WORDS YOU SHOULD KNOW

chance
experimental probability
event
likelihood
outcome
probability
probability generator
risk
sample
sample space
theoretical probability
tree diagram

The following table shows the percentage of the population living in the main urban areas of New Zealand in 1999.

If you met a person from New Zealand when overseas, what would be the estimated chance (probability) that the person lived in:

a Hamilton

b South Island urban area

Main Urban Area	*% population*
Auckland	28.6%
Wellington	9.1%
Christchurch	8.9%
Hamilton	4.4%
Napier/Hastings	3.0%
Dunedin	2.9%

a Since 4.4% of the population lives in Hamilton then the chance is 0.044

b Christchurch and Dunedin are in the South Island,

so the chance $= 2.9\% + 8.9\%$
$= 11.8\%$
$= 0.118$

EXERCISE 14C

1 For the data in **Example 3** above:

a Which urban area has the largest population?

b Which has the smallest population?

c Estimate the chance (probability) that a New Zealand person you met at random came from: **i** Napier/Hastings **ii** a North Island main urban area

d Estimate the probability that a New Zealand person met at random does not come from a main urban area in New Zealand.

2 Use the given table to estimate the probability that the next person you meet is:

- **a** Chinese
- **b** New Zealand Maori
- **c** Not European / Pakeha
- **d** from Asia

Ethnic Groups within New Zealand by %	
European/Pakeha	79.6%
New Zealand Maori	14.5%
Pacific Islands	5.6%
Chinese	2.2%
Indian	1.2%

3

2001 attendance at cinemas for persons aged 15 and over by age group	
Age group	*Percentage of total*
15 to 24	27.3
25 to 34	21.8
35 to 44	18.5
45 to 54	17.2
55 to 64	8.3
65+	6.9

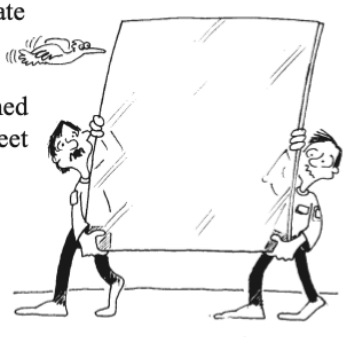
If you met a person aged over 14 who attended the cinema in 2001, what is your estimate that the person will be:

- **a** between 45 and 54 years of age
- **b** less than 35 years of age
- **c** over 54 years of age
- **d** between 25 and 54 years of age?

4 637 randomly chosen people were asked how many children were in their family. The results are shown in the table:

Number of children	*Frequency*
0	217
1	218
2	124
3	52
4	17
5	5
6 or more	4

- **a** What was the sample size?
- **b** Copy the table and add to it a 'Relative frequency' column.
- **c** Assuming that the results are representative of New Zealand family sizes, estimate the probability that a randomly selected family has:
  - **i** 2 children
  - **ii** less than 3 children
  - **iii** 3 or more children.
- **d** If a city has 5630 families, what is your best estimate of the number of families with 4 or more children?



5 Fifty window-sized sheets of glass were carefully examined for observable flaws (errors). The number of flaws per sheet was recorded below:

0	1	0	2	1	2	0	3	0	1	0	1	3
1	0	1	0	5	1	0	0	1	0	0	0	0
1	4	1	1	0	0	0	0	2	1	0	0	
0	1	0	0	0	0	0	1	2	0	0	1	

a Complete a Frequency/Relative frequency table for this information.

b What is the probability that, if examined, a sheet of glass will have:

i no flaws **ii** 1 flaw **iii** 8 flaws

iv less than 2 flaws **v** more than 3 flaws?

Tree diagrams

WALT draw tree diagrams

Success Criteria I know..

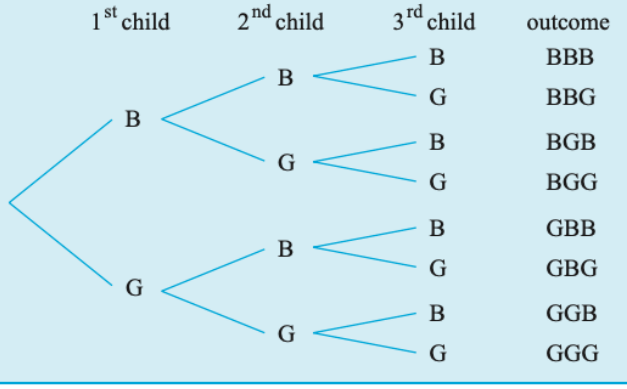
- How to list progression of events

A very good way to find all possible outcomes of an experiment is to draw a **tree diagram**, which if done correctly automatically gives you all the outcomes.

The first set of 'branches' we see show the first set of choices. The second set of 'branches' show the next set of choices, and so on.

Example 5

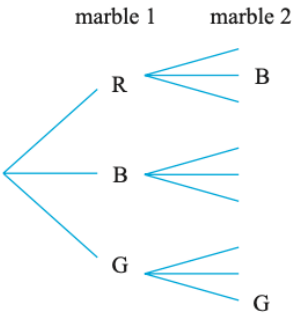
Draw a tree diagram to find all possible outcomes for a three child family.



EXERCISE 14E

1 A bag contains three red, five blue and six green marbles. Two marbles are selected one after the other without being replaced.

a Copy and complete the tree diagram alongside

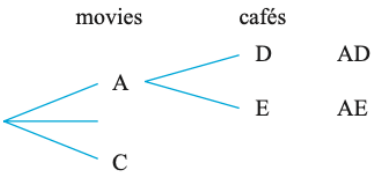


b List all the possible outcomes for the two marbles.

c How many outcomes had the two marbles the same colour?

2 Three friends, Marcia, Kim and Tamara wanted to go to a movie and then to a café for a cup of coffee. They had to choose from three movies A, B and C and two cafés D and E.

a Copy and complete the tree diagram below.



b List all the possible outcomes of movie and café the friends could have gone to.

c How many of these outcomes include movie B and café D?

3 Your class has to elect a class councillor and a deputy for the school council. Five students (A, B, C, D and E) have been nominated by your class and the vote is about to take place. The candidate with the highest vote will be the councillor and the next highest vote will be the deputy.

a Draw a tree diagram to show this situation, remembering it will need two tiers.

b How many possible outcomes are there?

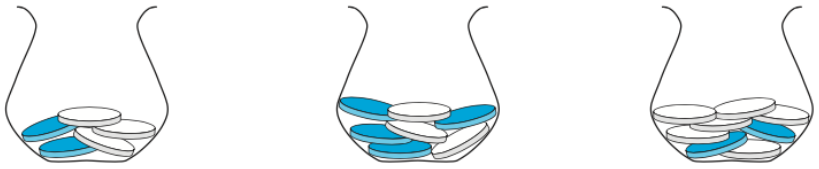
c How many of the outcomes had D as the deputy?

EXERCISE 14F

1 Blue and white discs are placed in a bag and one disc is randomly selected from it. For the following bags of discs given, answer the following questions:

- **i** How many of each disc are there in the bag?
- **ii** What is the probability of selecting a blue disc?
- **iii** What is the probability of selecting a white disc?

a **b** **c**



Example 7

A die has 4 faces painted blue and 2 faces painted grey. When the die is rolled, what is the chance that the uppermost face is:

a a blue **b** a grey?

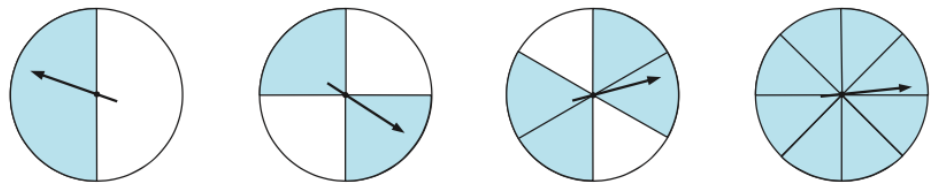
4 faces are blue, 2 faces are grey, 6 faces in all.

a $\Pr(\text{a blue}) = \frac{4}{6}$ ← total blue faces / total outcomes possible

b $\Pr(\text{a grey}) = \frac{2}{6}$

2 Determine the probability that the spinning needle will finish on blue in:

a **b** **c** **d**



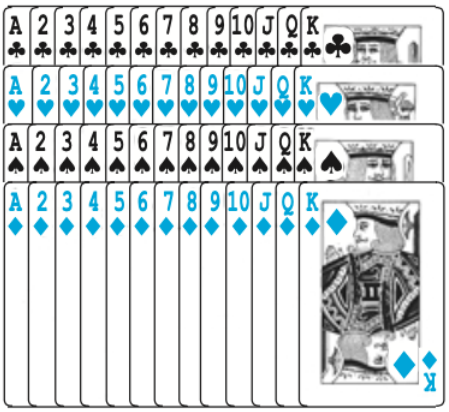
3 Consider the illustrated spinner (a regular octagon).
If the spinner is spun once, find the probability of getting:

a a 6 **b** a 3 or a 4 **c** a 1, 2 or 3
d a result less than 6 **e** a result more than 8

4 A hat contains 4 red, 3 white and 2 grey discs and one disc is randomly selected from it. Determine the likelihood that it is:

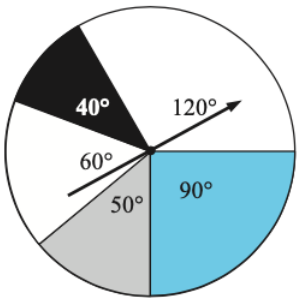
a red **b** white **c** grey
d green **e** not red **f** not white
g not grey **h** not red or grey **i** red, white or grey

5 This illustration shows a full pack of playing cards. For this exercise, the pack is well shuffled and placed face down. Hearts and diamonds are blue. Spades and clubs are black. Jacks, Queens and Kings are called picture cards. Frank picks one card at random from the shuffled pack. Determine the chance of getting:



a a heart ♥ **b** the 7 of ♥
c a club ♣ **d** a red 4
e a black ace **f** a 5 or a 6
g an ace **h** a picture card

6



The given spinner has sector angles of 120^o, 90^o, 50^o, 60^o and 40^o.

a After a spin, are the outcomes equally likely?
b What outcome do you expect to occur:
i most often **ii** least often?
c What is the probability of getting a blue?
d If you spin the spinner 1000 times, how often would you expect it to finish on the:
i blue **ii** white?

7 List the sample space when a 5-cent and a 20-cent coin are tossed simultaneously. Let HT represent "a head with the 5-cent coin and a tail with the 20-cent coin". Hence, if these coins are tossed simultaneously, determine the chance of getting:

a two heads **b** two tails **c** exactly one head **d** at least one head

8 List the possible two-child families, letting B represent a boy and G represent a girl. If these families occur with equal chance, determine the probability that a randomly selected two-child family consists of:

a two boys **b** at least one boy **c** children of the same sex

9 List the 8 possible 3-child families according to sex. One of them is GBB. Assuming that each of these is equally likely to occur, determine the probability that a randomly chosen 3-child family consists of:

a all boys **b** all girls **c** boy, then girl, then girl
d two girls and a boy **e** a girl for the eldest **f** at least one boy

10 3 seats are placed in a row and three children A, B and C enter the room and sit randomly on the chairs (one on each chair). List the sample space of all possible orderings. Hence, determine the probability that:

a A sits on the leftmost chair **b** they sit in order, BCA from left to right
c C sits in the middle **d** B does not sit in the middle

We can use tree diagrams to calculate probabilities. Earlier in the chapter you learnt how to construct a tree diagram. The next example shows you how to use it effectively.

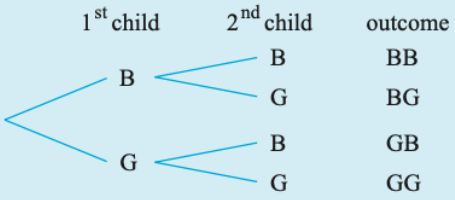
Example 8

From the tree diagram for a two child family, find the probability that the children will be:

a two boys

b at least one boy

c the same sex.



From this we can list four the possible outcomes. {BB BG GB GG}

a The probability of getting two boys is 1 out of 4 or $\frac{1}{4}$ since BB happens only once.

b The probability of at least one boy is 3 out of 4 or $\frac{3}{4}$ since BB, BG, GB, all have one or more boys.

c The probability of children of the same sex is 2 out of 4 or $\frac{2}{4}$ since BB and GG satisfy the question.

Check your answers with **Exercise 14F** question **8**.

EXERCISE 14G

Answer the following questions by drawing a tree diagram in each case.

1 A die is rolled and then a coin tossed.

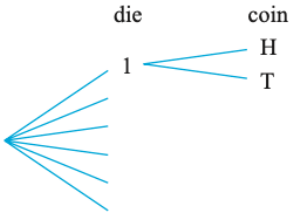
a How many outcomes are there in total?

b What is the probability that:

i a 4 and a head appears

ii an even number and a tail appears

iii a 3 or a 5 and a head appears?



2 A restaurant offers three main courses and two desserts on its menu. The main courses are lamb (L), chicken (C) and fish (F) while the desserts are pavlova (P) and fruit salad (S). I randomly choose one main course and one dessert.

a How many outcomes are there for a meal consisting of one main and one dessert?

b What is the probability that I have

i fish followed by pavlova?

ii lamb or chicken and fruit salad?

c What is the probability that I do not have lamb?

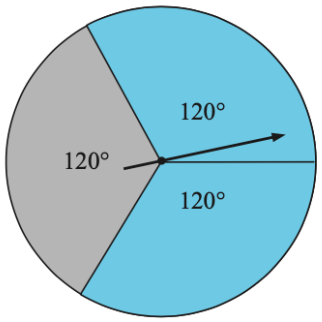
3 Repeat **Exercise 14F**, question **10** by drawing a tree diagram and using it to answer the questions. Which method do you find easier?

Suppose you wish to make a device which generates probabilities of $\frac{2}{3}$ and $\frac{1}{3}$,

that is, one event has a $\frac{2}{3}$ chance of occurring and an alternative event with $\frac{1}{3}$ chance of occurring.

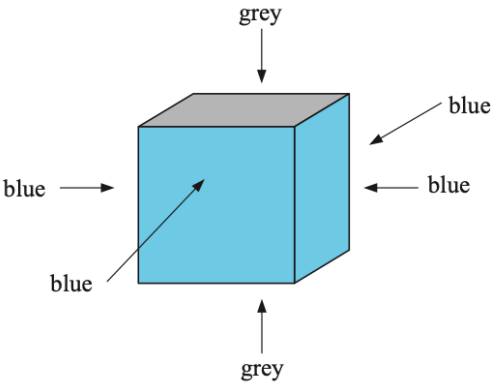
For example, $\Pr(\text{blue}) = \frac{2}{3}$ and $\Pr(\text{grey}) = \frac{1}{3}$.

We could use a **spinner** or a **die**.



120^o is $\frac{1}{3}$ of 360^o

240^o is $\frac{2}{3}$ of 360^o



4 faces are blue and
2 faces are grey

EXERCISE 14H

1 Design *two* devices which generate $\Pr(\text{A}) = \frac{1}{2}$ and $\Pr(\text{B}) = \frac{1}{2}$.

2 Design *two* devices which generate $\Pr(\text{red}) = \frac{2}{5}$ and $\Pr(\text{blue}) = \frac{3}{5}$.

3 Design *two* devices which generate $\Pr(\text{A}) = \frac{1}{6}$, $\Pr(\text{B}) = \frac{2}{6}$ and $\Pr(\text{C}) = \frac{3}{6}$.

4 Suppose you want to develop a way of generating probabilities in the ratio $1 : 2 : 4$ and do not wish to make a spinner.

a What simple way could you do this?

b If the things that are required are X, Y and Z in the ratio $1 : 2 : 4$, what are:

i $\Pr(\text{X})$ **ii** $\Pr(\text{Y})$ **iii** $\Pr(\text{Z})$?

5 Draw a circular dart-board (using three colours) where the chance of scoring:

a blue is $\frac{1}{2}$ and white is $\frac{1}{4}$

b blue is $\frac{1}{2}$ and white is $\frac{1}{3}$

c blue is $\frac{1}{4}$ and white is $\frac{1}{3}$

d blue is 0.2 and white is 0.3

e blue is 25% and white is 30%

f blue is 0 and white is $\frac{2}{3}$