[Do Now work on level 4,5 and 6]

WALT Solving Fractional equations

Success Criteria I know,that fractional equations can be simplified by finding the **least common denominator (LCD)** of the fractions. Each term is then multiplied by the fraction which makes the denominators the same (LCD) and then the numerators are equated.

Consider the following fractional equations:

$\frac{x}{2} = \frac{x}{3}$ has a LCD of $2 \times 3 = 6$ and $\frac{5}{2x} = \frac{3x}{5}$ has a LCD of $2x \times 5 = 10x$.

Example 5

Solve for x: **a** $\frac{x}{3} = \frac{3}{2}$ **b** $\frac{4}{x} = \frac{2}{3}$

a

$\frac{x}{3} = \frac{3}{2}$ has LCD of 6

$\frac{x \times 2}{3 \times 2} = \frac{3 \times 3}{2 \times 3}$ {to achieve a common denominator of 6}

$\therefore \quad 2x = 9$ {equating numerators}

$\therefore \quad \frac{2x}{2} = \frac{9}{2}$ {dividing both sides by 2}

$\therefore \quad x = 4\frac{1}{2}$ {simplifying}

b

$\frac{4}{x} = \frac{2}{3}$ has LCD $3x$

$\therefore \quad \frac{4 \times 3}{x \times 3} = \frac{2 \times x}{3 \times x}$ {to achieve a common denominator of $3x$}

$\therefore \quad 12 = 2x$ {equating numerators}

$\therefore \quad \frac{12}{2} = \frac{2x}{2}$ {dividing both sides by 2}

$\therefore \quad x = 6$ {simplifying}

EXERCISE 10B

1 Solve for x:

a $\frac{x}{2} = \frac{3}{5}$ **b** $\frac{x}{3} = \frac{2}{7}$ **c** $\frac{x}{5} = \frac{2}{3}$ **d** $\frac{x}{4} = -\frac{2}{3}$

e $\frac{3}{x} = \frac{1}{2}$ **f** $\frac{4}{x} = \frac{1}{7}$ **g** $\frac{5x}{3} = \frac{1}{2}$ **h** $\frac{7}{2x} = -\frac{1}{3}$

[Video to explain fractional equations]

CROSS MULTIPLICATION

If *two* fractions are equal we can use **cross multiplication** to form an equation without denominators.

Notice that $\frac{4}{6} = \frac{2}{3}$ and so $4 \times 3 = 2 \times 6$.

We can use cross multiplication of *two equal fractions* to avoid having to equalise denominators.

Example 6

Solve for x: $\frac{7}{x} = \frac{4}{3}$

$$\frac{7}{x} = \frac{4}{3}$$

$\therefore \quad 4 \times x = 7 \times 3$ {cross multiplying}

$\therefore \quad 4x = 21$ {simplifying}

$\therefore \quad \frac{4x}{4} = \frac{21}{4}$ {dividing both sides by 4}

$\therefore \quad x = 5\frac{1}{4}$ {simplifying}

Note: We are using the rule: "if $\frac{a}{b} = \frac{c}{d}$ then $a \times d = b \times c$".

2 Solve for x:

a $\frac{x}{7} = \frac{2}{5}$ **b** $\frac{x}{9} = \frac{1}{2}$ **c** $\frac{x}{8} = -\frac{1}{3}$ **d** $\frac{2x}{3} = \frac{1}{5}$

e $\frac{3}{x} = \frac{1}{2}$ **f** $\frac{5}{x} = \frac{3}{4}$ **g** $\frac{6}{7} = \frac{4}{x}$ **h** $-\frac{3}{4} = \frac{4}{3x}$

Example 7

Solve for x: $\dfrac{x}{2} = \dfrac{3+x}{5}$

Notice the insertion of brackets here.

$\dfrac{x}{2} = \dfrac{3+x}{5}$ has LCD $= 10$

$\therefore \dfrac{x \times 5}{2 \times 5} = \dfrac{2(3+x)}{2 \times 5}$ {to create a common denominator}

$\therefore 5x = 2(3+x)$ {equating numerators}

$\therefore 5x = 6 + 2x$ {expanding brackets}

$\therefore 5x - 2x = 6 + 2x - 2x$ {taking $2x$ from both sides}

$\therefore 3x = 6$

$\therefore x = 2$ {dividing both sides by 3}

or $\dfrac{x}{2} = \dfrac{3+x}{5}$

$\therefore 5x = 2(3+x)$ etc. {cross-multiplying}

3 Solve for x:

a $\dfrac{x+2}{2} = \dfrac{3}{7}$ **b** $\dfrac{3}{5} = \dfrac{x+1}{6}$ **c** $\dfrac{x}{5} = \dfrac{x-2}{3}$

d $\dfrac{x+1}{3} = \dfrac{2x-1}{8}$ **e** $\dfrac{2x}{3} = \dfrac{5-x}{4}$ **f** $\dfrac{3x+2}{3} = \dfrac{2x-5}{2}$

Challenge

Example 8

Solve for x: $\dfrac{2x+1}{3-x} = \dfrac{3}{4}$.

$$\frac{2x+1}{3-x} = \frac{3}{4} \qquad \text{has LCD } 4(3-x)$$

$\therefore \quad 4(2x+1) = 3(3-x)$ {cross multiplying}

$\therefore \quad 8x+4 = 9-3x$ {expanding the brackets}

$\therefore \quad 8x+4+3x = 9-3x+3x$ {adding $3x$ to both sides}

$\therefore \quad 11x+4 = 9$

$\therefore \quad 11x+4-4 = 9-4$ {subtracting 4 from both sides}

$\therefore \quad 11x = 5$

$\therefore \quad x = \frac{5}{11}$ {dividing both sides by 11}

More explanation video

4 Solve for x:

a $\dfrac{2x+3}{x+1} = \dfrac{4}{3}$ **b** $\dfrac{x+1}{1-2x} = \dfrac{3}{5}$ **c** $\dfrac{2x-1}{4-3x} = -\dfrac{2}{3}$

d $\dfrac{x+3}{2x-1} = \dfrac{1}{2}$ **e** $\dfrac{4x+3}{2x-1} = 6$ **f** $\dfrac{3x-2}{x+4} = -5$

g $\dfrac{6x-1}{3-2x} = 10$ **h** $\dfrac{5x-1}{x+4} = 5$ **i** $3 + \dfrac{2x+5}{x-1} = -1$

Extension

Example 9

Solve for x: $\frac{x}{3} - \frac{1-2x}{6} = {}^{-}4$

$$\frac{x}{3} - \frac{1-2x}{6} = {}^{-}4 \qquad \text{has LCD of 6}$$

$$\therefore \quad \frac{x}{3} \times \frac{2}{2} - \left(\frac{1-2x}{6}\right) = {}^{-}4 \times \frac{6}{6} \qquad \{\text{to create a common denominator}\}$$

$$\therefore \quad 2x - (1-2x) = {}^{-}24 \qquad \{\text{equating numerators}\}$$

$$\therefore \quad 2x - 1 + 2x = {}^{-}24 \qquad \{\text{expanding}\}$$

$$\therefore \quad 4x - 1 = {}^{-}24$$

$$\therefore \quad 4x - 1 + 1 = {}^{-}24 + 1 \qquad \{\text{adding 1 to both sides}\}$$

$$\therefore \quad 4x = {}^{-}23$$

$$\therefore \quad x = {}^{-}\tfrac{23}{4} \qquad \{\text{dividing both sides by 4}\}$$

Video explanation with multiple common denominators

5 Solve for x:

a $\frac{x}{2} - \frac{x}{6} = 4$

b $\frac{x}{4} - 3 = \frac{2x}{3}$

c $\frac{x}{8} + \frac{x+2}{2} = {}^{-}1$

d $\frac{x+2}{3} + \frac{x-3}{4} = 1$

e $\frac{2x-1}{3} - \frac{5x-6}{6} = {}^{-}2$

f $\frac{x}{4} = 4 - \frac{x+2}{3}$

g $\frac{2x-7}{3} - 1 = \frac{x-4}{6}$

h $\frac{x+1}{3} - \frac{x}{6} = \frac{2x-3}{2}$

i $\frac{x}{5} - \frac{2x-5}{3} = \frac{3}{4}$

j $\frac{x+1}{3} + \frac{x-2}{6} = \frac{x+4}{12}$

k $\frac{x-6}{5} - \frac{2x-1}{10} = \frac{x-1}{2}$

l $\frac{2x+1}{4} - \frac{1-4x}{2} = \frac{3x+7}{6}$