so the sums are done first

$= (x + 5)(y + 3)$ units^2 Simplify by writing without '×' sign

(iii) Write an expression for the total area by adding together the area of each small rectangle.



① Area $= x \times y$ $= xy$ units^2

② Area $= x \times 3$ $= 3x$ units^2

③ Area $= 5 \times y$ $= 5y$ units^2

④ Area $= 5 \times 3$ $= 15$ units^2

Total Area = Area ① + Area ② + Area ③ + Area ④

$= xy + 3x + 5y + 15$ units^2

(iv) Combine the answers to part (ii) and (iii) to write the area in both factorised and expanded forms.

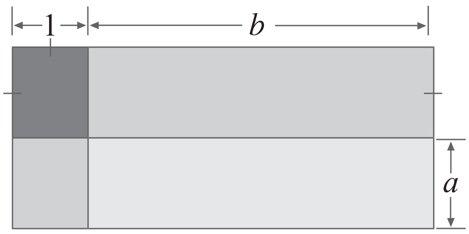
$\therefore$ Area of Rectangle $= (x + 5)(y + 3)$ units^2 Factorised form

$= xy + 3x + 5y + 15$ units^2 Expanded form

$\therefore (x + 5)(y + 3) = xy + 3x + 5y + 15$

Product of two parentheses with algebraic terms

1



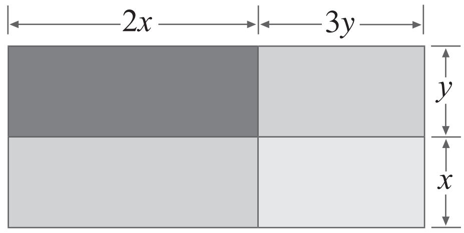
(i)

(ii)

(iii)

(iv)

2

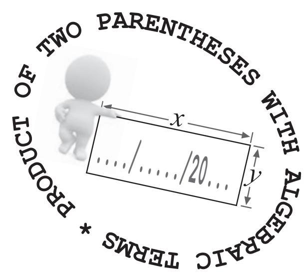


(i)

(ii)

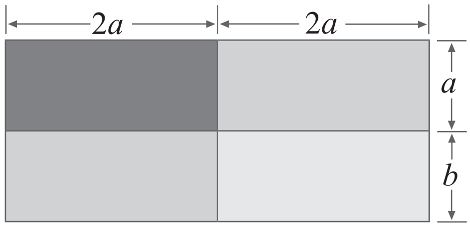
(iii) psst!: remember you can simplify if there are like terms

(iv)



Product of two parentheses

3



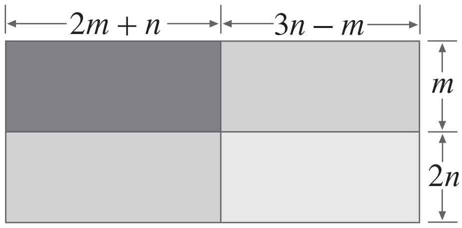
(i)

(ii)

(iii)

(iv)

4



(i)

(ii)

(iii)

(iv)

Expansion of two parentheses

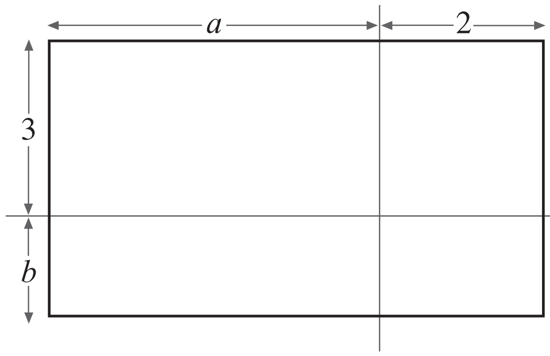
Drawing the areas represented by the product of two parentheses can assist with their expansion.

Starting with the area expression $(a+2)(3+b)$ units^2:

(i) Divide and label the large rectangle into four smaller rectangles using the area expression.

$$\text{Area of Rectangle} = \text{length} \times \text{width}$$
$$= (a+2)(3+b)$$

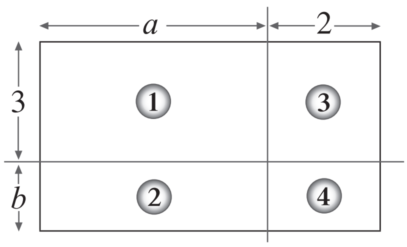
$\therefore$ One side of the rectangle $= a+2$ Other side of the rectangle $= 3+b$



A neat sketch is all that is needed. Just ensure that:

- The sides are correctly labelled, and
- the numerical lengths 'look' right.

(ii) Write an expression for the total area by adding together the area of each smaller rectangle.



Area ① $= 3a$ units^2

Area ② $= ab$ units^2

Area ③ $= 6$ units^2

Area ④ $= 2b$ units^2

Total Area $= 3a + ab + 6 + 2b$ units^2

(iii) Use the total area expression to write $(a+2)(3+b)$ in expanded form.

$$\therefore (a+2)(3+b) = 3a + ab + 6 + 2b$$

Expansion of two parentheses

For each of these questions:

(i) Divide and label each of these into four rectangles to represent the given product.
Sketch the divisions only

(ii) Write an expression for the total area by adding together the area of each smaller rectangle.

(iii) Use your answer to part (ii) to write the area expression in expanded form.

1. Area $= (x+3)(y+4)$ units^2

(i)

(ii)

(iii)

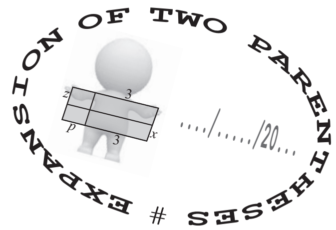
2. Area $= (1+m)(n+6)$ units^2

(i)

(ii)

(iii)

Expansion of two parentheses



For each of these questions:

(i) Divide and label each of these into four rectangles to represent the given product.
Sketch the divisions only

(ii) Write an expression for the total area by adding together the area of each smaller rectangle.

(iii) Use your answer to part (ii) to write the area expression in expanded form.

3 Area = $(2x+3)(x+2)$ units^2

(i)

(ii)

(iii)

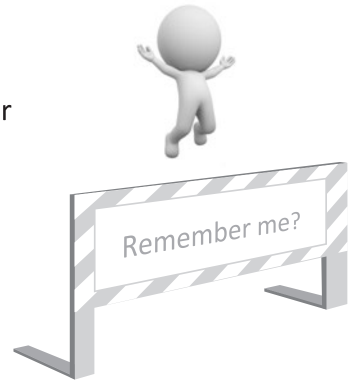
4 Area = $(4a+b)(a+2b)$ units^2

(i)

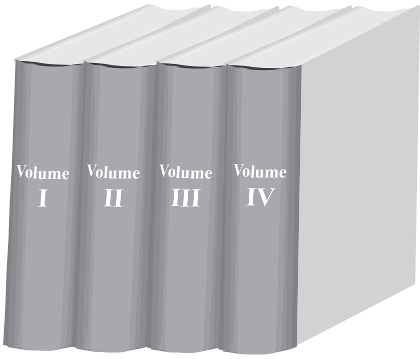
(ii)

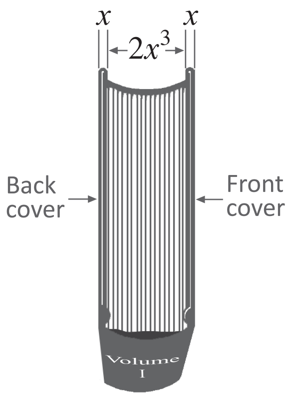
(iii)

Stuck in a book!



The picture below shows the first four volumes from a set of ten books in order on a shelf. All volumes are exactly the same size and all lengths are in units.





A small insect got caught **inside** the **front cover** of Volume I and ate through the books until it reached the **last page** of Volume IV.

1. Which expressions below represents the total distance the insect has travelled eating its way through the books to the last page of Volume IV?
   Work it out here!

   a $7x^4$ units   b $x(3+8x^2)$ units   c $2x(3+2x^2)$ units   d $8x(1+x^2)$ units

2. How far would the insect have travelled to get to the same destination (last page of Volume IV), if Volume I was incorrectly placed between Volumes VI and VII?
   Work it out here!

   a $6x(1+2x^2)$ units   b $2x(4x^2+3)$ units   c $2x(4+3x^2)$ units   d $8x(x^2+1)$ units

3. How far did it travel while eating in question 2 if $x = 1.5$?

Reflection Time

1. Reflect on the previous question by answering the three points below:

   - Think about your approach to this problem and write down any assumptions you made when answering this question that were incorrect.
   - What could you do to avoid making similar errors in future?
   - If you found the wording confusing, how would you have asked the same question?

2. Where do you think the skills you have learnt here will be useful and why?

Here is a summary of the things you need to remember for Expanding and Factorising

Expanding

Writing the same algebraic expression without parentheses.
This is achieved using the Distributive law:

The Distributive Law

Positive Number out the front

$$a(b+c) = a \times b \text{ and } a \times +c$$
$$= ab + ac$$

$$a(b-c) = a \times b \text{ and } a \times -c$$
$$= ab - ac$$

Negative Number out the front

$$-a(b+c) = -a \times b \text{ and } -a \times +c$$
$$= -ab - ac$$

$$-a(b-c) = -a \times b \text{ and } -a \times -c$$
$$= -ab + ac$$

Expanding and simplifying

After expanding, sometimes like terms can be collected to simplify the expression.

Highest common factor

The highest common factor (HCF) is the largest term that divides **exactly** into all the given terms.

Factorising

This is the opposite of expanding. Find the HCF then re-write the expression with the HCF out the front of a pair of parentheses and put the remaining parts inside.

$$2jk - 4j = 2j(\ ?\ )$$

HCF

Put the HCF out the front of a pair of parentheses

$$k - 2$$

Find what the HCF is multiplied by to get each term

Because $2j \times k = 2jk$ $2j \times (-2) = -4j$

$$\therefore 2jk - 4j = 2j(k-2)$$

Algebraic calculations

All the skills listed above can be used to simplify problems before performing calculations.

Mathletics

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