Do Now work on level 4,5 and 6

WALT Solving Fractional equations

Success Criteria I know, that fractional equations can be simplified by finding the least common denominator (LCD) of the fractions. Each term is then multiplied by the fraction which makes the denominators the same (LCD) and then the numerators are equated.

Consider the following fractional equations:

$$\frac{x}{2} = \frac{x}{3} \quad \text{has a LCD of} \quad 2 \times 3 = 6 \qquad \text{and} \qquad \frac{5}{2x} = \frac{3x}{5} \quad \text{has a LCD of} \quad 2x \times 5 = 10x.$$

Example 5

Solve for x: $\frac{x}{3} = \frac{3}{2}$ $\frac{4}{x} = \frac{2}{3}$

 $\frac{x}{3} = \frac{3}{2}$

 $\frac{x \times 2}{3 \times 2} = \frac{3 \times 3}{2 \times 3}$ {to achieve a common denominator of 6}

{equating numerators}

 $\therefore \quad \frac{2x}{2} = \frac{9}{2}$

{dividing both sides by 2}

 $\therefore x = 4\frac{1}{2}$ {simplifying}

has LCD of 6

has LCD 3x

 $\therefore \frac{4 \times 3}{x \times 3} = \frac{2 \times x}{3 \times x}$ {to achieve a common denominator of 3x}

12 = 2x

{equating numerators}

 $\therefore \frac{12}{2} = \frac{2x}{2}$

{dividing both sides by 2}

 $\therefore x = 6$

{simplifying}

EXERCISE 10B

Solve for x:

$$\frac{x}{2} = \frac{3}{5}$$

$$\frac{x}{3} = \frac{2}{7}$$

$$\frac{x}{5} = \frac{2}{3}$$

a
$$\frac{x}{2} = \frac{3}{5}$$
 b $\frac{x}{3} = \frac{2}{7}$ **c** $\frac{x}{5} = \frac{2}{3}$ **d** $\frac{x}{4} = -\frac{2}{3}$

$$\frac{3}{x} = \frac{1}{2}$$

$$\frac{4}{x} = \frac{1}{7}$$

$$\frac{5x}{3} = \frac{1}{2}$$

f
$$\frac{4}{x} = \frac{1}{7}$$
 g $\frac{5x}{3} = \frac{1}{2}$ h $\frac{7}{2x} = -\frac{1}{3}$

CROSS MULTIPLICATION

If two fractions are equal we can use cross multiplication to form an equation without denominators.

Notice that $\frac{4}{6} = \frac{2}{3}$ and so $4 \times 3 = 2 \times 6$.

We can use cross multiplication of two equal fractions to avoid having to equalise denominators.

Example 6

Solve for x: $\frac{7}{x} = \frac{4}{3}$

$$\frac{7}{x} = \frac{4}{3}$$

 \therefore 4 × x = 7 × 3 {cross multiplying}

 \therefore 4x = 21 {simplifying}

 $\therefore \quad \frac{4x}{4} = \frac{21}{4} \qquad \{\text{dividing both sides by 4}\}$

 $\therefore \quad x = 5\frac{1}{4} \quad \{\text{simplifying}\}\$



To keep the unknown on the left hand side multiply x by 4 first and then 3 by 7.

Note: We are using the rule:

"if $\frac{a}{b} = \frac{c}{d}$ then $a \times d = b \times c$ ".

- Solve for x:
 - $\frac{x}{7} = \frac{2}{5}$

- **b** $\frac{x}{9} = \frac{1}{2}$ **c** $\frac{x}{8} = -\frac{1}{3}$ **d** $\frac{2x}{3} = \frac{1}{5}$

- $\frac{3}{x} = \frac{1}{2}$
- $\frac{1}{x} = \frac{3}{4}$
- $\frac{6}{7} = \frac{4}{x}$
- $-\frac{3}{4} = \frac{4}{3x}$

Example 7

Solve for x: $\frac{x}{2} = \frac{3+x}{5}$

Notice the insertion of brackets here.

$$\frac{x}{2} = \frac{3+x}{5}$$

has LCD = 10

$$\therefore \quad \frac{x \times 5}{2 \times 5} = \frac{2(3+x)}{2 \times 5}$$

{to create a common denominator}

$$\therefore 5x = 2(3+x)$$

{equating numerators}

$$5x = 6 + 2x$$

{expanding brackets}

$$\therefore 5x - 2x = 6 + 2x - 2x$$

 $\therefore 5x - 2x = 6 + 2x - 2x$ {taking 2x from both sides}

$$\therefore$$
 $3x = 6$

$$\therefore \quad 3x = 0$$
$$\therefore \quad x = 2$$

{dividing both sides by 3}

or
$$\frac{x}{2} = \frac{3+x}{5}$$

$$5x = 2(3+x)$$
 etc. {cross-multiplying}

Solve for x:

$$\frac{x+2}{2} = \frac{3}{7}$$

b
$$\frac{3}{5} = \frac{x+1}{6}$$

$$\frac{x}{5} = \frac{x-2}{3}$$

$$\frac{x+1}{3} = \frac{2x-1}{8}$$

$$\frac{2x}{3} = \frac{5-x}{4}$$

$$\frac{3x+2}{3} = \frac{2x-5}{2}$$

Challenge

Example 8

Solve for
$$x$$
: $\frac{2x+1}{3-x} = \frac{3}{4}$.

$$\frac{2x+1}{3-x} = \frac{3}{4}$$

has LCD 4(3-x)

$$\therefore$$
 4(2x+1) = 3(3-x)

{cross multiplying}

$$\therefore 8x + 4 = 9 - 3x$$

{expanding the brackets}

$$\therefore 8x + 4 + 3x = 9 - 3x + 3x$$
 {adding 3x to both sides}

$$11x + 4 = 9$$

$$11x + 4 - 4 = 9 - 4$$

{subtracting 4 from both sides}

$$\therefore$$
 11 $x = 5$

$$\therefore x = \frac{5}{11}$$

{dividing both sides by 11}

More explanation video

Solve for x:

$$\frac{2x+3}{x+1} = \frac{4}{3}$$

$$\frac{x+1}{1-2x} = \frac{3}{5}$$

$$\frac{2x-1}{4-3x}=-\frac{2}{3}$$

$$\frac{x+3}{2x-1} = \frac{1}{2}$$

$$\frac{4x+3}{2x-1}=6$$

$$\frac{3x-2}{x+4} = -5$$

$$\frac{6x-1}{3-2x}=10$$

h
$$\frac{5x-1}{x+4} = 5$$

$$3 + \frac{2x+5}{x-1} = -1$$

Extension

Example 9

Solve for
$$x$$
: $\frac{x}{3} - \frac{1 - 2x}{6} = -4$

$$\frac{x}{3} - \frac{1-2x}{6} = -4$$
 has LCD of 6

$$\therefore \quad \frac{x}{3} \times \frac{2}{2} - \left(\frac{1-2x}{6}\right) = -4 \times \frac{6}{6} \qquad \text{{to create a common denominator}}$$

$$\therefore$$
 $2x - (1 - 2x) = -24$ {equating numerators}

$$\therefore 2x - 1 + 2x = -24$$
 {expanding}
$$\therefore 4x - 1 = -24$$

$$\therefore$$
 $4x-1+1=-24+1$ {adding 1 to both sides}

$$4x = -23$$

$$\therefore$$
 $x = -\frac{23}{4}$ {dividing both sides by 4}

Video explanation with multiple common denominators

5 Solve for x:

a
$$\frac{x}{2} - \frac{x}{6} = 4$$
 b $\frac{x}{4} - 3 = \frac{2x}{3}$

$$\frac{x}{8} + \frac{x+2}{2} = -1$$
 $\frac{x+2}{3} + \frac{x-3}{4} = 1$

i
$$\frac{x}{5} - \frac{2x-5}{3} = \frac{3}{4}$$
 i $\frac{x+1}{3} + \frac{x-2}{6} = \frac{x+4}{12}$

$$k \quad \frac{x-6}{5} - \frac{2x-1}{10} = \frac{x-1}{2} \qquad \qquad l \quad \frac{2x+1}{4} - \frac{1-4x}{2} = \frac{3x+7}{6}$$