Mental strategies

Teaching Sequence with examples

WALT apply mental strategies such as tidy numbers and distributive law
Success Criteria -
I know how to ….

- **I can make an easy number by splitting numbers**
- **I can apply the distributive law**

Strategy One - Make easy numbers

Strategy 1 – Make easy numbers

This strategy uses two properties of numbers:

1 The order in which you add or multiply any two numbers does not change the result.

For example: $2 \times 3 = 6$ and $3 \times 2 = 6$
$4 + 5 = 9$ and $5 + 4 = 9$

This is known as the **commutative law.**

2 The order in which three or more numbers are added or the order in which they are multiplied is not important. In the following examples, brackets are used to show which pair of numbers is added or multiplied first.

Multiplication:

$(2 \times 3) \times 5$
$= 6 \times 5$
$= 30$

and

$2 \times (3 \times 5)$
$= 2 \times 15$
$= 30$

Addition:

$(6 + 7) + 8$
$= 13 + 8$
$= 21$

and

$6 + (7 + 8)$
$= 6 + 15$
$= 21$

This is known as the **associative law.**

It is important to realise, however, that the commutative and associative laws do not apply to subtraction and division.

For example:

$(9 - 5) - 4$
$= 4 - 4$
$= 0$

but

$9 - (5 - 4)$
$= 9 - 1$
$= 8$

$(40 \div 10) \div 2$
$= 4 \div 2$
$= 2$

but

$40 \div (10 \div 2)$
$= 40 \div 5$
$= 8$

The order in which you add a group of numbers and the order in which you multiply a group of numbers makes no difference to the result. This is not true for subtraction or division.

Multiples of 10 are easy to add and multiply, so we can use the commutative and associative laws to 'shuffle' the calculation to create multiples of 10.

For example:

$7 + 9 + 3$ is the same as: $3 + 7 + 9$
$= 10 + 9$

$5 \times 37 \times 2$ is the same as: $5 \times 2 \times 37$
$= 10 \times 37$

We can split one number into two parts, then use the associative law to add one part to the other number to create a multiple of 10.

For example: $135 + 46$
$= 135 + 5 + 41$ (splitting 46 into 5 and 41)
$= 140 + 41$ (adding 5 to 135)

Worked Example 1

WE1

Calculate the following using the 'make easy numbers' strategy.

(a) $7 + 32 + 13$ **(b)** $2 \times 13 \times 5$ **(c)** $293 + 568$

Thinking	Working
(a) 1 Rearrange the addition to form 'easy' numbers, such as multiples of 10.	(a) $7 + 32 + 13$ $= 7 + 13 + 32$
2 Perform these calculations first.	$= 20 + 32$
3 Complete the question.	$= 52$
(b) 1 Rearrange the multiplication to form 'easy' numbers, such as multiples of 10.	(b) $2 \times 13 \times 5$ $= 2 \times 5 \times 13$
2 Perform these calculations first.	$= 10 \times 13$
3 Complete the question.	$= 130$
(c) 1 Split one number into two parts, then add one part to the other number to create an 'easy' number. (Here, we have split 7 away from 568 and added it to 293.)	(c) $293 + 568$ $= 293 + 7 + 561$ $= 300 + 561$
2 Perform the calculation to complete the question.	$= 861$

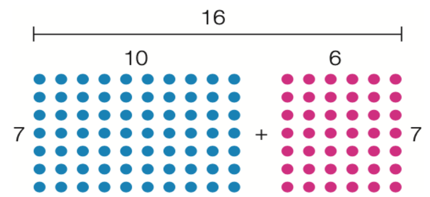
Strategy 2 – Use the distributive law

This strategy uses a property of numbers called the **distributive law**. The distributive law allows us to multiply a large number by splitting it up into 10s and 1s (or 100s, 10s and 1s), multiplying each part separately, then adding or subtracting each of the products. (When two numbers are multiplied together, the result is called the **product**.)

For example, we can break 7×16 down into 7 lots of 10 plus 7 lots of 6.

We can represent the multiplication in an array diagram:

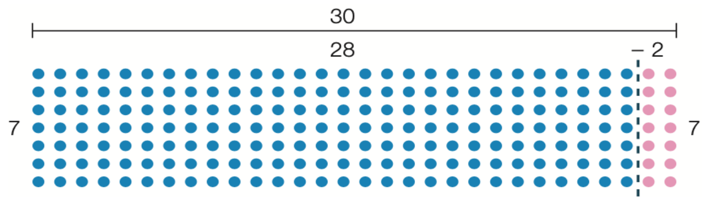
7×16
$= 7 \times (10 + 6)$ (because $16 = 10 + 6$)
$= 7 \times 10 + 7 \times 6$
$= 70 + 42$
$= 112$



In a similar way, we can write 19 as $20 - 1$, or 28 as $30 - 2$. For example, we can break 7×28 down into 7 lots of 30 minus 7 lots of 2.

We can represent this multiplication as an array diagram (the number subtracted is on the right of the dotted line).

7×28
$= 7 \times (30 - 2)$ (because $28 = 30 - 2$)
$= 7 \times 30 - 7 \times 2$
$= 210 - 14$
$= 196$



Check your fluency and understanding

Fluency

1 Calculate the following using the 'make easy numbers' strategy.

(a) $8 + 23 + 42$	**(b)** $15 + 57 + 35$	**(c)** $64 + 79 + 56$
(d) $5 \times 6 \times 2$	**(e)** $4 \times 6 \times 5$	**(f)** $2 \times 42 \times 5$
(g) $5 \times 7 \times 6$	**(h)** $5 \times 3 \times 8$	**(i)** $5 \times 14 \times 4$
(j) $47 + 73$	**(k)** $124 + 56$	**(l)** $211 + 169$
(m) $37 + 128 + 63$	**(n)** $77 + 78 + 23$	**(o)** $89 + 116 + 11$

2 Evaluate the following using the distributive law.

(a) 17×9	**(b)** 19×8	**(c)** 49×6
(d) 6×31	**(e)** 7×52	**(f)** 5×43
(g) 99×9	**(h)** 77×3	**(i)** 57×8
(j) 14×11	**(k)** 15×13	**(l)** 16×12
(m) 101×8	**(n)** 113×5	**(o)** 124×11

Challenge

3 Use any appropriate mental strategy to work out the following.

(a) $23 + 41 + 57$	**(b)** $347 + 156$	**(c)** $335 - 170$
(d) $8 \times 9 \times 5$	**(e)** 14×7	**(f)** 21×9
(g) 103×6	**(h)** 22×11	**(i)** 3×194
(j) $147 + 213$	**(k)** 19×14	**(l)** $4 \times 7 \times 15$

Extension and word problems

Understanding

4 Choose the correct answer to the following question.

23×7 could be calculated by:

A multiplying 3 and 7, then adding 20

B multiplying 20 and 7, then adding 3

C multiplying 20 and 7, multiplying 3 and 7, then adding the products together

D multiplying 20, 3 and 7 all together.

5 Bilal has completed the first 3 stages of a bike rally. He rode 87 km in Stage 1, 95 km in Stage 2, and 63 km in Stage 3. Use mental strategies to calculate:

(a) the total distance that Bilal has ridden so far

(b) how far Bilal still has to ride, if the total rally distance is 480 km.

6 The Year 7s at Mountain View Secondary College are undertaking a project to improve their environment. Each student will plant 5 seedlings of a native plant. Use a mental strategy to calculate how many seedlings 8 classes of 25 students will need.

7 Jason is saving $8 every week for some new cricket gear. Use mental strategies to calculate:

(a) how much Jason has saved after 17 weeks

(b) how much he still has to save if the cricket gear he wants costs $189.

8 Carlos is monitoring traffic on a busy road. Twelve cars go past him in 1 minute.

(a) Use a mental strategy to calculate how many cars Carlos can expect to go past in 1 hour, based on his 1 minute count.

(b) List two reasons why the actual number of cars might be less than your answer to **(a)**.

9 Jessica earns $5 every time she walks her neighbour's dog. If she walks the dog 3 times a week, how much will she earn in 6 months?

10 Alicia is ordering stationery for her office cupboard. Use mental strategies to calculate the cost of each of the following, in dollars.

(a) 8 notepads at 98 cents each

(b) 3 gluesticks at 77 cents each

(c) 12 pens at 59 cents each

(d) 5 boxes of paperclips at 82 cents each