Working with Decimals and Factions

Skills in working with fractions and decimals are important in all aspects of mathematics, so it is useful to revise and practise them.

EXPLANATION

Converting fractions to decimals

Fractions and decimals are used to represent parts of wholes. They are two different ways of writing a number. A fraction has a line or bar, which separates the top number (the numerator) from the bottom number (the denominator). This line can be thought of as a division sign, ÷.

So, $\frac{1}{2} = 1 \div 2$, or 1 shared between 2. If we perform this division, we obtain the decimal equivalent for $\frac{1}{2}$, 0.5.

$\frac{9}{5}$ means 9 wholes shared between 5, or $9 \div 5$, which is $1\frac{4}{5}$ or 1.8.

To convert a fraction to a decimal, perform the division; e.g. $\frac{3}{4} = 3 \div 4 = 0.75$.

Some fractions are used so often that it is useful to know the decimal equivalent. If you don't already know them, try to learn the following.

Common fractions and their decimal equivalents

$\frac{1}{2}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{5}$	$\frac{1}{8}$	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
0.5	$0.\dot{3}$	$0.\dot{6}$	0.25	0.75	0.2	0.125	0.1	0.01	0.001

Notice that the decimal equivalents for $\frac{1}{3}$ and $\frac{2}{3}$ have a dot above the decimal digit. This is to show that the digit is repeated forever; i.e. $\frac{1}{3} = 0.333\ldots$ These types of decimals are called **recurring decimals**. We will learn more about these in the next section.

In order to compare quantities expressed as fractions and decimals we need to change all values to the same format. It is often easier to convert all values to decimal form.

Fractions and decimals can most easily be compared by first converting the fractions to decimals.

Worked Example 1

WE1

Arrange the following list into ascending order (smallest to largest) by converting the fraction values to decimals.

$\frac{5}{8}, \frac{3}{5}, \frac{3}{4}$, 0.69, 0.686

Thinking	Working
1 Convert each of the fractions into decimals. (You could use your calculator for this step.)	$\frac{5}{8} = 5 \div 8 = 0.625$ $\frac{3}{5} = 3 \div 5 = 0.6$ $\frac{3}{4} = 3 \div 4 = 0.75$
2 List the numbers in order from smallest to largest, by comparing the decimal digits in each place value column.	0.6, 0.625, 0.686, 0.69, 0.75
3 Substitute the fraction values back into the list.	$\frac{3}{5}, \frac{5}{8}$, 0.686, 0.69, $\frac{3}{4}$

Converting decimals to fractions

Decimals use place value to represent the fractional parts of a number. Knowledge of place value allows you to convert decimals to fractions.

For example, we can write the number 13.2452 in a place value table, like this:

tens	ones		tenths	hundredths	thousandths	ten-thousandths
10	1	•	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$	$\frac{1}{10\,000}$
1	3	.	2	4	5	2

From this, 13.2452 can be written in expanded fractional form as

$13 + \frac{2}{10} + \frac{4}{100} + \frac{5}{1000} + \frac{2}{10\,000}$ or $13\frac{2452}{10\,000}$ (which simplifies to $13\frac{613}{2500}$ when we cancel a common factor of 4).

To convert a decimal to a fraction:

- use the place value column of the last digit to get the denominator of the fraction
- write all the digits of the decimal part in the numerator
- simplify the fraction if possible.

Further explanation

Worked Example 2

WE 2

Write the following decimal as a fraction in simplest form: 0.384

Thinking	Working
1 The last digit in the decimal is in the thousandths column, so 1000 is the denominator. Write the other digits as the numerator.	$0.384 = \frac{384}{1000}$
2 Simplify the fraction. (Here, we have cancelled common factors of 4 and 2.)	$= \frac{96}{250}$ $= \frac{48}{125}$

Fractions and decimals on the calculator

Scientific calculators have keys that enable you to enter fractions and convert between fraction and decimal form. They will usually look something like this [▣] or this [$a\frac{b}{c}$].

To enter a fraction, either:

- press [▣], then key in the numerator and the denominator, using the cursor to move between them; or key in the numerator, press [$a\frac{b}{c}$], then key in the denominator.

For example, to enter $\frac{13}{25}$, you would press [▣] [1] [3] [⊛] [2] [5] [=] or press [1] [3] [$a\frac{b}{c}$] [2] [5] [=].

To enter a mixed number, such as $1\frac{3}{7}$, either:

- press [SHIFT] [▣] [1] [⊛] [3] [⊛] [7] [=] or
- press [1] [$a\frac{b}{c}$] [3] [$a\frac{b}{c}$] [7] [=].

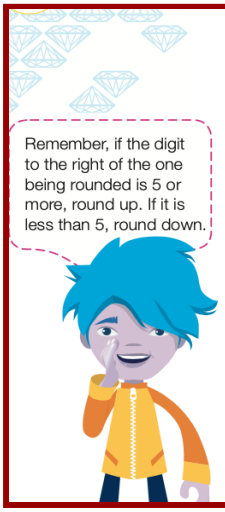
Pressing [S⇔D] or [$a\frac{b}{c}$] will switch a number entered in fraction form to decimal form, and vice versa.

Pressing [SHIFT] [S⇔D] or [SHIFT] [$a\frac{b}{c}$] will convert an improper fraction to a mixed number.

Rounding decimals

Some decimal numbers have a lot of decimal places. Often we don't need all of the decimal places or it doesn't make sense to use them. For example, we might calculate that we need 1.697 214 metres of wood for a project. The third decimal digit represents millimetres (thousandths of a metre). The fourth, fifth and sixth decimal places represent such a tiny amount that they are impossible to measure accurately and are not necessary. In these cases, we 'round' the number to a certain number of decimal places.

It is often important to state how many decimal places a number has been rounded to. We do this by using the abbreviation 'd.p.' We can also say that a rounded number has been written 'correct to' a certain number of decimal places.



To round a number to a given number of decimal places, look at the digit to the right of the one being rounded. If it is:

- 0, 1, 2, 3 or 4, round down. Keep the digit in the place to be rounded the same, and drop the digits following it;

  e.g. 1.697 214 rounded to 3 d.p. is 1.697.

- 5, 6, 7, 8 or 9, round up. Increase the digit in the place to be rounded by 1, and drop the digits following it. If it is a 9, write the zero of the 10, and carry the one across to the next column;

  e.g. 1.789 24 rounded to 2 d.p. is 1.79
  24.5999 rounded to 3 d.p. is 24.600

Working with remainders

Division calculations do not always give us neat, whole number answers. On a calculator, remainders are usually given in decimal form. It is important to be able to interpret what the remainder means for that particular question.

Worked Example 3 WE3

Use a calculator, if necessary, to answer the following question. A length of wood is $2\frac{1}{2}$ m long. How many pieces, each 600 mm long, can be cut from this length, and how much will be left over? Ignore any wastage that might occur in cutting.

Thinking	Working
1 Write the amounts using the same unit. (In most cases, the smaller unit is more convenient.)	$2\frac{1}{2}$ m = 2500 mm
2 Determine the operation required. 'How many' tells us this is a division question. Perform the division.	25ØØ ÷ 6ØØ = 25 ÷ 6 = 4.166 666 667
3 Write down the whole number of pieces.	4
4 Multiply the whole number of pieces (4) by the size of each piece (600 mm) and use this to calculate the number of mm left over.	4 × 600 = 2400 2500 − 2400 = 100
5 Write the answer.	We will get 4 pieces 600 mm long and will have 100 mm of wood left over.

In the Worked Example above, the decimal remainder after the division represents the length of wood left over after all the pieces of the required length (four 600 mm pieces) had been cut off. 0.166 666 667 of 2500 m is 100 mm.

Fluency

1 Arrange the following lists into ascending order (smallest to largest) by converting the fraction values to decimals. WE1

(a) $\frac{2}{5}$, 0.399, $\frac{4}{5}$, 0.382, $\frac{3}{4}$

(b) $\frac{9}{10}$, $\frac{9}{8}$, 0.88, 0.89, 0.899

(c) $\frac{1}{8}$, 0.112, 0.099, $\frac{1}{4}$, 0.07

(d) $\frac{2}{5}$, $\frac{1}{3}$, 0.3, $\frac{3}{8}$, 0.2

(e) $\frac{1}{2}$, 0.555, 0.58, $\frac{3}{5}$, 0.55

(f) 0.291, 0.302, $\frac{3}{10}$, $\frac{2}{9}$, $\frac{2}{3}$

(g) $2\frac{4}{5}$, $2\frac{3}{4}$, 2.278, 2.932, $2\frac{9}{10}$

(h) $1\frac{1}{5}$, $1\frac{2}{3}$, $1\frac{3}{8}$, 1.029, 1.243

(i) $4\frac{1}{5}$, $4\frac{1}{4}$, 4.295, 4.199, 4.201

(j) $3\frac{1}{5}$, 3.45, 3.439, $3\frac{2}{3}$, 3.482

2 Write the following decimals as fractions in simplest form. WE2

(a) 0.8 **(b)** 0.05 **(c)** 0.002 **(d)** 0.0009

(e) 0.14 **(f)** 0.62 **(g)** 0.31 **(h)** 0.85

(i) 0.711 **(j)** 0.684 **(k)** 0.625 **(l)** 0.128

(m) 0.203 **(n)** 0.094 **(o)** 0.1560 **(p)** 0.7009

3 Use a calculator, if necessary, to answer the following questions. WE3

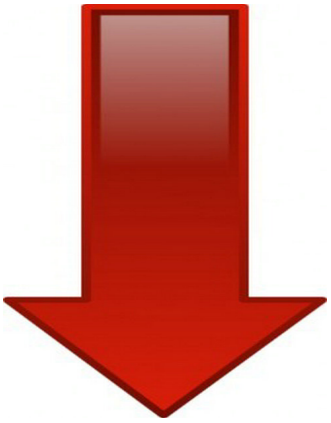
(a) A length of wood is 2.6 m long. How many pieces, each 40 cm long, can be cut from this length, and how much wood will be left over?

(b) A bag holds 750 g of flour. How many cups, each containing 120 g of flour, can be filled from the bag, and how much will be left over?

(c) A container holds $3\frac{1}{4}$ litres of juice. How many 200 mL cups can be filled from the container, and how much juice will be left over? (1 litre = 1000 mL)

(d) A bus can carry 52 passengers. If every bus except the last one is filled to capacity, how many buses will be needed to transport 650 people, and how many people will be on the last bus?

Challenge and Understanding



4 Write the following mixed numbers as decimals using a calculator where necessary.

(a) $51\frac{7}{10}$ **(b)** $5\frac{3}{20}$ **(c)** $7\frac{1}{8}$ **(d)** $67\frac{7}{25}$

(e) $39\frac{1}{16}$ **(f)** $28\frac{24}{200}$ **(g)** $14\frac{13}{80}$ **(h)** $24\frac{31}{400}$

5 Write the following decimals as mixed numbers in simplest form.

(a) 7.5 **(b)** 3.4 **(c)** 1.25 **(d)** 5.64

(e) 13.02 **(f)** 27.96 **(g)** 9.045 **(h)** 124.706

6 **(a)** $1\frac{5}{8}$ written as a decimal is:

A 0.16 **B** 1.58 **C** 1.625 **D** 1.63

(b) $2\frac{1}{3}$ written as a decimal, correct to two decimal places, is:

A 0.34 **B** 2.13 **C** 2.33 **D** 2.34

7 For each set of numbers, draw the number line below and indicate the position of each number with a labelled arrow. (Hint: First, determine the value of the smallest interval on the number line.)

(a) 0.5, 1.3, $1\frac{1}{2}$, $\frac{1}{5}$ **(b)** $\frac{2}{2}$, 0.8, 1.6, $1\frac{4}{5}$

8 For each set of numbers, draw the number line below and indicate the position of each number with a labelled arrow. (You may need to estimate the position of some numbers.)

(a) $\frac{3}{4}$, $\frac{7}{10}$, 0.34, 0.47, $1\frac{1}{3}$ **(b)** 1.05, $\frac{7}{8}$, 0.58, $\frac{8}{5}$, $\frac{6}{4}$

9 Use a calculator to convert the fractions to decimal form, then place a > (greater than), < (less than) or = (equal to) symbol between the pairs of numbers to make the following statements correct.

(a) 0.84 ____ $\frac{18}{21}$ **(b)** 2.29 ____ $2\frac{12}{39}$ **(c)** 0.912 ____ $\frac{114}{125}$

(d) 0.64 ____ $\frac{16}{25}$ **(e)** 1.83 ____ $1\frac{18}{23}$ **(f)** 0.97 ____ $\frac{98}{99}$

(g) $\frac{1}{3}$ ____ 0.3 **(h)** $\frac{2}{9}$ ____ 0.23 **(i)** $\frac{2}{3}$ ____ 0.67

For Question **10**, you can turn decimal numbers into whole numbers by multiplying by 10.

10 **(a)** Express each first quantity as a fraction of the second quantity. Write the fraction in simplest terms.

(i) first quantity 1.5 hours; second quantity 2.4 hours

(ii) first quantity 200 m; second quantity 322.5 m

(iii) first quantity 80.6 mL; second quantity 200 mL

(iv) first quantity 20.6 kg; second quantity 12.5 kg

(v) first quantity 18.6 m^2; second quantity 12 m^2

(vi) first quantity 14.4 g; second quantity 1.5 g

(b) Express each fraction from part **(a)** as a decimal, rounding your answers to three decimal places where necessary.

Further Challenge

11 Mr Scully is out buying materials to build the set for the school production. Calculate how much he will pay for the following materials. Round your answers to the nearest 5 cents.

(a) 6 lengths of timber, at $8.99 per length

(b) 9.4 metres of canvas, at $11.20 per metre

(c) 5.6 metres of ribbon, at $0.95 per metre

12 The exchange rate between the Australian and American dollars varies daily. Suppose that according to the current rate A$1 is worth US$0.72. How much are the following amounts worth in US dollars?

(a) A$10 **(b)** A$50 **(c)** A$100 **(d)** A$267 **(e)** A$1845

13 The most overdue library book ever was a copy of *Febrile Diseases* by Dr J. Currie. The book was borrowed from the University of Cincinnati Medical Library in 1823 by Mr M. Dodd, and was returned by his great-grandson in 1968. If the fines for late returns were $18.30 a year, how big was the fine his great-grandson had to pay?

14 The largest crab in the world is the giant spider crab, which is found off the south-eastern coast of Japan. It has a claw span of 2.74 m. If an average person has a width of 36 cm across the waist, how many people could fit in the claw span of the giant spider crab?

15 The highest wave ever recorded in Australia was 24.9 m, off Macquarie Island. If the average surfer is 180 cm tall, how many surfers standing on top of one another would it take to reach the top of the wave?

Reasoning

16 An insect was climbing a wall 2.7 m high. In the first 20 minutes the insect started at the bottom and climbed $\frac{1}{3}$ of the height of the wall. In the second 20 minutes it climbed $\frac{1}{4}$ of the remaining height and in the third 20 minutes it climbed $\frac{1}{5}$ of the remaining height.

(a) Calculate the distances climbed in each 20 minute period.

(b) Calculate how far the insect still had to climb to reach the top of the wall.

(c) Express the distance remaining as a fraction of the height of the wall.

17 Angela has to fill 50 gift bags with an equal amount of lollies in each. She has 4 packets of lollies that weigh 375 g each. Angela knows that the average mass of one lolly is 2.8 g.

(a) Explain how Angela can use this information to determine how many lollies she can put in each gift bag.

(b) Use your method from **(a)** to find the answer.

(c) About how many lollies will Angela have left over? Explain why this answer may not be exact.

18 The force of gravity varies from planet to planet and so the weight of an object can vary, depending on which planet it is on. Objects on Jupiter would weigh 2.6 times their weight on Earth. Objects on Mars would weigh 0.38 times their 'Earth weight'. Calculate the weight of a 5 kg bag of potatoes on **(a)** Jupiter **(b)** Mars.

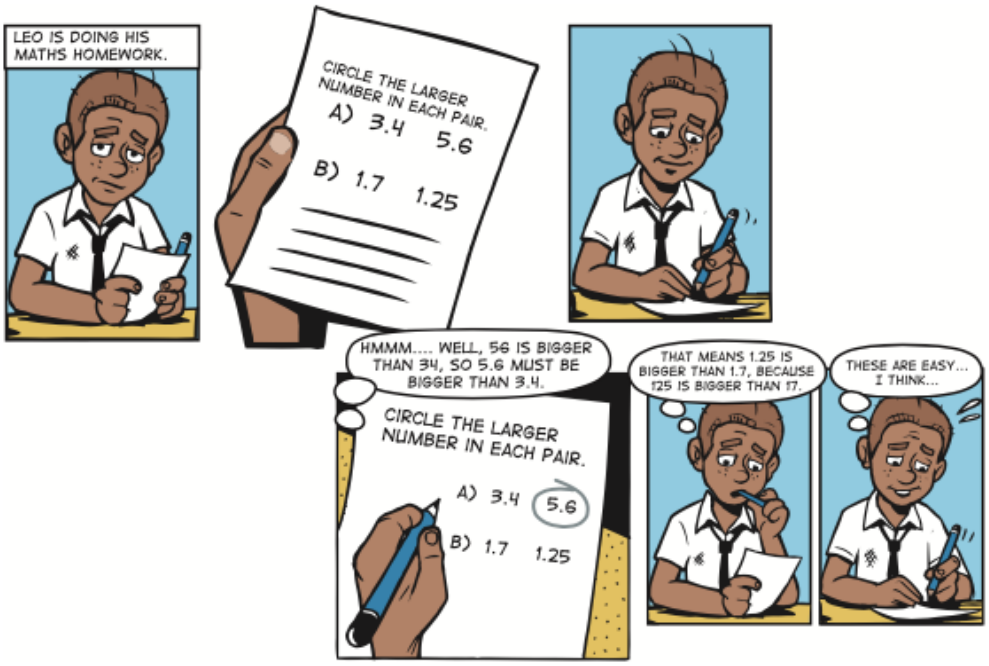
Extension

Open-ended

19 Write three fractions with three different denominators that have decimal values between 0.2 and 0.4.

20 Write three decimals, each with a different number of decimal places, that have fraction values between $\frac{9}{10}$ and $\frac{9}{8}$.

21



(a) Explain to Leo why his reasoning is incorrect.

(b) Describe to Leo how to compare two decimal numbers.