Walt Factorise quadratic expression

Success criteria I can use my knowledge of multiplication and addition facts to see the two numbers that factorise.

[Watch the video]

1. The **coefficient** (number in front) of x is the sum of the numbers a and b in the brackets.
2. The **constant term** (number at the end by itself) is the product of the numbers a and b in the brackets.

Therefore, working in reverse, to find the numbers a and b we want two numbers that:

1. add to the coefficient of x
2. multiply to the constant term.

Example

Factorise $x^2 + 8x + 12$.

Answer

To write $x^2 + 8x + 12$ in the form $(x \quad)(x \quad)$ we have to find a pair of numbers that add to 8 and multiply to 12.

Some pairs of numbers that add to 8	Some pairs of numbers that multiply to 12
$^{-}1$ 9	1 12
0 8	2 6
1 7	3 4
2 6	$^{-}1$ $^{-}12$
3 5	$^{-}2$ $^{-}6$
4 4	$^{-}3$ $^{-}4$

The pair we want is the one that is in *both* columns, i.e. 2   6.

Therefore, $x^2 + 8x + 12$ factorises to $(x + 2)(x + 6)$.

A competent Maths student explores the possibilities in their head rather than writing them all down. This takes some practice at first, but you will gradually speed up!

Example

Factorise $x^2 - x - 6$.

Answer

Here we want two numbers that add to $^{-}1$ and multiply to $^{-}6$. If they multiply to a negative number, the two numbers must have different signs – one positive and one negative.

$^{-}3$ and 2 are the only pair of numbers that have this property.

$x^2 - x - 6 = (x - 3)(x + 2)$

It does not matter which bracket comes first, e.g.
$(x - 3)(x + 2)$ is the same as $(x + 2)(x - 3)$.

Example

Factorise $x^2 - 26x + 48$.

Answer

Here we want two numbers that add to $^{-}26$ and multiply to 48. The two numbers must both be negative.

$^{-}24$ and $^{-}2$ are the only pair of numbers that have this property.

$x^2 - 26x + 48 = (x - 24)(x - 2)$

Expanding is the opposite of factorising.

$$x^2 + 3x - 10 \xrightarrow[\longleftarrow \text{ Expanding}]{\text{Factorising}} (x + 5)(x - 2)$$

EXERCISE 8.07

1–20 Factorise these quadratic expressions.

1 $x^2 + 8x + 15$
2 $x^2 + 10x + 24$
3 $x^2 + 4x + 3$
4 $x^2 - 2x - 15$
5 $x^2 - x - 12$
6 $x^2 + x - 56$
7 $x^2 - 7x + 12$
8 $x^2 - 13x + 42$
9 $x^2 - x - 2$
10 $x^2 - 8x - 20$
11 $x^2 - 15x + 56$
12 $x^2 + 16x + 39$
13 $x^2 + 11x - 60$
14 $x^2 - 6x - 40$
15 $x^2 - 22x - 23$
16 $x^2 - 13x - 48$
17 $x^2 + 15x + 50$
18 $x^2 - 3x - 70$
19 $x^2 - 17x + 72$
20 $x^2 + 5x - 36$

21–28 It is not always possible to factorise a quadratic expression. Four of the following factorise; the other four do not. Either factorise correctly or write "Cannot be factorised."

21 $x^2 - 2x + 8$
22 $x^2 + 12x - 13$
23 $x^2 - 5x - 6$
24 $x^2 + 2x + 3$
25 $x^2 - 4x + 18$
26 $x^2 + x - 7$
27 $x^2 + x - 6$
28 $x^2 + 13x - 14$

29 There are nine quadratic expressions in the left column. Eight of them have a corresponding factorisation placed in the right column, but they have been muddled up.

Use factorising or expanding to match up each expression (**a**–**i**) with its correct factorisation. For one of them write "Cannot be factorised."

	Expansion		Factorisation
a	$x^2 - 3x - 4$	A	$(x + 3)(x + 4)$
b	$x^2 - 5x + 4$	B	$(x + 4)(x - 1)$
c	$x^2 - x - 12$	C	$(x + 4)(x - 3)$
d	$x^2 - 7x + 12$	D	$(x - 1)(x - 4)$
e	$x^2 + 7x + 12$	E	$(x + 4)(x + 1)$
f	$x^2 + 5x + 4$	F	$(x - 3)(x - 4)$
g	$x^2 - 2x - 12$	G	$(x - 4)(x + 3)$
h	$x^2 + x - 12$	H	$(x + 1)(x - 4)$
i	$x^2 + 3x - 4$		

Two special cases

1 A **perfect square** is a quadratic expression where both brackets are the same. To write it as simply as possible we use squaring rather than repeating the same set of brackets.

Example

Factorise $x^2 - 6x + 9$.

Answer

The pair of numbers that multiply to 9 and add to $^-6$ are $^-3$ and $^-3$.

$x^2 - 6x + 9 = (x - 3)(x - 3)$

$= (x - 3)^2$

2 The **difference of two squares** factorises to two sets of brackets which are identical except for the signs; one + and the other −. It is called the difference of two squares because in its expanded form it is:

$x^2 - (\text{number})^2$

and there is no x term. If an x term was written in it would be $0x$.

Example

Factorise $x^2 - 64$.

Answer

$x^2 - 64 = x^2 + 0x - 64$

The two numbers that add to 0 and multiply to $^-64$ are 8 and $^-8$.

$x^2 - 64 = (x + 8)(x - 8)$

EXERCISE 8.08

Factorise these quadratic expressions. Each one is either a perfect square or a difference of two squares.

1 $x^2 - 16$
2 $x^2 + 4x + 4$
3 $x^2 - 10x + 25$
4 $x^2 - 1$
5 $x^2 + 6x + 9$
6 $x^2 - 36$
7 $x^2 - 18x + 81$
8 $x^2 - 24x + 144$
9 $x^2 - 64$
10 $x^2 - 121$
11 $x^2 + 40x + 400$
12 $x^2 - 169$
13 $x^2 - 36x + 324$
14 $x^2 - \frac{1}{9}$
15 $x^2 + \frac{1}{5}x + \frac{1}{100}$

Quadratics with a common factor

When the coefficient (number in front) of x^2 is not 1 the whole quadratic expression may have a common factor. Deal with this first, and then factorise the resulting quadratic.

Example

Factorise $4x^2 - 20x + 24$.

Answer

On inspection each term has a common factor of 4.

$4x^2 - 20x + 24 = 4(x^2 - 5x + 6)$

Then factorise the quadratic in brackets, *leaving* the 4 in place:

$= 4(x - 3)(x - 2)$

Factorise these quadratic expressions *completely*.

1 $3x^2 + 15x + 18$
2 $2x^2 - 24x + 72$
3 $5x^2 - 125$
4 $3x^2 - 12$
5 $3x^2 + 6x + 3$
6 $2x^2 + 2x - 4$
7 $4x^2 - 400$
8 $2x^2 - 8x + 8$
9 $5x^2 - 10x - 15$
10 $2x^2 + 12x + 10$
11 $3x^2 + 18x + 27$
12 $6x^2 - 6$
13 $4x^2 - 4x - 24$
14 $5x^2 - 40x + 35$
15 $3x^2 + 9x + 6$