A Complete Guide to ...



Utilising the objectives as written in

MATHEMATICS in the New Zealand CURRICULUM

for

Level 3

This resource contains:

- ☑ Table of contents
- ☑ Teaching notes
- ☑ In class activity sheets involving
 - worked examples
 - basic skills
 - word problems
 - problem solving
 - group work





☑ Homework / Assessment activity sheets

☑ Answers

These resources are supplied as PHOTOCOPY MASTERS

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This resource unit may be used as a master, and therefore can be photocopied, only by the school or institution that has purchased this resource unit.

Note from the author:



This resource ...

*A Complete Guide to Algebra

is one of a series of FIVE resources written utilising the objectives as stated in

Mathematics in the New Zealand Curriculum for Level 3.

With my experiences as a specialist mathematics teacher, I enjoyed mathematics as a subject, but I am aware that not all teachers feel the same way about mathematics. It can be a difficult subject to teach, especially if you are unsure of the content or curriculum and if resources are limited.

This series of resources has been written with you in mind. I am sure you will find this resource easy to use and of benefit to you and your class.



For more information about these and other resources, please contact ...



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Acknowledgement:

I would like to thank the staff and pupils of **Mairehau Primary School, Christchurch** for their assistance in making these resources possible.

This resource has been divided into EIGHT sections as listed below.

Although there are no page numbers, the sections follow in sequential order as listed.

Note: 'In-class' Worksheets Masters are lesson by lesson reuseable worksheets that can be photocopied or copied on to an OHP.

Homework / **Assessment Worksheets Masters** can be used as homework to reinforce work covered in class or they can be used for pupil assessment.





Algebra

The following are the objectives for Alegbra, Level 3, as written in the

MATHEMATICS in the New Zealand Curriculum document, first published 1992. [REFER PAGE 138]

Exploring patterns and relationships

Within a range of meaningful contexts, students should be able to:

- A1 describe in words, rules for continuing number and spatial patterns;
- A2 make up and use a rule to create a sequential pattern;
- A3 state the general rule for a set of similar practical problems;
- A4 use graphs to represent number , or informal, relations.

Exploring probability

Within a range of meaningful contexts, students should be able to:

• A5 solve problems of the type \square + 15 = 39.

At the top of each '**In-class' worksheet** and **Homework / Assessment worksheet**, the Algebra objective(s) being covered has been indicated. *Example:* **A1** means objective 1, **A2** means objective 2, etc.



The Mathematical Processes Skills: Problem Solving,

Developing Logic & Reasoning,

Communicating Mathematical Ideas,

are learned and assessed within the context of the more specific knowledge and skills of number, measurement, geometry, algebra and statistics. The following are the **Mathematical Processes Objectives** for **Level 3**.

Problem Solving Achievement Objectives [Refer page 24]

- MP1 pose questions for mathematical exploration;
- MP2 effectively plan mathematical exploration;
- **MP3** devise and use problem-solving strategies to explore situations mathematically;
- **MP6** use equipment appropriately when exploring mathematical ideas.

Developing Logic and Reasoning Achievement Objectives [Refer page 26]

- **MP8** classify objects, numbers and ideas;
- **MP9** interpret information and results in context;
- MP14 use words and symbols to describe and continue patterns.

Communicating Mathematical Ideas Achievement Objectives [Refer page 28]

- **MP15** use their own language and mathematical language and diagrams to explain mathematical ideas;
- **MP16** devise and follow a set of instructions to carry out a mathematical activity;
- MP18 record, in an organised way, and talk about the results of mathematical exploration.

Note:

The codes MP1, MP2, etc. have been created by numbering the Mathematical Processes Achievement Objectives in order as listed in the MATHEMATICS in the New Zealand Curriculum document. The numbering gaps occur as not all objectives are covered at Level 3. [REFER TO PAGES 23 - 29 OF THE CURRICULUM DOCUMENT]

'In-class' Algebra Worksheets Table of Worksheet Number / Objectives Covered

See the opposite page for details of each objective.

	Al	Algebra Objectives				Mat	hem	atica	l Pro	cess	es Ob	ojecti	ives		
Worksheet Number	A 1	A 2	A 3	A 4	A 5	МР 1	MP 2	MP 3	MP 6	MP 8	МР 9	MP 14	MP 15	MP 16	MP 18
1		Re	evisi	on				*			*			*	
2	*							*			*	*		*	
3	*	*						*			*	*			
4	*	*						*			*	*		*	
5		*						*			*	*		*	
6			*					*			*	*			
7		*			*			*			*				
8				*							*				
9				*							*		×		
10				*				*			*	*			
11					*			*						*	
12					*			*			*		*		
13					*			*			*		*		
14					*			*			*		*		
15					*			*			*			*	

Table of Contents for the 'In-class' Worksheet Masters for Algebra, Level 3

Worksheet Number	Торіс	Algebra Objective(s)			
1	Mathematical signs / Renaming numbers / Finite & infinite	Revision			
2	Continuing and describing shape patterns	A1			
3	Continuing and describing number patterns	A1 / A2			
4	Special number patterns / Creating patterns given a rule				
5	Using a rule to create a number sequence	A2			
6	Finding a rule given input / output numbers	A3			
7	Practical problems involving rules	A2 / A5			
8	Relationship graphs	A4			
9	Map grids	A4			
10	Map grid references / Mathematical graphs	A4			
11	'Guess the number' game problems	A5			
12	Introduction to equations	A5			
13	Solving equations involving + & -	A5			
14	Solving equations involving × & ÷	A5			
15	Using formulae	A5			
	Teaching Notes / Answers				



Copy each question, then replace the between each pair of numbers, with one of the three symbols for 'is equal to', 'is less than' and 'is greater than', to show that you understand what the signs mean.

1.	12 1	.6	2.	17	16	3.	21 2	1	4.	86	89
5.	27.6	27.7	6.	0.4	0.8	7.	45.36	45.41	8.	92.78	92.75
9.	9 + 7	8 + 8	10.	7 × 4	36 - 5	11.	48 ÷ 4	23 - 14	12.	16 + 15	5 4×8



Task 2

Renaming numbers:

It is possible to rename any number in many different ways, using all four mathematical signs +, –, ×, and $\div.$

Example: The number 12 can be written as ... 2 + 10, 17 - 5, 4 × 3, 24 ÷ 2

Rename each number **four** times, each time using a different mathematical sign ($+, -, \times$, and \div).

1.	8	2.	11	3.	13	4.	20
5.	17	6.	25	7.	32	8.	63
9.	75	10.	100	11.	120	12.	150

Finite and Infinite:

What do they mean?

If you stand in your classroom and look at the door, you could measure the distance from you to the door. This distance is said to be a **finite** distance. A finite distance can be measured.

If you look up into the night sky, some stars are so far away that it would be almost impossible to measure the distance from you to the stars. If you cannot measure the distance between two objects, the distance is said to be **infinite**.



Do you understand the difference between finite and infinite?

- Make a list of **five** things that are finite. Example: The number of pupils at your school.
- 2. Make a list of **three** things that are infinite. *Example:* The counting numbers.









 $|| \langle \langle \rangle \rangle | \langle \rangle$



9. **Draw** the next shape in this match stick pattern.







- 10. How many triangles will there be in the next shape of this pattern?
- 11. How many matchsticks will there be in the next shape?



Continuing and describing number patterns:

When a series of **numbers** form a pattern it is called a **sequence**. The numbers of a sequence are often found by adding or subtracting the same number to each previous number.

Example: Listing the number of squares in each diagram will create a sequence of numbers. How many squares are added to each previous diagram of this sequence? How many squares are in the next diagram?





Answer: Numbers would be 2, 4 and 6. Two squares are added to each previous diagram, with the next diagram having 8 squares.

What do you think the next 5 numbers will be in this sequence?

Task 5

Count the number of dots in each diagram and **write** your answers as a number sequence.

1.	•••	••••	•••••	2.	••	••••	
	00000	00000	00000		00000	00000	00000
	00000	00000	00000		00000	00000	00000
3.	00000	00000	00000	4.	00000	00000	00000
	00000	00000	00000	••	00000	00000	0000
	00000	000	0		00000	00	

5. **Describe** how each sequence was created and **write** the next **five** numbers in the sequence.

Look at each number sequence to work out each pattern, then write the next five numbers in each sequence. Describe how each of these number sequences has been created.

6.	5, 11, 17,	7.	63, 58, 53,	8.	3, 14, 25, 36,
9.	7, 16, 25,	10.	2.0, 2.5, 3.0,	11.	24.6, 23.9, 23.2, .

12. Karen has weekly meetings on Tuesdays, starting on the 3rd of July. If the meetings are to go on for six more weeks, what are the dates for each meeting?

Continuing and describing number patterns:

The numbers of a sequence can also be created by multiplying each previous number by the same number.

Example: The sequence, 2, 4, 8, 16, ... is created by multiplying each previous number by 2.

What do you think the next 5 numbers will be in this sequence?



Task 6

Look at each number sequence to work out each pattern, then write the next five numbers in each sequence. Describe how each of these number sequences has been created.

- 1. 4, 12, 36, ...
- 2. 1, 5, 25, ...

3. 3, 18, 108, ...

4. A bean plant doubles its height every week. If it was 7cm high after week one, how tall will it be at the end of each of the next 5 weeks?



Special number patterns:

Some number sequences are based on special numbers, such as odd or even, multiples, squares, cubes, triangular, pentagonal, hexagonal numbers.

Task 7

These diagrams show the first three triangular, square, pentagonal and hexagonal numbers. **Look** at each pattern. **Draw** the next 2 diagrams for each pattern, then **list** the number of dots in each diagram to continue each sequence.





Creating patterns given a rule:

Peter builds a book shelf out of bricks and planks of wood, as shown in this diagram. For every book shelf layer, Peter needs two bricks and one plank.

Write the sequence of numbers to show how many bricks are needed for the first 10 layers of this book shelf.

Answer: 2, 4, 6, 8, 10, 12, 14, 16, 18, 20

The rule for this sequence is 'the number of bricks needed is twice the number of layers in the book shelf'.

Task 8

Given each rule, work out the first **five** numbers in each sequence.

- 1. Start with 5, then multiply each new number by 2.
- 2. Start with 4, then add 13 to each new number.
- 3. Start with 100, then subtract 17 from each new number.
- 4. Make up **five rules** of your own. Check that your rules work by having a classmate create the first five numbers of your sequences using your rules.
- 5. New desks for a classroom can seat 4 pupils around each desk. Use the rule, 'the number of chairs needed is four times the number of desks', to work out the number of chairs needed if there are seven desks.
- 6. If there are 32 pupils in this class, how many desks and chairs are needed?







Using a rule to create a number sequence:

Example: Use the rule in the box to create part of a number sequence.



Use the **rules** in each box to find the missing 'output' numbers to create part of these number sequences.



- 5. The 'input' number is 13. If the rule is **'multiply by 3, then subtract 7'**, what is the output number?
- 6. The 'input' number is 24. If the rule is 'divide by 4, then add 9', what is the output number?





- 7. The 'input' number is 7. If the rule is **'add 3**, then multiply by 4' what is the output number?
- 8. The 'input' number is 19. If the rule is 'subtract 4, then divide by 5', what is the output number?

Task 10

Create five of your own diagrams with **4 'input' numbers** and a **rule**. Exchange your diagrams with a classmate, so that they can work out the 'output' numbers.



Finding a rule given the input / output numbers:

Example: Work out the rule that created these input / output numbers.



To work out the rule, look at each pair of numbers ... 1 and 8, 3 and 10, 5 and 12, 7 and 14.

What is common about the difference between these numbers?

Input

1 -

2 -

3 -

4

Rule

Output

4

7 10

13

The difference between each pair is 7. Answer: The rule is therefore, 'add 7'.

More difficult rules may require two steps, such as multiplying first, then adding or subtracting.

Can you work out the rule for this input / output diagram?

Rules of this type can be worked out by a 'trial and error' method.

Answer: Multiply by 3, than add 1.

Task 11

Work out the **rules** that would go in the boxes between these input / output numbers. All of these rules involve only adding or subtracting.



Work out the **rules** that would go in the boxes between these input / output numbers. These two rules involve multiplying first, followed by either adding or subtracting.



Task 12

Create five of your own rules and use them to create 4 input / output numbers, drawing diagrams as above. Exchange your 5 diagrams with a classmate, so that they can work out your rules.



When you buy anything from a shop, a rule can be written. Example: Buying hamburgers and chips.

One rule could be, *'each hamburger costs \$2.95'*.

The other rule could be, 'one scoop of chips costs \$1.20'.

Using these rules, we can work out the cost of buying any combinations of hamburgers and chips.





- 1. Use the **rules** in each box above to work out the cost of buying 1, 2, 3 and 4 hamburgers and the cost of buying 1, 2, 3, and 4 scoops of chips.
- 2. Use your answers above to work out the cost of buying 3 hamburgers and two scoops of chips.
- 3. How many scoops of chips could you buy with \$7.20?
- 4. How many hamburgers could you buy with \$29.50?
- 5. What would it cost to buy 5 hamburgers and 6 scoops of chips?



When you buy something by mail-order, there is often a charge for the postage. This diagram below shows the cost of CD's, plus postage for mail orders.



- 6. Work out the cost of buying 1, 2,3 and 4 CD's by mail order.
- 7. What would it cost to buy 12 CD's by mail-order?
- Sam spent \$65.00 on mail order CD's. How many CD's did he buy?

A school is running a fair on Saturday. They are going to have a barbecue and sell sausages. For every person they expect at the fair, they will buy 2 sausages and to that total will add 6 extra sausages.

- 9. Use the **rule** to work out how many sausages are needed if 50, 75, 100 and 140 people were to turn up.
- 10. If sausages cost 50 cents each, how much would it cost to buy sausages for 100 people?
- 11. If sausages sell for \$1.20 each, how much money would they raise if they sell 126 sausages?
- 12. How many people could they feed if they purchased 56 sausages?







Draw three relationship graphs of your own. Remember to name the relationship that you are drawing about. *Example: 'The moods I felt during a scary movie.'*





Map grids:

Have you ever looked for a street or a place on a map? How do you know which part of the map to look at? All maps usually have **grid references**, so that it makes it easier to find what you are looking for.

Example: On this grid all squares can be defined by a letter and a number, such as (A,4). The A means the first column and the 4 means the fourth row up.
(A,4) is known as a grid reference.

What shape is in the square (A,4)? Answer: The pentagon.

In which square is the hexagon? Answer: (F,2).

Name all the other shapes and the squares they are in.

Task 16

Miri draw a grid of her room, showing the major items.



What items are in the squares ... (D,1) (B,3) (A,1) and (D,3)? Write the grid references for the fireplace and the desk chair.

Use this grid to break the code and find out what this message says.

- (E,4), (B,6), (B,2), (E,5), (B,5), (E,4), (B,6), (B,2), (A,4),
 (D,6), (A,2) / (A,4), (A,2) / (D,5), (E,3), (B,5), (B,6),
 (B,2) / (C,5), (C,2), (A,3).
- Write your own message using this grid.
 Exchange messages with a classmate and see if you can decode their message.

Task 17

1. **Draw** a plan of your classroom on maths paper so that you can have a grid set up with letters across the bottom and numbers up the side.

1.

2.

2. Using your grid references, state where 10 items in your classroom can be found.





1	ШП	×	u y	z	
	5		u	v	vv
2	5	+		v	14/
3	n	0	р	q	r
4	i	j	k	Ι	m
5	d	e	f	g	h
6		۵	b	с	





Map grid references / mathematical graphs:

Grid references on a map refer to anything within the square or grid named.

Example: On this grid the square (B,2) is shaded. If this was a map, there could be many streets, buildings, places etc. within the shaded square.

On a **mathematical graph** we have numbers on both sides, where the first number always means across to the right, and the second number always means up.

Example: The letter A marks the point (3,2) on this graph. This means 3 lines to the right and 2 lines up. An X is marked on the graph, where the two lines cross, and it is this point only that (3,2) refers to. A dot could be used instead of an X.

What two numbers would you use to refer to the points **B**, **C** and **D**?

Answer: B = (4,1), C = (2,4), D = (1,3). Remember, the order is important.







On this graph there are **five** points marked. What two numbers are used to refer to each point? **Draw** a graph that is the same as drawn here. On your graph, **mark** the points.

F = (5,1) G = (1,4) H = (2,3) I = (4,5) J = (0,2)

Use this graph to break the code and find out what this message says.

- (1,1), (5,1) / (10,3), (5,1,), (7,1) / (3,0), (5,5), (5,1), (8,2) / (0,4), (4,6), (4,6) / (10,3), (5,1), (7,1), (6,2) / (1,6), (0,4), (7,6), (3,4), (1,3) / (5,5), (7,1), (4,3), (1,6), (2,8), (6,2) / (2,5), (0,4), (1,3), (7,4), (7,6)?
- Write your own message using this graph.
 Exchange messages with a classmate and see if you can decode their message.





The line on this graph shows the relationship between the cost of a chocolate bar and the number of chocolate bars bought.

- Point A would be written as (1, \$1.00). What would you write for the points B, C, D and E.
 - What is the rule for this relationship?
 - Use your rule to work out the cost of buying 12 chocolate bars.
 - How many chocolate bars could you buy with \$16.00?





'Guess the number' game:

Consider these problems ...

Think of a number	'Trial and err	or' is one v	vay to work out th	iese problems.
I add 13. The result is 25. What is the number?	<i>Example:</i> Т Т Т	ry 10. ry 14. ry 12.	10 + 13 = 23 14 + 13 = 27 12 + 13 = 25	The result is not 25. The result is not 25. The result is 25, so 12 was the number.
Think of a number. I multiply by 2. then add 5	Use 'trial <i>Example:</i>	and error Try 2.	' to work out this 2 × 2 + 5 = 9	problem. The result is not 13.

I multiply by 2, then add 5. The result is 13. What is the number?

ial c	and error' ·	to work out this pro	blem.
le:	Try 2.	2 × 2 + 5 = 9	The result is not 13.
	Try 5.	5 × 2 + 5 = 15	The result is not 13.
	Try 4.	4 × 2 + 5 = 13	The result is 13, so 4 was
			the number.

Can you think of another way to work these problems out?

Task 19

Use a 'trial and error' method to 'guess' these numbers.

1.	Think of a number.	2.	Think of a number.	3.	Think of a number.		
		I add 10.		I subtract 9.		I subtract 17.	
	The result is 27.			The result is 14.		The result is 8.	
	What	is the number?		What is the number?		What is the number?	
4.	Thin	k of a number.	5. (Think of a number.	6.	Think of a number.	
	Im	ultiply by 7.		I divide by 6.		I multiply by 12.	
	The result is 56.			The result is 9.		The result is 108.	
What is the number?			What is the number?		What is the number?		
	7.	Think of	a number.	8. (Think of a	number.	
		I multiply by	3. then add 4.		I divide by 2.	then add 5.	
		The res	sult is 13.		The resu	lt is 13.	
		What is th	he number?		What is the	number?	
	9.	Think o	f a number.	10.	Think of	a number.	
		I multiply by 2	, then sub trac	<i>t 5.</i>	I divide by 3, t	hen subtract 5.	
		The re	esult is 13.		The result is 3.		
		What is	the number?	J	What is th	e number?	

- 11. Make up **5** of your own problems, following these steps.
 - Step 1: Choose a number less than 12.
 - Step 2: Multiply by a number less than 8.
 - Step 3: Add a number less than 25. The result is ...?
 - Step 4: Write your problem in words as above.

Give your problems to a classmate to work out or solve. You might like to make up some more problems that involve dividing and subtracting instead of multiplying and adding.



Introduction to equations:



This diagram is of a set of scales.



The two circles are not as heavy as one square. These scales

are **not balanced** as there are different weights on each side.

What is needed to balance these scales?

On this set of balance scales, the three circles are the same weight as one square. These scales are **balanced** because there is the same weight on both sides.

If there were two squares on one side, how many circles would you need on the other side to balance the scales?

Answer: 6 circles

Task 20

questions below.

These three scales are



These scales have the number 5 on the left side and the number 9 on the right side. We could write this as $5 + \boxtimes = 9$, where the \boxtimes is the number missing, needed to balance these scales.

ℬ

5 +
$$\boxtimes$$
 = **9** is called **an equation**.
This equation can also be written as **9** – \boxtimes = **5**.

What number needs to be added to the left side to balance these scales? Finding this number is called **solving the equation**.

Answer: The number 4, because 5 + 4 = 9 and 9 - 4 = 5.

balanced. Use the information ወወ in these scales to answer these

What **shapes** need to be **added** to the left side of these scales, so that each **scale balances**? Find more than one way that each scale can be balanced.



Below are some more scales that can be written as equations. *Example:* $6 + \mathbb{E} = 14$ or $14 - \mathbb{E} = 6$ Write two equations for each set of scales, then work out or solve each equation.



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Solving equations:

 $12 - \boxtimes = 5$ $\boxtimes + 14 = 21$ $9 - 5 = \boxtimes$ $\boxtimes - 14 = 21$ are all equations.



The symbol \boxtimes represents the missing number, that you are trying to find. However, usually a letter is used in an equation. *Example:* 12 + d = 21.

Solving an equation means, 'find the missing number' that would replace the letter or symbol so that both sides equal each other. Equations can be worked out using a 'trial and error' method.

Can you come up with any other methods for solving equations?

Task 21

Solve these equations.

1.	12 + a = 26	2.	b + 17 = 31	3.	24 - c = 19	4.	d - 9 = 23
5.	f + 24 = 49	6.	24 - 15 = g	7.	49 - 23 = h	8.	i + 26 = 26
9.	41 + 27 = j	10.	k + 28 = 49	11.	47 - l = 29	12.	37 - m = 5
13.	65 + n = 81	14.	81 - 47 = o	15.	p - 54 = 13	16.	64 + 87 = q
17.	r + 45 = 102	18.	92 - s = 27	19.	105 - 59 = t	20.	u + 51 = 119

Writing and solving equations:

Consider this problem.

Jim has \$12.00 and was given some more money for his birthday. If he now has \$26.00, how much was he given for his birthday?

Write this information as an equation, then solve it.

2.

Answer: \$12.00 + x = \$26.00, where 'x' equals the birthday money given. x = \$14.00, as \$12.00 + \$14.00 = \$26.00.

or it could be written as \$26.00 - \$12.00 = x , where 'x' equals the birthday money given. x = \$14.00, as \$26.00 - \$12.00 = \$14.00.

Task 22

Write an equation for each of these word problems, then work out the answer.

1. In two classes at a school there are 54 pupils. If there are 29 pupils in one class, how many are in the other class?



- Mr Smith has been in a meeting for 34 minutes. The meeting is expected to last for a total of 75 minutes. How long before the meeting ends?
- Kerry bought 52 soccer cards, but gave away 18 to his friends. How many soccer cards does he now have?



Make up **5** of your own word problems, similar to those above, that can be written as equations. Give your questions to a classmate to solve.







Solving more equations:

These equations below involve multiplication or division. Can you solve them?

6 × a = 24	a = ?
<i>b</i> × 8 = 40	b = ?
$12 \times 4 = c$	<i>c</i> = ?
24 ÷ <i>d</i> = 3	<i>d</i> = ?
<i>e</i> ÷ 4 = 5	e = ?
32 ÷ 2 = f	<i>f</i> = ?

Explain your method for solving these equations above. Did you use a 'trial and error' method?

Answers: a = 4, b = 5, c = 48, d = 8, e = 20, f = 16,

Task 23

Solve these equations.

1.	4 × a = 36	2.	b × 7 = 56	3.	64 ÷ c = 8	4.	d ÷ 9 = 5
5.	f × 9 = 108	6.	42 ÷ 7 = g	7.	9 × 8 = h	8.	i × 7 = 77
9.	12 × 8 = j	10.	k ÷ 11 = 11	11.	84 ÷ l = 12	12.	150 ÷ m = 30
13.	15 × n = 45	14.	63 ÷ 9 = o	15.	p ÷ 12 = 6	16.	60 × 8 = q
17.	r × 25 = 200	18.	60 ÷ s = 15	19.	105 ÷ 5 = t	20.	u × 30 = 360

Writing and solving equations:

Consider this problem.

Mary buys 6 new books for the school library. If each book costs the same price and the total cost was \$30.00, how much did each book cost?

Write this information as an equation, then solve it.

Answer: $6 \times x = 30.00 , where 'x' equals the cost of one book. x = \$5.00, as $6 \times $5.00 = 30.00 .

or it could be written as \$30.00 \div 6 = \pmb{x} , where ' \pmb{x} ' equals the cost of one book. \pmb{x} = \$5.00, as \$30.00 \div 6 = \$5.00.

Task 24

Write an equation for each of these word problems, then work out the answer.

 David has been running for 72 minutes around a park. How many laps of the park has he done, if each lap takes 6 minutes?



- A chocolate bar is to be divided up equally among 6 pupils. If there are 54 squares of chocolate in the bar, how many squares does each pupil get?
- 27 pupils are going on the school camp.
 If each pupil has to pay \$30.00, what is the total cost of the trip?



Make up **5** of your own word problems, similar to those above, that can be written as equations. Give your questions to a classmate to solve.







Using a formula:

A formula is like a rule and can be used to work things out.

Example: Chocolate bars cost \$1.00 each. A formula can be written, that can be used to work out the total cost of any number of chocolate bars bought.

Total cost of chocolate bars = $1.00 \times \text{Number of chocolate bars bought}$

This can be written as letters rather than in words.

Let **C** = 'cost of chocolate bars'

and **N** = 'number of chocolate bars bought'

A simple formula can be written,

, $\mathbf{C} = \$1.00 \times \mathbf{N}$

Use this formula to find the cost of 25 chocolate bars.

Answer: Replace the N with 25.

C = \$1.00 × 25 [Note: Working goes down the page, not across.] C = \$25.00

Task 25

Use each formula given to work out these problems.

The cost of hamburgers (H) is given by the formula,

 $\mathbf{H} = \$2.50 \times \mathbf{N}$ where N is the number of hamburgers bought.

- 1. How much would it cost to buy 3 hamburgers?
- 2. How much would it cost to buy 9 hamburgers?

4.

5. 6.

10.

3. If Michael spent \$12.50 on hamburgers, how many hamburgers did he buy?



The cost of ice-creams (C) is given by the formula,

 $(\mathbf{C} = \$0.80 \times \mathbf{S})$ where S is the number of scoops.

- How much would a two scoop ice-cream cost?
- How much would a three scoop ice-cream cost?
- How much would it cost to buy 3 two scoop ice-creams?

The cost of going to the movies (M) is given by the formula,

 $\mathbf{M} = \mathbf{\$6.50 \times P}$ where P is the number of people who buy tickets.

- 7. How much would it cost for 12 people to go to the movies?
- 8. How much would it cost for 25 people to go to the movies?
- 9. If \$65.00 was spent on tickets, how many people went to the movies?



The area (A) of a rectangle is given by the formula,

 $\mathbf{A} = \mathbf{B} \times \mathbf{H}$ where B is the base and H is the height.

- What is the area of a rectangle if the base is 10cm and the height is 8cm?
- 11. What is the area of a rectangle if the base is 20cm and the height is 9cm?

The cost (C) of time on the Internet is given by the formula,

 $\mathbf{C} = \mathbf{\$2.50 \times H}$ where H is time in hours.

12. How much would it cost if you were on the Internet for $5\frac{1}{2}$ hours?









'In-class' Worksheet

Teaching Notes & Answers

How to use this section:

Teaching notes are enclosed in a box with a 'push-pin' at the top left corner. The teaching notes precede the answers for each worksheet / task. The teaching notes have been included to provide assistance and background information about each topic or unit of work.

Introduction:

T

Worksheet 1

The topic of algebra is often considered difficult, but it is not as bad as it sounds. Algebra is concerned with creating patterns and sequences, finding relationships and drawing graphs, solving equations and using formulae. Many of the problems pupils already do are in fact solved using algebra skills without them realising it.

Worksheet 1, Task 1 is concerned with the revision and correct use of the signs '=', '<', and '>'.

Task 2 is to encourage pupils to think about different ways that we can write numbers. There will be an infinite number of answers for these questions and this leads on to **Task 3**.

Task 3 is concerned with what the words 'finite' and 'infinite' mean.

There are no model answers for Task 2.

ά	'act	1

1.	12 < 16	2.	17 > 16	3.	21 = 21	4.	86 < 89
5.	27.6 < 27.7	6.	0.4 < 0.8	7.	45.36 < 45.41	8.	92.78 > 92.75
9.	9 + 7 = 8 + 8	10.	7×4 < 36-5	11.	48÷4 > 23 - 14	12.	16 + 15 < 4 × 8

Task 3

J

Possible finite things Pos Number of pupils in your class. Number of people in New Zealand How much food you can eat at one time, etc.

Possible infinite things The counting numbers The different ways you can write the number 8 Time ? etc.

Continuing and describing shape patterns:

Worksheet 2

When shapes are drawn and you can see a pattern occurring, a sequence of shapes has been created. After working out how the diagram pattern was created, the sequence of diagrams can be continued.

Task 4 is designed to give practice at continuing diagram patterns, then describing in words how the pattern was created.



3.	0000	00000	00000	4.	\times	\times
	4th shape	5th shape	6th shape	4th shape	5th shape	6th shape

- 5. Question 1: The pattern was created by turning the arrows upside down for each new diagram. Question 2: Each new diagram had one circle added to end of each arm.
 - Question 3: Each new diagram had a rectangle with an oval inside and a diagonal line across, added to each new diagram.

11.

17 matches

- Question 4: Each new diagram had two circles and two lines added, in the same pattern.
- 6.

7.





10.

The pattern in these boxes was created by placing the three shapes in each row and column so that there was a different shape in each row and column.

Worksheet 3

9.

Continuing and describing number patterns:

When a series of numbers form a pattern it is called a sequence. Each number in the sequence is called a term. For number sequences involving addition or subtraction, there is a repeating pattern whereby each new term in the sequences is increased or decreased by the same number. Other sequences can be created by multiplying each new term by a common number.

8 triangles

Task 5 is designed to give practice at finding what the repeated addition or subtraction pattern is, describing each pattern in words, then continuing the sequence.

Task 6 is designed to give practice at finding what the repeated multiplication pattern is, describing each pattern in words, then continuing the sequence.

2.

Task 5

- 1. 3, 5, 7 2. 4, 7, 10 3. 25, 23, 21
- Question 1: Add 2 to each new term. Next 5 terms: 9, 11, 13, 15, 17 Question 3: Subtract 2 from each new term Next 5 terms: 19, 17, 15, 13, 11
- 6. 23, 29, 35, 41, 47: add 6
- 8. 47, 58, 69, 80, 91: add 11
- 10. 3.5, 4.0, 4.5, 5.0, 5.5: add 0.5

4. 25, 22, 19

Question 2: Add 3 to each new term. Next 5 terms: 13, 16, 19, 22, 25 Question 4: Subtract 3 from each new term. Next 5 terms: 16, 13, 10, 7, 4

- 7. 48, 43, 38, 33, 28: subtract 5
- 9. 34, 43, 52, 61, 70: add 9
- 11. 22.5, 21.8, 21.1, 20.4, 19.7: subtract 0.7

125, 625, 3125, 15625, 78125: multiply by 5

12. 3rd July, 10th July, 17th July, 24th July, 31st July, 7th August, 14th August

Task 6

- 1. 108, 324, 972, 2916, 8748: multiply by 3
- 3. 648, 3888, 23328, 139968, 839808: multiply by 6
- 4. 14cm, 28cm, 56cm, 112cm, 224cm





- 1. Rule: Add 6 2. Rule: Subtract 2
- 5. Rule: Multiply by 4, then add 1
- 3. Rule: Subtract 11 4. Rule: Add 9
- 6. Rule: Multipy by 2, then subtract 3

Worksheet 7

Worksheet 8

Practical problems involving rules:

The questions on this worksheet utilise rules to work out practical everyday problems. In everyday life, many problems can be worked out using algebraic skills and most times, pupils would do this without realising that algebra was involved. *Example:* Going shopping.

Task 13 is designed to give practice at using rules to solve practical everyday problems.

Task 13

1.	1 hamburger: \$2.95	1 scoop of chips: \$1.20							
	2 hamburgers: \$5.90	2 scoops of chips: \$2.40							
	3 hamburgers: \$8.85	3 scoops of chips: \$3.60							
	4 hamburgers: \$11.80	4 scoops of chips: \$4.80							
2.	\$8.85 + \$2.40 = \$11.25	3. \$7.20 ÷ \$1.20 = 6 scoops							
4.	\$29.50 ÷ \$2.95 = 10 hamburgers	5. \$14.75 + \$7.20 = \$21.95							
6.	1 CD: \$15.00 2 CD's: \$2	5.00 3 CD's: \$35.00 4 CD's: \$45.00							
7.	12 CD's: \$125.00 8. \$65.0)0 less postage divided by CD cost = 6 CD's							
9.	50 people: 106 sausages	75 people: 156 sausages							
	100 people: 206 sausages	140 people: 286 sausages							
10.	206 sausages × 50 cents = \$103.00	11. 126 × \$1.20 = \$151.20							
12.	25 people								

r

Relationship graphs:

Algebra is involved with sequences etc. where there is a relationship between each term in the sequence or the diagram in a pattern. For many situations where there is a 'connection', a relationship graph can be drawn. This worksheets introduces the idea of relationship graphs. This will lead on to relationship graphs that involve numbers.

Task 14 is designed to give practice at understanding relationship graphs.

Task 15 is designed to give practice at drawing relationship graphs.

Task 14

- 1. 2. John and Richard
- 3. John married Joanne, their children are Andrew, James and Kaye
- 4. Jan is George's grand daughter or George is Jan's grand father
- 5. Jan and Donald are brother and sister
- 6. James and Donald are cousins
- 7. Richard is Kaye's uncle or Kaye is Richard's niece
- 8. How Kerry felt about each question in a test
- 9. He thought it was very hard
- 10. He thought they were very easy
- 11. question 2: easy, question 4: ok, question 7: hard

Map grids:

Map grids are used to help locate places, therefore there is a relationship between the map grid reference and the place. So that we all locate the same place, the grid system was created. In the example in worksheet 9, the order in which the references are listed is the same as the order used when listing 'ordered pairs' on a mathematical graph. That is, across first followed by up. This has been done to make it easier when mathematical graphs are looked at in the next worksheet. Grid references refer to a 'box' on the map etc, within which the street, city etc can be located. It does not pin-point the exact spot, but gives a general area.

Task 16 is designed to give practice at using map grids.

Task 17 is designed to give pupils practice at drawing and using map grids.

Task 16

- 1. bed, soft chair, desk, telephone
- 3. Mathematics is great fun

2. (C,4) & (B,1) 4 -

4

. Mamemanes is great ful

Worksheet 10

Worksheet 11

x

Worksheet 9

 Map grid references / mathematical graphs:
 worksneet references

 The mathematical graphs (Cartesian graphs) in this worksheet only have the postive numbers. These

graphs are used to show relationships between a pair of numbers, called an 'ordered pair'. The axes are labelled. The vertical axis is labelled 'y' and horizontal axis is labelled 'x'. The order the numbers in the ordered pair are listed, is most important. It is always across left / right, followed by up / down. Again this is done so that the same position on the graph can be located by everyone. At this **y** level, only positive numbers are used, therefore numbers refer to (right, up).

Task 18 is designed to give practice at using mathematical graphs.

Task 18

- 1. A = (1,1), B = (4,2), C = (3,4), D = (2,2), E = (1,5) 2.
- 3. Do you know all your basic number facts?
- 5. B = (2 ,\$2.00), C = (3, \$3.00), D = (4,\$4.00), E = (5,\$5.00)
- 6. Rule: Cost of chocolate bars is \$1.00 multiplied by the number of bars bought
- 7. 12 × \$1.00 = \$12.00 8. 16 chocolate bars

'Guess the number' games:

Guess the number game is an introduction to equations. The ability to work these out in a logical way will be of benefit, as equations can be solved the same way. Trial and error is a good method and may be the most common at this level.

A mathematical way to work out the number is as follows. This will also apply to solving equations. *Example: Think of a number.*

l add 13. The result is 25 What is the number? This could be written as an equation x + 13 = 25

Addition and subtraction are opposite operations, as are multiplication and division. To solve, start with the 'result'. Then use the opposite operation to that listed. If it said add, then subtract. In the example above that means, 25 minus 13. Therefore, 12 is the number. This can be checked by going through the problem using this number.

Task 19 is designed to give practice at problem solving in a fun way.

Task 19

1.	17 (27 - 10 = 17)	2.	23 (14 + 9 = 23)	3.	25 (8 + 17 = 25)
4.	8 (56 ÷ 7 = 8)	5.	54 (9×6=54)	6.	9 (108 ÷ 12 = 9)
7.	3 (13 - 4 = 9, 9 ÷ 3 =	3)	8. 16 (13 -	5 = 8, 8	× 2 = 16)
9.	9 (13 + 5 = 18, 18 ÷ 2	= 9)	10. 24 (3 + 5	= 8, 8,	< 3 = 24)
11 -					

Worksheet 12

Introduction to equations:

An equation is a group of numbers, mathematical signs, variables (letter), plus an equals sign. Equations are like a set of old fashioned weighing scales where weights were used. When the scales were 'balanced', there was the same weight on both sides. The aim of solving equations is to find the 'missing' number or object that would 'balance, both sides. This can be done by taking off or adding the same to each side, until you come up with the answer. 'To solve' means to find the missing number.

Task 20 is designed to give practice at balancing scales using objects, then numbers and writing equations from the information displayed. Note: $6 + \mathbf{X} = 14$ is the same as $\mathbf{X} + 6 = 14$.

Task 20



Solving equations:

Worksheet 13 & 14

Solving equations can be done in several ways. Introducing equations through balancing scales is designed to reinforce the idea that information on both sides of the equal sign MUST balance. The aim is to find the value of the number which is being represented by a letter or shape.

One method of solving is 'trial and error'. Pupils replace or substitute a number for the letter, then work out that side of the equation.

A more formal method is to perform 'opposite operations'. Addition and subtraction are opposite operations, as are multiplication and division.

Example: x + 25 = 52. This equation is saying, *"a number plus 25 equals 52"*. This can be turned around to read, *"a number equals 52 minus 25"*. This would be written as x = 52 - 25. This method will work for most equations that involve addition, subtraction and multiplication.

Solving equations continued:

Example: $y \times 5 = 30$. This equation is saying, *"a number multiplied by 5 equals 30"*. This can be turned around to read, *"a number equals 30 divided by 5"*. This would be written as $y = 30 \div 5$.

Equations involving division, where the unknown or letter is being divided by a number, will work this way.

Example: $x \div 6 = 4$. This equation is saying, "*A number divided by 6 equals 4*". This can be turned around to read "*a number equals 6 multiplied by 4*". This would be written as $x = 6 \times 4$.

However, if a number is being divided by a letter, or has a number subtracted from it, then the method of using opposite operations does not work.

Example: $24 \div y = 3$. This equation is saying *"24 divided by a number equals 3"*. In this case, it can be rewritten to read "a number equals 24 divided by 3". This would be rewritten as $x = 24 \div 3$.

Example: 15 - y = 8. This equation is saying *"15 minus a number equals 8"*. In this case, it can be rewritten to read "a number equals 15 minus 8". This would be rewritten as y = 15 - 8.

As all of these equations only involve one step, showing working may not be necessary. However if working is shown, it is better to work down the page, with equal signs in line, rather than across the page. Developing good setting out habits will be most benefical when pupils learn to solve more complicated equations in later years.

Tasks 21 & 23 are designed to give practice at solving equations.

Tasks 22 & 24 are designed to give practice at writing equations given a word problem, then solving the equation created.

Task 21

1.	a = 14	2.	b = 14	3.	c = 5	4.	d = 32	5.	f = 25	6.	g = 9
7.	h = 26	8.	i = 0	9.	j = 68	10.	k = 21	11.	l = 18	12.	m = 32
13.	n = 16	14.	o = 34	15.	p = 67	16.	q = 151	17.	r = 5718.	s = 6	65
19.	t = 46	20.	u = 68								

Task 22

1. Let x = the number of pupils in the second class. x + 29 = 54 or x = 54 - 29, x = 25 pupils

2. Let y = the number of minutes left in the meeting. y + 34 = 75 or y = 75 - 34, y = 41 minutes

Let w = the number of cards Kerry has left. w + 18 = 52 or w = 52 - 18, w = 34 cards
 -

Task 23

1.	a = 9	2.	b = 8	3.	c = 8	4.	d = 45	5.	f = 12	6.	g = 6
7.	h = 72	8.	i = 11	9.	j = 96	10.	k = 11	11.	=7	12.	m = 5
13.	n = 3	14.	o = 7	15.	p = 72	16.	q = 480	17.	r = 8	18.	s = 4
19.	† = 21	20.	u = 12								

Task 24

1. Let z = the number of laps David has run. $z = 72 \div 6$ or $z \times 6 = 72$, z = 12 laps

- 2. Let y = the number of chocolate squares each pupil gets. $y = 54 \div 6$ or $y \times 6 = 54$, y = 9 squares
- 3. Let w = the total cost of the school camp. $w = 30×27 or $w \div 30 = 27$, w = \$810

4. -

Using formulae:

J

Worksheet 15

A formula is like a rule and can be used to work things out. Formulae are written like equations with letters, sometimes numbers, and an equals sign.

Example: A = bh, $A = \frac{1}{2} bh$

Formulae are worked out by replacing or substituting the letters with numbers. Many everyday situations involve calculations that could be written as formulae.

Task 25 is designed to give practice at using formulae for practical everyday problems.

Task 25

- \$7.50 1. 2. \$22.50 5 hamburgers 3. 4. \$1.60 5. \$2.40 6. \$4.80 7. \$78.00 8. \$162.50 9. 10 people
- 10. 80 cm^2 11. 180 cm^2 12. \$13.75

Table of Content for the Homework / Assessment Worksheet Masters for Algebra, Level 3

Worksheet Number	Торіс	Algebra Objective(s)			
1	Mathematical signs / renaming / finite & infinite / continuing diagram patterns	Revision A1			
2	Continuing number sequences / input & output diagrams / creating sequences given a rule	A2 / A3			
3	Relationship graphs / Map references / Mathematical graphs	A4			
4	Balancing scales / Solving equations / 'Guess the number	A5			
5	Using word and letter formulae	A5			
	Answers				



AWS



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			J	Ш
	Homer		LIMA ESTERIO	
		WUIK ASSC		m
				$\overline{}$
A : 1. 2. 3. 4. 5. 6. 7. 8. 9. 10.	10 'Quick Questions' 4.36 + 18.2 = 100 - 57.6 = 115 × 11 = 480 ÷ 40 = List the factors of 24 $$3.25 \times 9 =$ What is the time on this clock? Find 25% of \$6.40 How many grams in 5.6kg? If 8 books cost \$17.20, what does each book cost?	B: 1 Use the word for The cost of buyin <i>for every dozen</i> 1. What woul 2. What woul 2. What woul 3. The cost of buying formula, <i>the num</i> <i>plus \$2.50 poster</i> 5. What would order?	 Using a word formula ormula to work out these problems. ving eggs is given by the formula, <i>n eggs you buy it costs \$2.60'</i>. uld it cost to buy 2 dozen eggs? \$	en
l	what does each book cost?	6. How many	y book were bought with \$22.50?	
Use prob The is giv	C: Using a form each formula given to work out elems. cost of cartons of juice (J) ven by the formula, $J = $1.65 \times N$, where N cartons of	ula It these is the number of finitian boundst	D: More formulae To calculate the area of a square or rectangle, us the formula, $\mathbf{A} = \mathbf{b} \times \mathbf{h}$, where b = base and h = height. What is the area of these squares and rectangle	se ght
2. 3. In ec pupil for c 4. 5.	What would 5 cartons of jui $J = \dots$ What would 9 cartons of jui $J = \dots$ How many cartons of juice c with \$9.90? N = ach carton of juice there is end a class party is given by the for $C = P \div 3$, where P is the pupils in a How many juice cartons are class of 27 pupils? $C =$ Jim's class needed 11 carton many pupils in this class? P	ce cost? 	$\mathbf{A} = \frac{1}{2} \times \mathbf{b} \times \mathbf{h},$ where b = base and h = height. $\mathbf{A} = \frac{1}{2} \times \mathbf{b} \times \mathbf{h},$ what is the area of this triangle if the base is 12 cm and the height is 8 cm? $\mathbf{A} = \dots $	es? mm m ² es l ² . m se
1. 2. 3. In ea pupil for a 4. 5.	What would 5 cartons of jui $J = \dots$ What would 9 cartons of jui $J = \dots$ How many cartons of juice c with \$9.90? N = ach carton of juice there is en s. The number of cartons of class party is given by the for $C = P \div 3$, where P is th pupils in a How many juice cartons are class of 27 pupils? C = Jim's class needed 11 carton many pupils in this class? P Comments:	ce cost? 	$ \begin{array}{c} & & & & & & & & & & & & & & & & & & &$	es? mm m ² ess r ² . m

Homework / Assessment Worksheet

Answers

Worksheet 1

A: 1. 39.2 2. 328 3. 945 4. 90 5. 1,2,3,4,6,12 6. \$27.60 7. 10 past 10 8. \$1.40 9. 3500L 10. \$10.35 B: 1. < 2. =3. > 5. < 6. > 7. < 8. = 9. < 10. < 4. > C: 1. C 2. I 3. F 4. J 5. A 6. H 7. E 8. B 9. G 10. D D: 1. infinite 2. finite 3. infinite 4. finite, although it would be impossible to count 5. - 6. -E: 1. a ¼ turn to the right each time 2. add two circles and a rectangle 3. add four matches, forming two more triangles Worksheet 2 A: 1. 16.22 2. 254 3. 2628 4. 4 5. 7,14,21,28,35 6. \$20.00 7. 14:45 8. \$2.85 9. 4.7kg 10. \$11.25 **B**: 1. 8, 10, 12, ... 18, 20, 22 counting in two's, even numbers, adding two to each new number 2. 16, 32, 64, ... 512, 1024, 2048 multiplying each new number by 2 3. 88, 84, 80, ... 68, 64, 60 subtracting 4 from each new number 4. 24, 31, 38, ... 59, 66, 73 adding 7 to each new number 5. 125, 625, 3125, ... 78125, multiplying each new number by 5 C: 1. 16, 25, 34, 43, 52 2. 12, 48, 192, 768, 3072 3. 87, 74, 61, 48, 35 D: 1. A = 9, B = 13, C = 17, D = 21 2. Subtract 5 E: 1. \$23.90 2. \$47.80 3. \$119.50 4. \$298.75 5. \$16.00 6. \$40.00 7. \$88.00 8. \$124.00 9.8 CD's Worksheet 3 A: 1. 53.83 2. 247 4. 60 5. 1,2,3,6,9,18 6. \$25.55 7. 25 to 4 8. \$4.75 9. 1500mm 3. 1925 10. \$5.60 B: 5. Friday 1. Ben 2. Ben is the youngest 3. John, Bill, Jack, Ben 4. Tuesday 6. sick C: 1. (F,6) = Ripon Street, (C,3) = Norwich Quay 2. (A,4)3. (C,4) 4. (F,2) 5. (B,2) 6. (F,1)

D:

1. A = (6,7), B = (1,6), C = (3,0), D = (2,5), E = (0,4) 2. check pupil's graph 3. pentagon (5 sides)

A:

1. 16.16 2. 14.3 3. 3949 4. 12 5. 9,18,27,36,45 6. \$38.70 7. 06:25 8. \$2.65 9. 25.3cm 10. \$10.40

B:



3. X+12 = 23 or 23 - X = 12, X = 11

C:

1. a = 14 2. b = 48 3. c = 7 4. d = 240 5. e = 5 6. f = 96 7. g = 4 8. h = 3 9. i = 3510. j = 28**D:** 1. 31 2. 26 3. 24 4. 5 **E:** 1. 19 girls 2. 5 kgs of meat 3. \$119.40 4. \$42.75

Worksheet 5

A:

1. 22.56 2. 42.4 3. 1265 4. 12 5. 1,2,3,4,6,8,12,24 6. \$29.25 7. 5 to 5 8. \$1.60 9. 5600g 10. \$2.15 **B:** 1. \$5.20 2. \$1.30 3. 240 bricks 4. 15 metres long 5. \$37.50 6. 4 books **G:** 1. \$8.25 2. \$14.85 3. 6 cartons 4. 9 cartons 5. 33 pupils **D:** 1. 48 cm² 2. 72 cm² 3. 90 mm² 4. 81 cm² 5. 48 cm²

Tracking Sheet: 'In-class' Activity Sheets

	Comments								
Workshoot	Objectives								
15	A5								
14	A5								
13	A5								
12	A5								
11	A5								
10	A4								
9	A4								
8	A4								
7	A2 / A5								
6	A3								
5	A2								
4	A1 / A2								
3	A1 / A2								
2	A1								
1	Revision								
1. Celhec	Name								

Comments Worksheet Objectives 5 A5 A5 4 З **A4** A2 / A3 2 1 **A1** LACCORC Name

Tracking Sheet: Homework / Assessment Worksheets