# COMPOUND INTEREST

If the interest earned each year on an investment is reinvested, the size of the principal on which interest is calculated increases each year. Hence the amount of interest earned also increases. This is called **compound interest**.

### Compound interest formula

$$A = P(1 + R)^n$$

where $\$P$ is the initial amount invested (principal)

$R = \frac{r}{100}$ and $r$ is the percentage interest rate per compounding period

$n$ is the number of compounding periods

$\$A$ is the final amount (includes principal and interest).

Use the compound interest formula to calculate:

**a** the amount to which \$10 000 grows if invested for 8 years at 4.5% p.a. compound interest

**b** the amount of interest earned over this period.

| | Solve | Think | Apply |
|---|---|---|---|
| **a** | $A = \$10\,000\left(1 + \frac{4.5}{100}\right)^8$<br>$= \$10\,000(1.045)^8$<br>$= \$14\,221.01$ | $A = P(1 + R)^n$<br>$P = 10\,000$<br>$r = 4.5$<br>$n = 8$ | Substitute the values into the compound interest formula $A = P(1 + R)^n$.<br>The amount is always the original amount plus interest. |
| **b** | Interest $= \$14\,221.01 - \$10\,000$<br>$= \$4221.01$ | Subtract the original amount invested. | |

**1** Complete the following using the compound interest formula.

**a** Find the amount to which \$6000 grows if it is invested for 5 years at 3% p.a. compound interest.

$A = \_\_\_\left(1 + \frac{\square}{100}\right)^5 = \_\_\_(1.03)^{\square} = \_\_\_$

**b** The total amount of interest earned over this period = ___ − \$6000 = ___

**2** Use the compound interest formula to calculate:

**a** the amount to which \$18 000 grows if it is invested for 7 years at 6% p.a. compound interest

**b** the amount of interest earned over this period.

**3** **a** If I invest \$25 000 at 6.5% p.a. compound interest, how much will I have in 10 years time?

**b** Calculate the amount of interest earned over this period.

**4** **a** If I invest \$5000 at 4.7% p.a. compound interest, how much will I have in 8 years time?

**b** Calculate the amount of interest earned over this period.

Use the compound interest formula to calculate the amount to which \$10 000 grows if it is invested for 5 years at 9% p.a. interest, compounded:

**a** monthly **b** quarterly **c** six-monthly.

| | Solve | Think | Apply |
|---|---|---|---|
| **a** | $A = \$10\,000\left(1 + \frac{0.75}{100}\right)^{60}$<br>$= \$10\,000(1.0075)^{60}$<br>$= \$15\,656.81$ | In this case the time period is monthly.<br>Hence $r = \frac{9}{12}\% = 0.75\%$ per month<br>and $n = 5 \times 12 = 60$ months. | The compound interest formula $A = P(1 + R)^n$ where $R = \frac{r}{100}$ can be adapted for this question by using $r$ as the interest rate per time period and $n$ as the number of time periods.<br>*Note:* The more often the interest is reinvested, the greater the final value of the investment. |
| **b** | $A = \$10\,000\left(1 + \frac{2.25}{100}\right)^{20}$<br>$= \$10\,000(1.0225)^{20}$<br>$= \$15\,605.09$ | The time period is quarterly.<br>Hence $r = \frac{9}{4}\% = 2.25\%$ per quarter<br>and $n = 5 \times 4 = 20$ quarters. | |
| **c** | $A = \$10\,000\left(1 + \frac{4.5}{100}\right)^{10}$<br>$= \$10\,000(1.045)^{10}$<br>$= \$15\,529.69$ | The time period is six-monthly.<br>Hence $r = \frac{9}{2}\% = 4.5\%$ per 6 months<br>and $n = 5 \times 2 = 10$ six-monthly periods. | |

**5** Complete the following using the compound interest formula to calculate the amount to which \$10 000 grows if it is invested for 3 years at 12% p.a. interest, compounded:

**a** monthly: $r = \frac{\square}{12}\% =$ ___% per month and $n = 3 \times$ ___ = ___ months

$A = 10\,000\left(1 + \frac{\square}{100}\right)^{\square} = 10\,000($___$)^{36} =$ ___

**b** quarterly: $r = \frac{12}{\square}\% =$ ___ per quarter and $n =$ ___ $\times 4 =$ ___ quarters

$A = 10\,000\left(1 + \frac{\square}{100}\right)^{12} = 10\,000($___$)^{12} =$ ___

**c** six-monthly: $r = \frac{\square}{2}\% =$ ___ per 6 months and $n =$ ___ $\times 2 =$ ___ six-month periods

$A = 10\,000\left(1 + \frac{\square}{100}\right)^{\square} = 10\,000($___$)^{6} =$ ___

**6** Use the compound interest formula to calculate the amount to which \$5000 grows if it is invested for 4 years at 6% p.a. interest, compounded:

**a** monthly **b** quarterly **c** six-monthly

**7** **a** Use the compound interest formula to calculate the amount to which \$20 000 grows if invested for 5 years at 3% p.a. interest, compounded:

**i** monthly **ii** quarterly **iii** six-monthly

**b** Calculate the amount of interest earned in each case of part **a**. Which time period of compounding (reinvesting) gives the greatest amount of interest?

**8** Which of the following investments will earn the greatest amount of interest?

**A** \$5000 invested at 6% p.a. for 4 years interest compounded yearly

**B** \$5000 invested at 6% p.a. for 4 years interest compounded six-monthly

**C** \$5000 invested at 6% p.a. for 4 years interest compounded quarterly

**D** \$5000 invested at 6% p.a. for 4 years interest compounded monthly

**E** \$5000 invested at 6% p.a. for 4 years simple interest

It is possible to obtain cash advances, up to a certain limit, using a credit card. In these cases compound interest is charged daily from the time the cash is withdrawn. Calculate the interest charged on a cash advance of $400 for 27 days when the annual interest rate is 16%.

| Solve | Think | Apply |
|---|---|---|
| $A = \$400\left(1 + \left(\frac{16}{365} \div 100\right)\right)^{27}$<br>$= \$404.76$ (to the nearest cent)<br>Interest charged $= \$404.76 - \$400 = \$4.76$ | Use the compound interest formula:<br>daily interest rate $= \frac{16}{365}\%$ | Divide the percentage rate by 365 to make it daily, then by 100 to make it a decimal. |

**9** Complete the following to find the interest charged on a credit card cash advance of $500 for 11 days when the annual interest rate is 23%.

Daily interest rate $= \frac{\square}{365}\%$

Using the compound interest formula:

$A = \_\_\_\left(1 + \left(\frac{\square}{\square} \div 100\right)\right)^{\square}$

$= \_\_\_$

Interest charged $= \_\_\_$