Probability Calculations

Conditional Probability

Often when dealing with probability we are only interested in some of the outcomes from an experiment and not all of them. Probability problems that use a restricted or reduced sample space are called **conditional** probabilities.

Consider a set of cards numbered 1 to 10. One card is drawn out at random.

The probability of it being even is $\frac{5}{10} = \frac{1}{2}$.

However, if we were asked to find the probability that the number drawn out is an even number given that it is greater then 5, we have reduced the sample space to just the cards numbered 6, 7, 8, 9, and 10 and only 3 of these are even i.e. 6, 8 and 10.

Hence we get the answer $\frac{3}{5}$.

Example

Two dice are tossed. What is the probability, if the two numbers uppermost total less than 10, that the numbers are the same?

We begin by listing all 36 possible outcomes of throwing two dice. See the table below.

	1	2	3	4	5	6
1	1,1	1,2	1,3	1,4	1,5	1,6
2	2,1	2,2	2,3	2,4	2,5	2,6
3	3,1	3,2	3,3	3,4	3,5	3,6
4	4,1	4,2	4,3	4,4	4,5	4,6
5	5,1	5,2	5,3	5,4	5,5	5,6
6	6,1	6,2	6,3	6,4	6,5	6,6

We now **circle** those outcomes that total less than 10 and we find our sample space has been reduced to 30.

We then cross those circled outcomes that have both numbers the same. In total we have 4 of these.

$$\text{P(Same if <10)} = \frac{\text{Number of correct outcomes}}{\text{Total number of outcomes}}$$

$$= \frac{4}{30} \quad \text{as only 30 less than 10.}$$

$$= \frac{2}{15} \quad (0.1333)$$

Extra Notes

Look to see if the possible answers (sample space) is restricted as this can mean a conditional probability problem. Look for the words 'given' and 'when'.

If you have a table listing the possible results, cross all those that do not apply so you only focus on the restricted sample space.

Example

The table below contains information about the Year 9, 10 and 11 students at a school and what elective they have chosen for Term I.

	Year 9	Year 10	Year 11	Total
Drama	170	165	150	485
Chess	54	32	24	110
Sports	94	85	62	241
Total	318	282	236	836

a) Find the probability that a randomly chosen student is taking the drama elective.

b) Given that a randomly chosen student is in Year 11, find the probability they are taking the drama elective.

In part a) the sample space is all 836 students and those taking the drama elective total 485.

a) $$\text{P(Takes drama)} = \frac{\text{485 take drama}}{\text{836 students}}$$

$$= \frac{485}{836} \quad (0.5801)$$

b) Sample space is restricted to all Year 11 students i.e. 236 and 150 of these take drama, so

$$\text{P(Y11 that take chess)} = \frac{\text{150 Y11s take drama}}{\text{236 in Y11}}$$

$$= \frac{75}{118} \quad (0.6356)$$

Expected Value

If the probability of an event occurring, p, is known and you have n of such events, then you would expect that n x p of such events would be successful.

Expected number = n x p

where n is the number of events and p is the probability that one event is successful.

Example

When tossing a six-sided die the probability that it lands with a 5 up is

$$P(x = 5) = \frac{1}{6}$$

a) Calculate the expected number of 5s if the die was tossed 120 times.

b) If two dice were thrown at once 120 times, how many times would you expect to get two 5s?

a) Expected $= np$
$= 120 \times \frac{1}{6}$
$= 20$

b) P(two 5s) $= \frac{1}{6} \times \frac{1}{6}$
$= \frac{1}{36}$

Expected double 5s $= 120 \times \frac{1}{36}$
$= 3.33$
$= 3$ or 4

Achievement / Merit – Use probability concepts to calculate and justify statements and findings.

8. A spinner has the numbers 1 to 10 on it. If the spinner is spun once, find the probability

a) given it is an even number, that the result is 4.

b) given it is 7 or less, that the result is 1 or 2.

c) given it is greater than 5, that the result is 9.

d) given it is an odd number, that the result is 5 or 7.

9. The table below gives a breakdown of tourist numbers (age and country) to a particular attraction on a random day.

	Aust.	Jpn.	Eng.	Fr.	USA	Other	Total
≤ 20	9	16	3	2	6	21	57
21 – 39	25	18	10	4	12	57	126
≥ 40	47	35	24	9	15	68	198
Total	81	69	37	15	33	146	381

a) Find the probability that a randomly chosen tourist is French.

b) Find the probability, given a tourist is 40 or over, that they are from Japan.

c) A tourist is from Australia, find the probability that they are 20 years of age or under.

d) A tourist is NOT from Australia, find the probability that they are between 21 – 39.

10. Calculate the expected number for each problem.

a) The probability that you get a 'head' when tossing a coin is 0.5. If you toss two coins at once 100 times, how many times would you expect them to both land on 'heads'?

b) In shooting from the free throw line, Julia can score a goal 40% of the time. How many times would you expect her to miss in the next 30 attempts?

11. A calculator can generate two-digit random numbers from 00 to 99.

a) What is the probability that a two-digit random number is divisible by 5?

b) In 40 trials how many would you expect to be divisible by 5?

c) If you know one or both of the digits is a 9, how many out of 40 trials would you expect to be divisible by 5?

12. Six discs are placed face down on a table. Three discs have the letters A, B, and C on them and the other three, the numbers 1, 2, and 3. Two discs are selected at random from the table.

a) Complete the table below listing the set of all possible outcomes of the two discs.

	A	B	C	1	2	3
A		AB				
B	BA					
C						
1						
2						
3						

b) Find the probability, given that two letters are drawn out, that one of them is a vowel (i.e. A).

c) Find the probability, given that two numbers are drawn out, that they add to 4.

d) Find the probability, given a C disc is drawn out, that the other disc is a number.

e) Find the probability, given an A disc is drawn out, that the other disc is a B.

13. A six-sided die has the numbers 1 through to 6 on its faces. A red and a blue die are tossed and the numbers displayed are multiplied together.

a) Record the results by completing this table.

Red \ Blue	1	2	3	4	5	6
1	1	2				
2	2	4				
3						
4						
5						
6						

b) Find the probability that the result is at least 20.

c) In 200 throws how many would you expect to have a product of at least 20?

d) If we know the blue die is greater than 2, find the expected number of throws out of 200 where the product is at least 20.

e) If we know that at least one die is a 6, find the expected number of throws out of 200 where the product is at least 20.

Answers

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8. a) P(4 given even) $= \frac{1}{5}$ (0.2)

b) P(1 or 2 given ≤ 7)
$= \frac{2}{7}$ (0.286)

c) P(9 given > 5) $= \frac{1}{5}$ (0.2)

d) P(5 or 7 given O) $= \frac{2}{5}$ (0.4)

9. a) P(Fr.) $= \frac{5}{127}\left(\frac{15}{381}\right)$ (0.039)

b) P(Japan given ≥ 40)
$= \frac{35}{198}$ (0.177)

c) P(≤ 20 given Aust.)
$= \frac{1}{9}\left(\frac{9}{81}\right)$ (0.111)

d) P(21-39 given not Aust)
$= \frac{101}{300}$ (0.337)

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10. a) Expect 25 times.
b) Miss 18 times.

11. a) P($\div$ by 5) = 0.2
b) Expect 8 times.
c) 90 and 95 out of 19.
P($\div$ by 5) $= \frac{2}{19}$ (0.105)
Expect 4 or 5 to be divisible by 5 in 40 trials.

12. a)

	A	B	C	1	2	3
A		AB	AC	A1	A2	A3
B	BA		BC	B1	B2	B3
C	CA	CB		C1	C2	C3
1	1A	1B	1C		12	13
2	2A	2B	2C	21		23
3	3A	3B	3C	31	32	

b) P(one vowel given 2 letters)
$= \frac{2}{3}\left(\frac{4}{6}\right)$ (0.667)

c) P(add to 4 given 2 numbers)
$= \frac{1}{3}\left(\frac{2}{6}\right)$ (0.333)

d) P(number given C)
$= \frac{3}{5}\left(\frac{6}{10}\right)$ (0.6)

e) P(B given A) $= \frac{1}{5}\left(\frac{2}{10}\right)$ (0.2)

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13. a)

	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	6	9	12	15	18
4	4	8	12	16	20	24
5	5	10	15	20	25	30
6	6	12	18	24	30	36

b) $P(X \geq 20) = \frac{8}{36}$ (0.2222)

c) Expect = 44 or 45

d) $P(X \geq 20) = \frac{8}{24}$ (0.3333)

Expect = 66 or 67

e) $P(X \geq 20) = \frac{5}{11}$

Expect = 90 or 91