## Integers

WALT understand using integers and applying the integers rule when adding, subtracting, multiplying and dividing
Success Criteria I know how to multiply and divide negative and positive, negative and negative add and subtract negative and negative, positive and positive

## Watch the Video

The set of all integers is $I=\left\{\ldots,{ }^{-} 4,{ }^{-} 3,{ }^{-} 2,{ }^{-} 1,0,1,2,3,4, \ldots.\right\}$.
Integers can be used when dealing with

- bank statements when owing say $\$ 1500$ could be represented by ${ }^{-1500}$.
- temperatures where $23^{\circ} \mathrm{C}$ is $23^{\circ}$ above zero and $-23^{\circ} \mathrm{C}$ is $23^{\circ}$ below zero.

We can show these numbers on a number line. Zero is neither positive nor negative.


We can classify integers as follows:


## ADDING AND SUBTRACTING INTEGERS

Movement along the number line can be used for adding and subtracting integers.

$$
\begin{aligned}
& \text { Move } \rightarrow \text { for adding a positive or } \text { subtracting a negative. } \\
& \text { Move } \leftarrow \text { for adding a negative } \text { or } \quad \text { subtracting a positive. }
\end{aligned}
$$

We could use previously developed rules for handling addition and subtraction of integers. These are:

$$
\begin{aligned}
& +(\text { positive })=(\text { positive }) \\
& -(\text { positive })=(\text { negative }) \\
& +(\text { negative })=(\text { negative }) \\
& -(\text { negative })=(\text { positive })
\end{aligned}
$$

| Fxample 10 |  |  |  |
| :---: | :---: | :---: | :---: |
| Find the value of: <br> a $5+{ }^{-7}$ | b $5--7$ | c $-6+-4$ | d $\quad-6-4$ |
| $\text { a } \begin{aligned} & 5+{ }^{-} 7 \\ = & 5-7 \\ = & -2 \end{aligned}$ | $\text { b } \begin{aligned} & 5-{ }^{-7} \\ = & 5+7 \\ = & 12 \end{aligned}$ | $\text { c } \begin{aligned} & -6+-4 \\ = & -6-4 \\ = & -10 \end{aligned}$ | $\text { d } \begin{aligned} & -6--4 \\ = & -6+4 \\ = & -2 \end{aligned}$ |

1 Find the value of:
a $6+{ }^{-3}$
b $6-{ }^{-} 3$
c $-6+{ }^{-} 3$
d $-6-{ }^{-} 3$
e $12+{ }^{-7}$
f $12-{ }^{-7}$
g $-12+-7$
h $-12-{ }^{-7}$

2 Find the value of:

| a | $3+7$ | b | $-3+2$ | c | $14-2$ | d | $-13-6$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| e | $5+{ }^{-} 6$ | $\mathbf{f}$ | $-3+{ }^{-} 5$ | g | $6-{ }^{-} 4$ | h | $-3-{ }^{-} 7$ |
| i | $-4+-6$ | j | $3-{ }^{-} 7$ | $\mathbf{k}$ | $24+-8$ | I | $5-20$ |
| m | $-5--9$ | n | $-6-10$ | ○ | $-3+10$ | P | $6-12$ |

## MULTIPLYING AND DIVIDING INTEGERS

We have previously developed these rules:

```
(positive) }\times(\mathrm{ positive) =(positive) (positive) % (positive) = (positive)
(positive) }\times(\mathrm{ negative) }=(\mathrm{ negative )
(negative) }\times(\mathrm{ positive) }=(\mathrm{ negative)
(negative)}\times(\mathrm{ negative ) = (positive)
```

```
(positive) \div(negative) = (negative)
```

(positive) \div(negative) = (negative)
(negative) \div (positive) = (negative)
(negative) \div (positive) = (negative)
(negative) \div(negative) }=(\mathrm{ (positive)

```
(negative) \div(negative) }=(\mathrm{ (positive)
```


## Example 11

Find the value of:
a $3 \times 5$
b $3 \times{ }^{-5}$
c $-3 \times 5$
d $-3 \times-5$


1 Find the value of:
a $6 \times 7$
e $5 \times 8$
i $7 \times 9$
$\begin{array}{cc}\text { b } & 6 \times-7 \\ \text { f } & 5 \times-8 \\ \text { j } & -7 \times 9\end{array}$
$\begin{array}{ll}\text { C } & -6 \times 7 \\ \text { g } & -5 \times 8 \\ \text { k } & 7 \times-9\end{array}$
d $-6 \times-7$
h $-5 \times-8$

## Challenge

## Example 12

Find the value of:
a $16 \div 2$
b $16 \div-2$
C $-16 \div 2$
d $-16 \div-2$
a $\quad 16 \div 2$
$=8$

b. | 16 |
| :---: |
| $=$ |

C $\begin{gathered}-16 \\ =\end{gathered}$
d $\begin{aligned} & -16 \div-2 \\ = & 8\end{aligned}$

2 Find the value of:
a $15 \div 3$
b $15 \div-3$
c $-15 \div 3$
d $-15 \div-3$

| e $24 \div 8$ | f | $24 \div{ }^{-} 8$ | g | $-24 \div 8$ | h | $-24 \div-8$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| i $\frac{4}{8}$ | j | $\frac{-4}{8}$ | k | $\frac{4}{-8}$ | I | $\frac{-4}{-8}$ |

## Extension

## Example 13

Find the value of:
a $-3^{2}$
b $(-3)^{2}$
c $-2^{3}$
d $(-2)^{3}$

b $(-3)^{2}$
c $\begin{aligned} & -2^{3} \\ = & -(2 \times 2 \times 2)\end{aligned}$ d $\begin{aligned} & (-2)^{3} \\ = & -2 \times-2 \times-2 \\ = & -8\end{aligned}$
$=-8$

3 Find the value of:
a $-5^{2}$
$(-5)^{2}$
$-2^{4}$
c $(-1)^{3}$
d ${ }^{-} 1^{3}$
e $-2 \times(-3)^{2}$
h $\left({ }^{-} 3\right)^{2} \times\left({ }^{-} 2\right)^{2}$

4 Find the value of:
a $6 \times 2 \times-1$
b $-3 \times-5 \times 2$
c $-3 \times-2 \times-1$
d $2 \times(-3)^{2}$
e $\frac{6 \times-4}{3}$
f $\frac{-3 \times-4}{-6}$
g $\frac{12 \times-5}{1-5}$
h $\frac{-6-{ }^{-} 14}{(-2)^{2}}$

## Some more practice

## Integers

Here is the set of integers:
$I=\{\ldots,-4,-3,-2,-1,0,1,2,3,4,5, \ldots\}$
Integers include:

- the counting numbers, $N=\{1,2,3,4, \ldots\}$
- zero $\{0\}$
- the opposites of the counting numbers $\{-1,-2,-3,-4, \ldots\}$

You may need some familiarisation with integer questions although you have already met them in previous years!

$$
\begin{aligned}
& \text { Examples } \\
& -4+-6=-10 \\
& 16--13=16+13=29 \\
& -8 \times-2=16 \\
& 39 \div-3=-13
\end{aligned}
$$



## Five different digits

A five-digit number has these properties:

- all five digits are different
- the last digit is 6 more than the first digit.

1 What is the smallest number with these properties?
2 What is the largest number with these properties?

## EXERCISE 1.01

| a | $3+-2$ | b | $2 \times-5$ |
| :--- | :--- | :--- | :--- |
| c | $3--2$ | d | $-10+8$ |
| e | $8 \div-2$ | $\mathbf{f}$ | $-14--3$ |
| g | $-24 \div-6$ | h | $4+-7$ |
| $\mathbf{i}$ | $1 \times-4$ | j | $-2-8$ |
| $\mathbf{k}$ | $-5 \times-7$ | $\mathbf{1}$ | $-4--7$ |
| $\mathbf{m}$ | $-72 \div 6$ | n | $-8+-7$ |


| o | $35 \div-7$ | p | $60 \div-2$ |
| :--- | :--- | :--- | :--- |
| q | $-8 \times 2$ | r | $-12+1$ |
| s | $-5-2$ | t | $-18+-2$ |
| u | $6 \times-4$ | v | $-13-{ }^{-} 8$ |
| w | $-100 \div 25$ | x | $-21+7$ |
| y | $-8-1$ | z | $-4 \times-15$ |

2 Calculate the answers to these integer expressions.
$p=-5 \quad q=-7 \quad r=35 \quad s=-3$

| $\mathbf{a}$ | $p+q$ | $\mathbf{b}$ | $p-q$ |
| :--- | :--- | :--- | :--- |
| $\mathbf{c}$ | $s-r$ | $\mathbf{d}$ | $r+q$ |
| $\mathbf{e}$ | $r q$ | $\mathbf{f}$ | $p s$ |
| $\mathbf{g}$ | $r \div q$ | $\mathbf{h}$ | $p-q-s$ |
| $\mathbf{i}$ | $(p+q) \div s$ | $\mathbf{j}$ | $(q+s) \div p$ |
| $\mathbf{k}$ | $p(q+s)$ | $\mathbf{l}$ | $p+q+r+s$ |
| $\mathbf{m}$ | $p-(q+s)$ | $\mathbf{n}$ | $q(p-s)$ |
| $\mathbf{o}$ | $p-q+r-s$ |  |  |

3 Starting with the number 3 at the beginning of this maze, work through it to find which route leads to the answer 18 at the end.


4 At Scott Base, Antarctica, the maximum temperature ever recorded is $7^{\circ} \mathrm{C}$, and the minimum temperature ever recorded is $-57^{\circ} \mathrm{C}$. How much warmer is the maximum temperature compared with the minimum?

5 New Zealanders travel a total of about 8 billion km to and from work each year. If 2000000 New Zealanders are in work, and each one works on average 250 days per year, how far does the typical New Zealander travel to and from work each day?

## Answers

$\begin{array}{lllllllllllll}\mathbf{1} & \mathbf{a} & 3 & \mathbf{b} & 9 & \mathbf{c} & { }^{-} 9 & \mathbf{d} & { }^{-} 3 & \text { e } & 5 & \mathbf{f} & 19 \\ & \mathbf{g} & { }_{1} 19 & \mathbf{h} & { }_{-}\end{array}$
$\begin{array}{lllllllllllll} & 2 & \mathbf{a} & 10 & \mathbf{b} & { }^{-} 1 & \mathbf{c} & 12 & \mathbf{d} & & -19 & \mathbf{e} & { }^{-} 1\end{array} \mathbf{f}^{-} 8$
$\begin{array}{llllllllllll}\mathbf{g} & 10 & \mathbf{h} & 4 & \mathbf{i} & { }^{-} 10 & \mathbf{j} & 10 & \mathbf{k} & 16 & \mathbf{l} & { }^{-15}\end{array}$
$\begin{array}{llllllll}\mathbf{m} & 4 & \mathbf{n} & -16 & \mathbf{o} & 7 & \mathbf{p} & { }^{-} 6\end{array}$

## EXERCISE 1 D. 2

$\mathbf{1}$ a 42 b ${ }^{-} 42$ c -42 d 42 e 40 f -40
g ${ }^{-} 40 \quad \mathbf{h} 40 \quad \mathbf{i} 63 \quad \mathbf{j}-63 \quad \mathbf{k}{ }^{-} 63 \quad \mathbf{l} 63$
$\begin{array}{lllllllllllll} & \mathbf{a} & 5 & \mathbf{b} & -5 & \mathbf{c} & -5 & \mathbf{d} & 5 & \mathbf{e} & 3 & \mathbf{f} & -3\end{array}$

$\begin{array}{lllllllllll} & 3 & \mathbf{a} & - \\ -25 & \mathbf{b} & 25 & \mathbf{c} & { }^{-} 1 & \mathbf{d} & & { }^{-1} & \text { e } & & \\ & 18\end{array}$
$\begin{array}{llllll} & \mathbf{f} & -16 & \mathbf{g} & 16 & \mathbf{h}\end{array}$
$\begin{array}{lllllllllllll}4 & \mathbf{a} & -12 & \mathbf{b} & 30 & \mathbf{c} & -6 & \mathbf{d} & 18 & \mathbf{e} & -8 & \mathbf{f} & -2\end{array}$
$\begin{array}{llll}\mathbf{g} & 15 & \mathbf{h} & 2\end{array}$

## 1 Number concepts

## PUZFALE

## Five different digits

$110237 \quad 238769$
EXERCISE 1.01


PUZZZIF
Unfriendly number net

(Other answers are possible.)

