Do now

Expand and simplify:

a	$(x+1)^2$	**b**	$(x+3)^2$	**c**	$(x-2)^2$	**d**	$(x-5)^2$
e	$(2+x)^2$	**f**	$(2-x)^2$	**g**	$(2x+1)^2$	**h**	$(2x-1)^2$
i	$(3x+2)^2$	**j**	$(3x-2)^2$	**k**	$(x+y)^2$	**l**	$(x-y)^2$

Expand and simplify:

a	$(x-1)(x+1)$	**b**	$(x+4)(x-4)$	**c**	$(x+5)(x-5)$
d	$(2x+1)(2x-1)$	**e**	$(4-x)(4+x)$	**f**	$(3-2x)(3+2x)$

Why did the x-terms disappear in the expansions of question **6**?

WALT: Factorise algebraic expressions

Success Criteria: I know in expansions we have to remove brackets whereas in factorisation we have to insert brackets

1 Copy and complete:

a	$3x+6=3(x+\ldots)$	**b**	$4a-12=4(a-\ldots)$
c	$20-5p=5(\ldots-p)$	**d**	$16x+12=4(\ldots+3)$
e	$3x^2-9x=3x(x-\ldots)$	**f**	$2m+8m^2=2m(\ldots+4m)$

2 Copy and complete:

a	$4x+12=4(\ldots+\ldots)$	**b**	$9+3d=3(\ldots+\ldots)$
c	$3c-3=3(\ldots-\ldots)$	**d**	$cd+de=d(\ldots\quad\ldots)$
e	$6a+8ab=\ldots(3+4b)$	**f**	$4x-2x^2=\ldots(2-x)$
g	$4ab-4a=\ldots(b-1)$	**h**	$4ab-6bc=\ldots(2a-3c)$

3 Fully factorise:

a	$5a+5b$	**b**	$2x-4$	**c**	$7d+14$
d	$21-14x$	**e**	$6x-12$	**f**	$12+3x$
g	$ac+bc$	**h**	$12y-6a$	**i**	$2a+ab$
j	$bc-3cd$	**k**	$2x-xy$	**l**	$xy+y$
m	$a+ab$	**n**	$ab-bc$	**o**	$2an+ab$
p	$ab-a$	**q**	$ab+bc$	**r**	$2x+xy-4$

Factorising Quadratics - Challenge

Example 17

Factorise: $x^2 + 11x + 24$

We need to find two numbers which have sum $= 11$, product $= 24$.
Pairs of factors of 24:

Factor product	1×24	2×12	3×8	4×6
Factor sum	25	14	11	10

↑ this one

The numbers we want are 3 and 8.

So, $x^2 + 11x + 24$
$= (x + 3)(x + 8)$

Most of the time we can find these two numbers mentally.

Note: Only the last two lines of this example need to be shown in your working.

2 Factorise:

a	$x^2 + 4x + 3$	**b**	$x^2 + 11x + 24$	**c**	$x^2 + 10x + 21$
d	$x^2 + 15x + 54$	**e**	$x^2 + 9x + 20$	**f**	$x^2 + 8x + 15$
g	$x^2 + 10x + 24$	**h**	$x^2 + 9x + 14$	**i**	$x^2 + 6x + 8$
j	$x^2 + 11x + 18$	**k**	$x^2 + 9x + 18$	**l**	$x^2 + 13x + 42$
m	$x^2 + 11x + 24$	**n**	$x^2 + 15x + 26$	**o**	$x^2 + 29x + 100$

Example 15

Fully factorise: $^{-}2a + 6ab$

$$
\begin{aligned}
& {}^{-}2a + 6ab \\
&= 6ab - 2a && \{\text{Rewrite with } 6ab \text{ first. Why?}\} \\
&= 2 \times 3 \times a \times b \; - \; 2 \times a \\
&= 2a(3b - 1) && \{\text{as } 2a \text{ is the HCF}\}
\end{aligned}
$$

5 Fully factorise:

a	$^{-}2a + 2b$	**b**	$^{-}3 + 6b$	**c**	$^{-}4a + 8b$
d	$^{-}3c + cd$	**e**	$^{-}a + ab$	**f**	$^{-}7x^2 + 14x$
g	$^{-}6x + 12x^2$	**h**	$^{-}4b^2 + 2ab$	**i**	$^{-}a + a^2$

Example 16

Fully factorise: $^{-}2x^2 - 4x$

$$
\begin{aligned}
& {}^{-}2x^2 - 4x \\
&= {}^{-}2 \times x \times x + {}^{-}2 \times 2 \times x \\
&= {}^{-}2x(x + 2) && \{\text{as HCF is } {}^{-}2x\}
\end{aligned}
$$

6 Fully factorise:

a	$^{-}3a - 3b$	**b**	$^{-}4 - 8x$	**c**	$^{-}3y - 6b$
d	$^{-}5c - cd$	**e**	$^{-}x - xy$	**f**	$^{-}5x^2 - 10x$
g	$^{-}4y - 12y^2$	**h**	$^{-}6a^2 - 3ab$	**i**	$^{-}8x^2 - 24x$

Example 18

Factorise: $x^2 - 7x + 12$

sum $= {}^-7$ and product $= 12$

$\therefore$ numbers are ${}^-3$ and ${}^-4$

So, $x^2 - 7x + 12$

$= (x - 3)(x - 4)$

3 Factorise:

a	$x^2 - 3x + 2$	**b**	$x^2 - 4x + 3$	**c**	$x^2 - 5x + 6$
d	$x^2 - 14x + 33$	**e**	$x^2 - 16x + 39$	**f**	$x^2 - 19x + 48$
g	$x^2 - 11x + 28$	**h**	$x^2 - 14x + 24$	**i**	$x^2 - 20x + 36$
j	$x^2 - 7x + 12$	**k**	$x^2 - 17x + 30$	**l**	$x^2 - 11x + 30$
m	$x^2 - 13x + 36$	**n**	$x^2 - 13x + 42$	**o**	$x^2 - 17x + 60$

Example 19

Factorise: **a** $x^2 - 2x - 15$ **b** $x^2 + x - 6$

a sum $= {}^-2$ and product $= {}^-15$

$\therefore$ numbers are ${}^-5$ and 3

So, $x^2 - 2x - 15$

$= (x - 5)(x + 3)$

b sum $= 1$ and product $= {}^-6$

$\therefore$ numbers are 3 and ${}^-2$

So, $x^2 + x - 6$

$= (x + 3)(x - 2)$

4 Factorise:

a	$x^2 - 7x - 8$	**b**	$x^2 + 4x - 21$	**c**	$x^2 - x - 2$
d	$x^2 - 2x - 8$	**e**	$x^2 + 5x - 24$	**f**	$x^2 - 3x - 10$
g	$x^2 + 3x - 54$	**h**	$x^2 + x - 72$	**i**	$x^2 - 4x - 21$
j	$x^2 - x - 6$	**k**	$x^2 - 7x - 60$	**l**	$x^2 + 7x - 60$

Example 20

Fully factorise by first removing a common factor: $3x^2 + 6x - 72$

$3x^2 + 6x - 72$ {first look for a **common factor**}
$= 3(x^2 + 2x - 24)$ {sum = 2, product = $^-24$ i.e., 6 and $^-4$}
$= 3(x + 6)(x - 4)$

5 Fully factorise by first removing a common factor:

a	$2x^2 + 10x + 8$	**b**	$3x^2 - 21x + 18$	**c**	$2x^2 + 14x + 24$
d	$2x^2 - 44x + 240$	**e**	$4x^2 - 8x - 12$	**f**	$3x^2 - 42x + 99$
g	$2x^2 - 2x - 180$	**h**	$3x^2 - 6x - 24$	**i**	$2x^2 + 18x + 40$
j	$x^3 - 7x^2 - 8x$	**k**	$x^3 - 3x^2 - 28x$	**l**	$x^4 + 2x^3 + x^2$

"the difference of two squares".

Example 21

Fully factorise:
a $x^2 - 4$ **b** $1 - 25y^2$

a $x^2 - 4$
$= x^2 - 2^2$
$= (x + 2)(x - 2)$

b $1 - 25y^2$
$= 1^2 - (5y)^2$
$= (1 + 5y)(1 - 5y)$

EXERCISE 11E

1 Fully factorise:

a	$c^2 - d^2$	**b**	$m^2 - n^2$	**c**	$n^2 - m^2$	**d**	$m^2 - x^2$
e	$x^2 - 16$	**f**	$x^2 - 81$	**g**	$a^2 - 9$	**h**	$4x^2 - 1$
i	$4x^2 - 9$	**j**	$9y^2 - 25$	**k**	$64 - x^2$	**l**	$16 - 9a^2$
m	$9x^2 - 1$	**n**	$4a^2 - 9b^2$	**o**	$16a^2 - x^2$	**p**	$9x^2 - 16b^2$

F PERFECT SQUARE FACTORISATION (EXTENSION)

Recall that

$(a + b)^2$
$= (a + b)(a + b)$
$= a^2 + ab + ab + b^2$
$= a^2 + 2ab + b^2$

and

$(a - b)^2$
$= (a - b)(a - b)$
$= a^2 - ab - ab + b^2$
$= a^2 - 2ab + b^2$