

STANDARD DIVISIBILITY TESTS

Number Divisibility test

- 2** If the last digit is 0 or even, then the original number is divisible by 2.
- 3** If the sum of the digits is divisible by 3, then the original number is divisible by 3.
- 5** If the last digit is 0 or 5 the number is divisible by 5.
- 7** Write down all the digits except the last one. Take twice the last digit from this number. Keep doing this until you end up with a 2 digit number. If this 2 digit number is divisible by 7, the original number is divisible by 7.
- 11** Add the digits in odd positions. Add the digits in the even positions. Find the difference between your two answers. If the difference is 0 or a multiple of 11, the original number is divisible by 11.

INVESTIGATION 3

DIVISIBILITY BY 4 AND 9



One of the joys of mathematics comes from investigating and discovering things for yourself. In this investigation you should discover rules for divisibility by 4 and by 9.

What to do:

- 1** Copy the following table:

<i>Number</i>	<i>Divisibility by 4 (Yes/No)</i>	<i>Last 2 digits</i>
81		
154		
252		
3624		
8185		
9908		

- 2** Fill out the second column using your calculator (or using simple division) and fill out the third column, writing down the last two digits of each number .

- 3** Copy and complete: “A natural number is divisible by 4 if”.

- 4** Copy and complete the following table:

<i>Number</i>	<i>Divisibility by 9 (Yes/No)</i>	<i>Sum of its digits</i>
81		$8 + 1 = 9$
154		
252		
3624		
8185		
9908		

- 5** Copy and complete: “A natural number is divisible by 9 if”.

EXERCISE 2E

- Answer *true* or *false* for the following:
 - 45 is divisible by 5
 - 70 is divisible by 5
 - 402 is divisible by 5
 - 75 is divisible by 2
 - 96 is divisible by 2
 - 2338 is divisible by 2
 - 92 is divisible by 3
 - 126 is divisible by 3
 - 56 235 is divisible by 3
- Which of the following are divisible by 3?
 - 87
 - 153
 - 512
 - 861
 - 977
 - 1002
 - 111 111
 - 56 947
 - 12 321
 - 778 899
 - 123 456 789
 - 124 124 124
- Decide whether the following are divisible by 4: (Use **Investigation 3** result.)
 - 1250
 - 4234
 - 30 420
 - 315 422
- Decide whether the following are divisible by 9: (Use **Investigation 3** result.)
 - 801
 - 2979
 - 35 533
 - 59 283
- Determine by which of the numbers 2, 3, 4, 5, 9, 10 the following are divisible:
 - 120
 - 616
 - 960
 - 1443
- Find all possible values of the missing digit if the following are divisible by 3:
 - $1\square3$
 - $\square36$
 - $6\square34$
 - $89\square12$
- Discuss and then write down concise divisibility tests for divisibility by:
 - 10
 - 6
 - 8
 - 12
 - 24
- A four digit number has digit form ' $a2b4$ ' and is divisible by 3. What are the possible values of $a + b$?
- I am the smallest positive integer which is divisible by the numbers 2, 3, 7, 10, 15 and 21. What am I?
- Find all possible values for the digits p and q if the number with digit form ' $p132q$ ' is divisible by 24 .
- Write down the smallest positive integer which has a remainder of 1 when it is divided by 3, 4 and 5.

Factors of Natural Numbers

EXERCISE 2F.1

- 1 **a** List all the factors of 9. **b** List all the factors of 12.
 c Copy and complete this equation: $12 = 2 \times \dots$
 d Write another pair of factors which multiply to give 12.
- 2 List *all* the factors of each of the following numbers:
- | | | | |
|-------------|-------------|-------------|-------------|
| a 10 | b 18 | c 30 | d 35 |
| e 44 | f 56 | g 50 | h 84 |
| i 39 | j 42 | k 66 | l 75 |
- 3 Complete the factorisations below:
- | | | |
|---------------------------------|----------------------------------|--------------------------------|
| a $24 = 6 \times \dots$ | b $25 = 5 \times \dots$ | c $28 = 4 \times \dots$ |
| d $100 = 5 \times \dots$ | e $88 = 11 \times \dots$ | f $88 = 2 \times \dots$ |
| g $36 = 2 \times \dots$ | h $36 = 3 \times \dots$ | i $36 = 9 \times \dots$ |
| j $49 = 7 \times \dots$ | k $121 = 11 \times \dots$ | l $72 = 6 \times \dots$ |
| m $60 = 12 \times \dots$ | n $48 = 12 \times \dots$ | o $96 = 8 \times \dots$ |
- 4 Write the largest factor (not itself) of each of the following numbers:
- | | | | |
|-------------|-------------|-------------|-------------|
| a 12 | b 18 | c 27 | d 48 |
| e 44 | f 75 | g 90 | h 39 |

EXERCISE 2F.2

- 1 **a** Beginning with 8, write three consecutive even numbers.
 b Beginning with 17, write five consecutive odd numbers.
- 2 **a** Write two even numbers which are not consecutive and which add to 10.
 b Write all the pairs of two non-consecutive odd numbers which add to 20.
 c Write all the triplets of three different even numbers which add to 20.
- 3 Use the words “even” and “odd” to complete these sentences correctly:
- | |
|---|
| a The sum of two even numbers is always |
| b The sum of two odd numbers is always |
| c The sum of three even numbers is always |
| d The sum of three odd numbers is always |
| e The sum of an odd number and an even number is always |
| f When an even number is subtracted from an odd number the result is |
| g When an odd number is subtracted from an odd number the result is |
| h The product of two odd numbers is always |
| i The product of an even and an odd number is always |

HIGHEST COMMON FACTOR

A number which is a factor of two or more other numbers is called a **common factor** of these numbers.

For example, 7 is a common factor of 28 and 35.

We can use the method of finding prime factors to find the **highest common factor (HCF)** of two or more natural numbers.

Example 12

Find the highest common factor (HCF) of 18 and 24.

$$\begin{array}{r|l} 2 & 18 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array}
 \qquad
 \begin{array}{r|l} 2 & 24 \\ \hline 2 & 12 \\ \hline 2 & 6 \\ \hline 3 & 3 \\ \hline & 1 \end{array}
 \qquad
 \begin{array}{l} 18 = 2 \times 3 \times 3 \\ 24 = 2 \times 2 \times 2 \times 3 \end{array}$$

2×3 is common to both and $2 \times 3 = 6$

$\therefore 6$ is the highest common factor of 18 and 24.

EXERCISE 2F.4

1 Find the highest common factor of:

a 9, 12

b 8, 16

c 18, 24

d 14, 42

e 18, 30

f 24, 32

g 12, 36

h 15, 33

2 Find the highest common factor of:

a 25, 50, 75

b 22, 33, 44

c 21, 42, 84

d 39, 13, 26

3 Find the highest common factor of:

a 25, 35, 50, 60

b 36, 44, 52, 56

c 10, 18, 20, 36

d 32, 56, 72, 88

Answers

EXERCISE 2E

1 a true b true c false d false e true
f true g false h true i true

2 a yes b yes c no d yes e no f yes
g yes h no i yes j yes k yes l yes

3 a no b no c yes d no

4 a yes b yes c no d yes

5 a 2, 3, 4, 5, 10 b 2, 4 c 2, 3, 4, 5, 10 d 3

6 a 2, 5, 8 b 3, 6, 9 c 2, 5, 8 d 1, 4, 7

- 7 a last digit is 0
 b divisible by 2 and divisible by 3
 c number formed by last three digits is divisible by 8
 d divisible by 3 and divisible by 4
 e divisible by 3 and divisible by 8
- 8 3, 6, 9, 12, 15, 18 9 210

10

<i>p</i>	1	3	4	6	7	9
<i>q</i>	8	0	8	0	8	0

11 61

EXERCISE 2F.1

- 1 a 1, 3, 9 b 1, 2, 3, 4, 6, 12 c $12 = 2 \times 6$
 d 3×4
- 2 a 1, 2, 5, 10 b 1, 2, 3, 6, 9, 18
 c 1, 2, 3, 5, 6, 10, 15, 30 d 1, 5, 7, 35
 e 1, 2, 4, 11, 22, 44 f 1, 2, 4, 7, 8, 14, 28, 56
 g 1, 2, 5, 10, 25, 50
 h 1, 2, 3, 4, 6, 7, 12, 14, 21, 28, 42, 84
 i 1, 3, 13, 39 j 1, 2, 3, 6, 7, 14, 21, 42
 k 1, 2, 3, 6, 11, 22, 33, 66
 l 1, 3, 5, 15, 25, 75
- 3 a 4 b 5 c 7 d 20 e 8 f 44 g 18
 h 12 i 4 j 7 k 11 l 12 m 5 n 4
 o 12
- 4 a 6 b 9 c 9 d 24 e 22 f 25 g 45
 h 13

EXERCISE 2F.2

- 1 a 8, 10, 12 b 17, 19, 21, 23, 25
- 2 a $2 + 8$ b $1 + 19, 3 + 17, 5 + 15, 7 + 13$
 c $0 + 2 + 18, 0 + 4 + 16, 0 + 6 + 14, 0 + 8 + 12,$
 $2 + 4 + 14, 2 + 6 + 12, 2 + 8 + 10, 4 + 6 + 10$
- 3 a even b even c even d odd e odd
 f odd g even h odd i even

EXERCISE 2F.3

- 1 a 2, 3, 5, 7, 11, 13, 17, 19, 23, 29
 b No, a prime has exactly two factors, 1 and itself
 c Yes, 2
- 2 a $5485 = 5 \times 1097$ b $8230 = 2 \times 4115$
 c $7882 = 2 \times 3941$ d $999 = 3 \times 333$
- 3 a $2 \times 2 \times 2 \times 3$ b $2 \times 2 \times 7$ c $3 \times 3 \times 7$
 d $2 \times 2 \times 2 \times 3 \times 3$ e $2 \times 2 \times 2 \times 17$
 f $2 \times 2 \times 3 \times 7$ g $2 \times 2 \times 2 \times 3 \times 3 \times 3$
 h $2 \times 2 \times 2 \times 2 \times 3 \times 11$ i $3 \times 3 \times 3 \times 3 \times 5$
 j $2 \times 2 \times 2 \times 2 \times 7 \times 7$
- 4 a 3 b 11 c 15 d 17, 71, 35, 53 e 5

EXERCISE 2F.4

- 1 a 3 b 8 c 6 d 14 e 6 f 8 g 12 h 3
- 2 a 25 b 11 c 21 d 13
- 3 a 5 b 4 c 2 d 8