WALT convert decimals to fractions and vice versa
SUCCESS CRITERIA I understand place value and I can divide numbers

I MUST KNOW THIS

Common fractions and their decimal equivalents									
$\frac{1}{2}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{5}$	$\frac{1}{8}$	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
0.5	$0.\dot{3}$	$0.\dot{6}$	0.25	0.75	0.2	0.125	0.1	0.01	0.001

Notice that the decimal equivalents for $\frac{1}{3}$ and $\frac{2}{3}$ have a dot above the decimal digit. This is to show that the digit is repeated forever; i.e. $\frac{1}{3} = 0.333\ldots$ These types of decimals are called **recurring decimals**. We will learn more about these in the next section.

In order to compare quantities expressed as fractions and decimals we need to change all values to the same format. It is often easier to convert all values to decimal form.

Fractions and decimals can most easily be compared by first converting the fractions to decimals.

Arrange the following list into ascending order (smallest to largest) by converting the fraction values to decimals.

$\frac{5}{8}, \frac{3}{5}, \frac{3}{4}, 0.69, 0.686$

Thinking	Working
1 Convert each of the fractions into decimals. (You could use your calculator for this step.)	$\frac{5}{8} = 5 \div 8 = 0.625$ $\frac{3}{5} = 3 \div 5 = 0.6$ $\frac{3}{4} = 3 \div 4 = 0.75$
2 List the numbers in order from smallest to largest, by comparing the decimal digits in each place value column.	0.6, 0.625, 0.686, 0.69, 0.75
3 Substitute the fraction values back into the list.	$\frac{3}{5}, \frac{5}{8}, 0.686, 0.69, \frac{3}{4}$

Let's look at the example

Converting decimals to fractions

Decimals use place value to represent the fractional parts of a number. Knowledge of place value allows you to convert decimals to fractions.

For example, we can write the number 13.2452 in a place value table, like this:

tens 10	ones 1	•	tenths $\dfrac{1}{10}$	hundredths $\dfrac{1}{100}$	thousandths $\dfrac{1}{1000}$	ten-thousandths $\dfrac{1}{10\,000}$
1	3	.	2	4	5	2

From this, 13.2452 can be written in expanded fractional form as

$13 + \dfrac{2}{10} + \dfrac{4}{100} + \dfrac{5}{1000} + \dfrac{2}{10\,000}$ or $13\dfrac{2452}{10\,000}$ (which simplifies to $13\dfrac{613}{2500}$ when we cancel a common factor of 4) tel:10%20100%201000%2010%20000

To convert a decimal to a fraction:

- use the place value column of the last digit to get the denominator of the fraction
- write all the digits of the decimal part in the numerator
- simplify the fraction if possible.

Write the following decimal as a fraction in simplest form: 0.384

Thinking	Working
1 The last digit in the decimal is in the thousandths column, so 1000 is the denominator. Write the other digits as the numerator.	$0.384 = \dfrac{384}{1000}$
2 Simplify the fraction. (Here, we have cancelled common factors of 4 and 2.)	$= \dfrac{96}{250}$ $= \dfrac{48}{125}$

Equipment required: A calculator may be used for Questions **1**, **3**, **4**, **9**, **13–15** and **17**

1 Arrange the following lists into ascending order (smallest to largest) by converting the fraction values to decimals.

(a) $\frac{2}{5}$, 0.399, $\frac{4}{5}$, 0.382, $\frac{3}{4}$

(b) $\frac{9}{10}$, $\frac{9}{8}$, 0.88, 0.89, 0.899

(c) $\frac{1}{8}$, 0.112, 0.099, $\frac{1}{4}$, 0.07

(d) $\frac{2}{5}$, $\frac{1}{3}$, 0.3, $\frac{3}{8}$, 0.2

(e) $\frac{1}{2}$, 0.555, 0.58, $\frac{3}{5}$, 0.55

(f) 0.291, 0.302, $\frac{3}{10}$, $\frac{2}{9}$, $\frac{2}{3}$

(g) $2\frac{4}{5}$, $2\frac{3}{4}$, 2.278, 2.932, $2\frac{9}{10}$

(h) $1\frac{1}{5}$, $1\frac{2}{3}$, $1\frac{3}{8}$, 1.029, 1.243

(i) $4\frac{1}{5}$, $4\frac{1}{4}$, 4.295, 4.199, 4.201

(j) $3\frac{1}{5}$, 3.45, 3.439, $3\frac{2}{3}$, 3.482

2 Write the following decimals as fractions in simplest form.

(a) 0.8
(b) 0.05
(c) 0.002
(d) 0.0009

(e) 0.14
(f) 0.62
(g) 0.31
(h) 0.85

(i) 0.711
(j) 0.684
(k) 0.625
(l) 0.128

(m) 0.203
(n) 0.094
(o) 0.1560
(p) 0.7009

3 Use a calculator, if necessary, to answer the following questions.

(a) A length of wood is 2.6 m long. How many pieces, each 40 cm long, can be cut from this length, and how much wood will be left over?

(b) A bag holds 750 g of flour. How many cups, each containing 120 g of flour, can be filled from the bag, and how much will be left over?

(c) A container holds $3\frac{1}{4}$ litres of juice. How many 200 mL cups can be filled from the container, and how much juice will be left over? (1 litre = 1000 mL)

(d) A bus can carry 52 passengers. If every bus except the last one is filled to capacity, how many buses will be needed to transport 650 people, and how many people will be on the last bus?

Understanding

4 Write the following mixed numbers as decimals using a calculator where necessary.

(a) $51\frac{7}{10}$

(b) $5\frac{3}{20}$

(c) $7\frac{1}{8}$

(d) $67\frac{7}{25}$

(e) $39\frac{1}{16}$

(f) $28\frac{24}{200}$

(g) $14\frac{13}{80}$

(h) $24\frac{31}{400}$

5 Write the following decimals as mixed numbers in simplest form.

(a) 7.5

(b) 3.4

(c) 1.25

(d) 5.64

(e) 13.02

(f) 27.96

(g) 9.045

(h) 124.706

6 (a) $1\frac{5}{8}$ written as a decimal is:

    A 0.16   **B** 1.58   **C** 1.625   **D** 1.63

  (b) $2\frac{1}{3}$ written as a decimal, correct to two decimal places, is:

    A 0.34   **B** 2.13   **C** 2.33   **D** 2.34

7 For each set of numbers, draw the number line below and indicate the position of each number with a labelled arrow. (Hint: First, determine the value of the smallest interval on the number line.)

(a) $0.5, 1.3, 1\frac{1}{2}, \frac{1}{5}$

(b) $\frac{2}{2}, 0.8, 1.6, 1\frac{4}{5}$

8 For each set of numbers, draw the number line below and indicate the position of each number with a labelled arrow. (You may need to estimate the position of some numbers.)

(a) $\frac{3}{4}, \frac{7}{10}, 0.34, 0.47, 1\frac{1}{3}$

(b) $1.05, \frac{7}{8}, 0.58, \frac{8}{5}, \frac{6}{4}$

9 Use a calculator to convert the fractions to decimal form, then place a > (greater than), < (less than) or = (equal to) symbol between the pairs of numbers to make the following statements correct.

(a) $0.84 \underline{\hphantom{xx}} \frac{18}{21}$

(b) $2.29 \underline{\hphantom{xx}} 2\frac{12}{39}$

(c) $0.912 \underline{\hphantom{xx}} \frac{114}{125}$

(d) $0.64 \underline{\hphantom{xx}} \frac{16}{25}$

(e) $1.83 \underline{\hphantom{xx}} 1\frac{18}{23}$

(f) $0.97 \underline{\hphantom{xx}} \frac{98}{99}$

(g) $\frac{1}{3} \underline{\hphantom{xx}} 0.3$

(h) $\frac{2}{9} \underline{\hphantom{xx}} 0.23$

(i) $\frac{2}{3} \underline{\hphantom{xx}} 0.67$

10 (a) Express each first quantity as a fraction of the second quantity. Write the fraction in simplest terms.

   (i) first quantity 1.5 hours; second quantity 2.4 hours

   (ii) first quantity 200 m; second quantity 322.5 m

   (iii) first quantity 80.6 mL; second quantity 200 mL

   (iv) first quantity 20.6 kg; second quantity 12.5 kg

   (v) first quantity 18.6 m^2; second quantity 12 m^2

   (vi) first quantity 14.4 g; second quantity 1.5 g

(b) Express each fraction from part **(a)** as a decimal, rounding your answers to three decimal places where necessary.

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11 Mr Scully is out buying materials to build the set for the school production. Calculate how much he will pay for the following materials. Round your answers to the nearest 5 cent

  (a) 6 lengths of timber, at $8.99 per length

  (b) 9.4 metres of canvas, at $11.20 per metre

  (c) 5.6 metres of ribbon, at $0.95 per metre

12 The exchange rate between the Australian and American dollars varies daily. Suppose that according to the current rate A$1 is worth US$0.72. How much are the following amount worth in US dollars?

  (a) A$10   **(b)** A$50   **(c)** A$100   **(d)** A$267   **(e)** A$1845

13 The most overdue library book ever was a copy of *Febrile Diseases* by Dr J. Currie. The book was borrowed from the University of Cincinnati Medical Library in 1823 by Mr M. Dodd, and was returned by his great-grandson in 1968. If the fines for late returns were $18.30 a year, how big was the fine his great-grandson had to pay?

14 The largest crab in the world is the giant spider crab, which is found off the south-eastern coast of Japan. It has a claw span of 2.74 m. If an average person has a width of 36 cm across the waist, how many people could fit in the claw span of the giant spider crab?

15 The highest wave ever recorded in Australia was 24.9 m, off Macquarie Island. If the average surfer is 180 cm tall, how many surfers standing on top of one another would it take to reach the top of the wave?

Reasoning

16 An insect was climbing a wall 2.7 m high. In the first 20 minutes the insect started at the bottom and climbed $\frac{1}{3}$ of the height of the wall. In the second 20 minutes it climbed $\frac{1}{4}$ of the remaining height and in the third 20 minutes it climbed $\frac{1}{5}$ of the remaining height.

   (a) Calculate the distances climbed in each 20 minute period.

   (b) Calculate how far the insect still had to climb to reach the top of the wall.

   (c) Express the distance remaining as a fraction of the height of the wall.

17 Angela has to fill 50 gift bags with an equal amount of lollies in each. She has 4 packets of lollies that weigh 375 g each. Angela knows that the average mass of one lolly is 2.8 g.

   (a) Explain how Angela can use this information to determine how many lollies she can put in each gift bag.

   (b) Use your method from **(a)** to find the answer.

   (c) About how many lollies will Angela have left over? Explain why this answer may not be exact.

18 The force of gravity varies from planet to planet and so the weight of an object can vary, depending on which planet it is on. Objects on Jupiter would weigh 2.6 times their weight on Earth. Objects on Mars would weigh 0.38 times their 'Earth weight'. Calculate the weight of a 5 kg bag of potatoes on **(a)** Jupiter **(b)** Mars.

Open-ended

19 Write three fractions with three different denominators that have decimal values between 0.2 and 0.4.

20 Write three decimals, each with a different number of decimal places, that have fraction values between $\frac{9}{10}$ and $\frac{9}{8}$.

21

   (a) Explain to Leo why his reasoning is incorrect.

   (b) Describe to Leo how to compare two decimal numbers.

Outside the Square | Puzzle

Sudoku

Equipment required: 1 brain, 1-cm grid paper

To solve a Sudoku you need to use the digits 1–9 to fill in the blank squares so that each row, column and small 3 × 3 box contains the digits 1–9 only once.

Copy the grid into your book and complete it.

5		8	2				6	
	7	2			6			5
6			1			4		
	8					2		4
			8		1			
	6	4	3			5	8	
			7		8	1		3
2						6		
		1	6		3		7	2

1 (a) $0.382, 0.399, \dfrac{2}{5}, \dfrac{3}{4}, \dfrac{4}{5}$

(b) $0.88, \; 0.89, 0.899, \dfrac{9}{10}, \dfrac{9}{8}$

(c) $0.07, 0.099, 0.112, \dfrac{1}{8}, \dfrac{1}{4}$

(d) $0.2, 0.3, \dfrac{1}{3}, \dfrac{3}{8}, \dfrac{2}{5}$

(e) $\dfrac{1}{2}, 0.55, 0.555, 0.58, \dfrac{3}{5}$

(f) $\dfrac{2}{9}, 0.291, \dfrac{3}{10}, 0.302, \dfrac{2}{3}$

(g) $2.278, 2\dfrac{3}{4}, 2\dfrac{4}{5}, 2\dfrac{9}{10}, 2.932$

(h) 1.029, $1\frac{1}{5}$, 1.243, $1\frac{3}{8}$, $1\frac{2}{3}$

(i) 4.199, $4\frac{1}{5}$, 4.201, $4\frac{1}{4}$, 4.295

(j) $3\frac{1}{5}$, 3.439, 3.45, 3.482, $3\frac{2}{3}$

2 (a) $\frac{4}{5}$ (b) $\frac{1}{20}$ (c) $\frac{1}{500}$ (d) $\frac{9}{10\,000}$

(e) $\frac{7}{50}$ (f) $\frac{31}{50}$ (g) $\frac{31}{100}$ (h) $\frac{17}{20}$

(i) $\frac{711}{1000}$ (j) $\frac{171}{250}$ (k) $\frac{5}{8}$ (l) $\frac{16}{125}$

(m) $\frac{203}{1000}$ (n) $\frac{47}{500}$ (o) $\frac{39}{250}$ (p) $\frac{7009}{10\,000}$

3 (a) 6 pieces; 20 cm remaining

(b) 6 cups; 30 g remaining

(c) 16 cups; 50 mL remaining

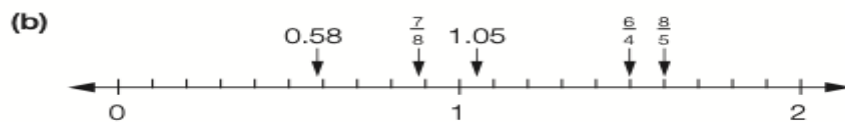(d) 13 buses; 26 people on the thirteenth bus
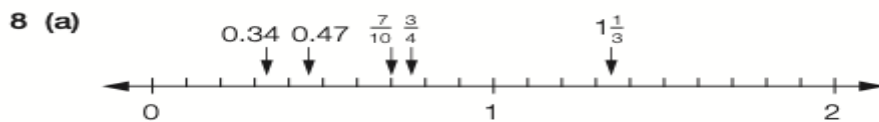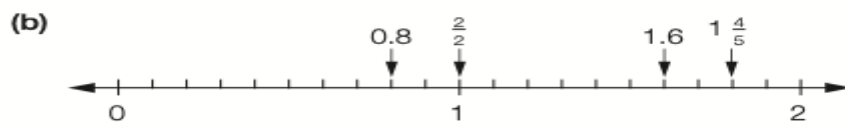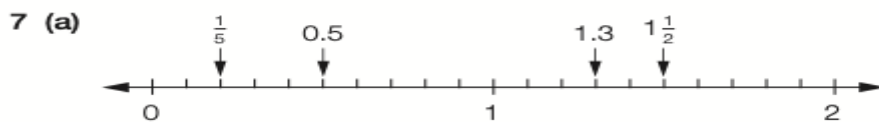
4 (a) 51.7   (b) 5.15   (c) 7.125   (d) 67.28

(e) 39.0625   (f) 28.12   (g) 14.1625   (h) 24.0775

5 (a) $7\frac{1}{2}$ (b) $3\frac{2}{5}$ (c) $1\frac{1}{4}$ (d) $5\frac{16}{25}$

(e) $13\frac{1}{50}$ (f) $27\frac{24}{25}$ (g) $9\frac{9}{200}$ (h) $124\frac{353}{500}$

6 (a) C   (b) C

7 (a)

(b)

8 (a)

(b)

9 (a) <   (b) <   (c) =   (d) =   (e) >

(f) <   (g) >   (h) <   (i) <

10 (a) (i) $\frac{5}{8}$ (ii) $\frac{80}{129}$ (iii) $\frac{403}{1000}$ (iv) $1\frac{81}{125}$

(v) $1\frac{11}{20}$ (vi) $9\frac{3}{5}$

(b) (i) 0.625   (ii) 0.620   (iii) 0.403   (iv) 1.648

(v) 1.55   (vi) 9.6

11 (a) $53.95   **(b)** $105.30   **(c)** $5.30

12 (a) US$7.20   **(b)** US$36   **(c)** US$72

     (d) US$192.24   **(e)** US$1328.40

13 $2653.50

14 7

15 14

16 (a) 0.9 m, 0.45 m, 0.27 m   **(b)** 1.08 m   **(c)** $\frac{2}{5}$

17 (a) Sample answer: Angela can calculate how many lollies are in each packet to find out how many she has in total, and then divide that amount among the 50 gift bags.

     (b) 10

     (c) Sample answer: 36; this answer may not be exact as we can't be sure how many lollies are in each packet.

18 (a) 13 kg   **(b)** 1.9 kg

Open-ended – Sample answers

19 $\frac{1}{4}, \frac{3}{10}, \frac{7}{20}$

20 0.95, 0.99999, 1.102

21 (a) Leo is treating the decimals as whole numbers and not comparing place value. He needs to compare the place value of each digit in the number. 1.7 has 7 in the tenths place value, whereas 1.25 only has 2 in that same position. Leo's answer to the first question is only correct because the numbers have a digit in the same place value position (units and tenths).

     (b) Compare the whole number parts of the two decimal numbers first. If one is larger than the other, then that decimal number is larger. If the whole number parts are the same, compare the digits in the tenths place value column. If one is larger than the other, then that decimal number is larger. If they are the same, compare the digits in the hundredths place value column. Keep moving down the place value columns comparing digits until you find one digit larger than the other corresponding digit. This digit belongs to the larger number.