Trigonometry

TRIGONOMETRY

SOLUTIONS

SERIES J

MATHLETICS

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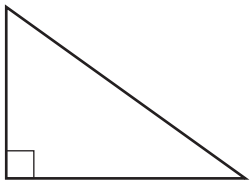
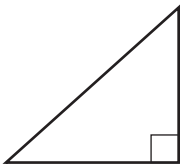
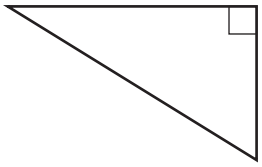
Page 3 questions

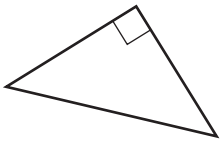
1. Identify if the following triangles are right angled or not.

Triangles a, c, d, e are right angled indicated by the small square in the corner of each triangle.

a 

c 

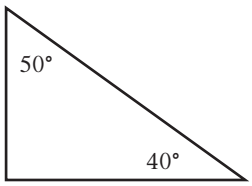
d 

e 

2. What is the sum of the interior angles of a triangle?

The sum of the interior angles of a triangle is 180°.

3. Identify if the following triangles are right angled.

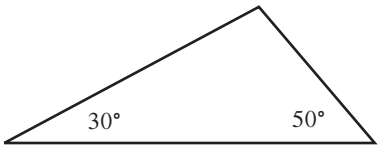


Sum of the interior angles is 180°. The last angle must be 90° for the triangle to be right angled.

Therefore,

$$180^\circ - 40^\circ - 50^\circ = 90^\circ$$

Therefore, the triangle is right angled since the last angle is 90°

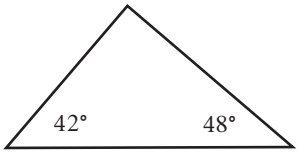


Sum of the interior angles is 180°. The last angle must be 90° for the triangle to be right angled.

Therefore,

$$180^\circ - 30^\circ - 50^\circ = 100^\circ$$

Therefore, the triangle is not right angled since the last angle is 100°.



Sum of the interior angles is 180°. The last angle must be 90° for the triangle to be right angled.

Therefore,

$$180^\circ - 42^\circ - 48^\circ = 90^\circ$$

Therefore, the triangle is right angled since the last angle is 90°

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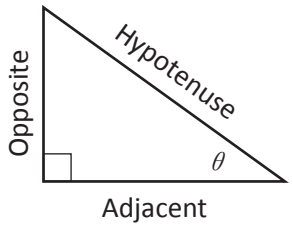
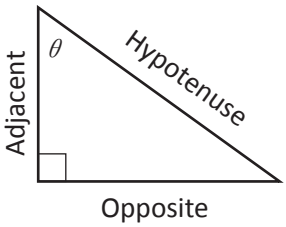
4. Label the opposite, adjacent and hypotenuse in each of the following triangles.

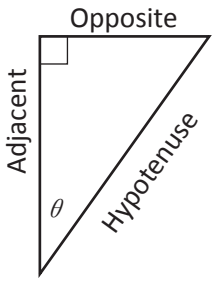
a



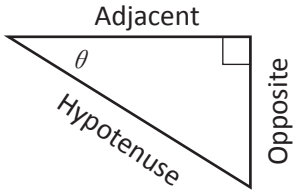
b



c



d



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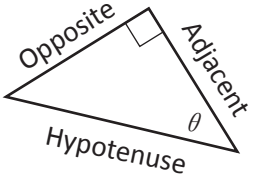
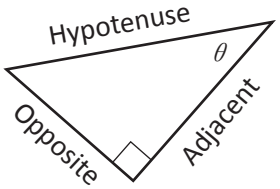


f



5. Use the following 5 triangles to fill in the correct sides in the table below:

Triangle	Opposite to θ	Adjacent to θ	Opposite to α	Adjacent to α	Hypotenuse
ΔABC	AC	BC	BC	AC	AB
ΔDEF	EF	DE	DE	EF	DF
ΔLMN	MN	LM	ML	MN	LN
ΔPQR	PR	QR	QR	PR	PQ
ΔWXY	WX	WY	WY	WX	XY

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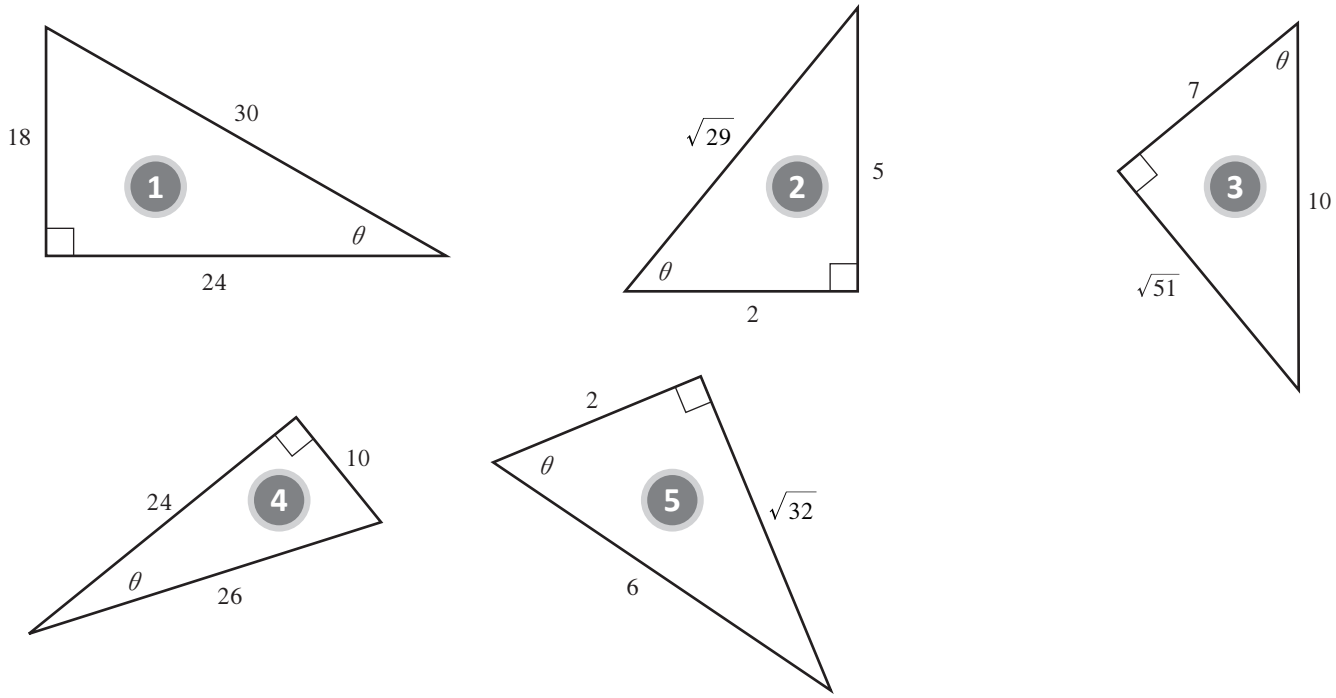
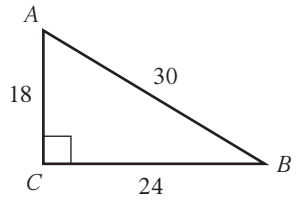
1. Use the triangles to complete the table below:



Triangle	Opposite to θ	Adjacent to θ	Hypotenuse	$\sin\theta$	$\cos\theta$	$\tan\theta$
1	18	24	30	$\frac{18}{30} = \frac{3}{5}$	$\frac{24}{30} = \frac{4}{5}$	$\frac{18}{24} = \frac{3}{4}$
2	5	2	$\sqrt{29}$	$\frac{5}{\sqrt{29}}$	$\frac{2}{\sqrt{29}}$	$\frac{5}{2}$
3	$\sqrt{51}$	7	10	$\frac{\sqrt{51}}{10}$	$\frac{7}{10}$	$\frac{\sqrt{51}}{7}$
4	10	24	26	$\frac{10}{26} = \frac{5}{13}$	$\frac{24}{26} = \frac{12}{13}$	$\frac{10}{24} = \frac{5}{12}$
5	$\sqrt{32}$	2	6	$\frac{\sqrt{32}}{6}$	$\frac{2}{6} = \frac{1}{3}$	$\frac{\sqrt{32}}{2}$

Page 9 questions

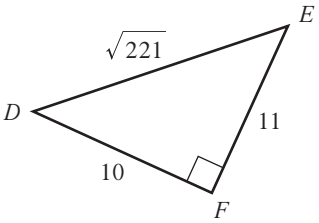
2. Complete the following for each triangle:



$\sin\angle A = \frac{O}{H}$ $= \frac{24}{30}$ $= \frac{4}{5}$

$\cos\angle B = \frac{A}{H}$ $= \frac{24}{30}$ $= \frac{4}{5}$

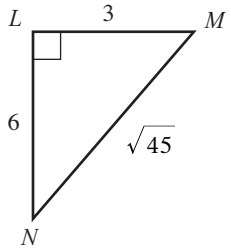
$\tan\angle B = \frac{O}{A}$ $= \frac{18}{24}$ $= \frac{3}{4}$



$\cos\angle E = \frac{A}{H}$ $= \frac{11}{\sqrt{221}}$

$\tan\angle D = \frac{O}{A}$ $= \frac{11}{10}$

$\sin\angle D = \frac{O}{H}$ $= \frac{11}{\sqrt{221}}$



$\tan\angle N = \frac{O}{A}$ $= \frac{3}{6}$ $= \frac{1}{2}$

$\sin\angle N = \frac{O}{H}$ $= \frac{3}{\sqrt{45}}$

$\cos\angle N = \frac{A}{H}$ $= \frac{6}{\sqrt{45}}$

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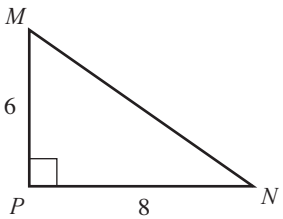
3. Find the missing side in each right angled triangle, and then find the ratios that follow:

a



Find MN, use Pythagoras $MN^2 = MP^2 + PN^2$ if $MP = 6, \quad PN = 8$

$MN^2 = 6^2 + 8^2$
$MN^2 = 36 + 64$
$MN^2 = 100$
$MN = \pm\sqrt{100}$
$MN = \pm 10$
$MN = 10$ (since length is positive)

$\sin\angle N = \frac{O}{H}$ $= \frac{6}{10}$ $= \frac{3}{5}$

$\tan\angle M = \frac{O}{A}$ $= \frac{8}{6}$ $= \frac{4}{3}$

$\cos\angle M = \frac{A}{H}$ $= \frac{6}{10}$ $= \frac{3}{5}$

$\tan\angle N = \frac{O}{A}$ $= \frac{6}{8}$ $= \frac{3}{4}$

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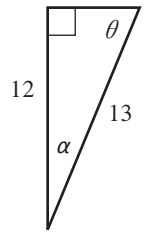
b

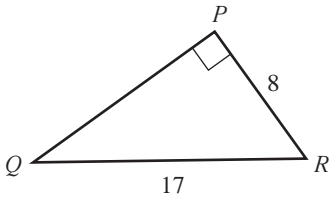


Find the missing side, use Pythagoras $c^2 = a^2 + b^2$ if $a = 12, \quad c = 13$

$$13^2 = 12^2 + b^2$$
$$169 = 144 + b^2$$
$$b^2 = 169 - 144$$
$$b^2 = 25$$
$$b = \pm\sqrt{25}$$
$$b = \pm 5$$
$$b = 5$$

$$\sin\theta = \frac{O}{H} = \frac{12}{13} \qquad \tan\alpha = \frac{O}{A} = \frac{5}{12} \qquad \sin\alpha = \frac{O}{H} = \frac{5}{13} \qquad \cos\theta = \frac{A}{H} = \frac{5}{13}$$

c

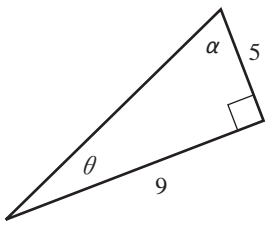


Find PQ, use Pythagoras $QR^2 = PR^2 + PQ^2$ if $PR = 8, \quad QR = 17$

$$17^2 = 8^2 + PQ^2$$
$$289 = 64 + PQ^2$$
$$PQ^2 = 289 - 64$$
$$PQ^2 = 225$$
$$PQ = \pm\sqrt{225}$$
$$PQ = \pm 15$$
$$PQ = 15$$

$$\sin\angle Q = \frac{O}{H} = \frac{8}{17} \qquad \cos\angle Q = \frac{A}{H} = \frac{15}{17} \qquad \cos\angle R = \frac{A}{H} = \frac{8}{17} \qquad \tan\angle R = \frac{O}{A} = \frac{15}{8}$$

d



Find Hypoenuse, use Pythagoras $c^2 = a^2 + b^2$ if $a = 5, \quad b = 9$

$$c^2 = 5^2 + 9^2$$
$$c^2 = 25 + 81$$
$$c^2 = 106$$
$$c = \pm\sqrt{106}$$
$$c = \sqrt{106}$$

$$\tan\theta = \frac{O}{A} = \frac{5}{9} \qquad \cos\alpha = \frac{A}{H} = \frac{5}{\sqrt{106}} \qquad \sin\theta = \frac{O}{H} = \frac{5}{\sqrt{106}} \qquad \tan\alpha = \frac{O}{A} = \frac{9}{5}$$

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4. Evaluate the following, to 3 decimal places:

a $\sin 40^\circ = 0.643$ (3 d.p.)

b $\cos 30^\circ = 0.866$ (3 d.p.)

c $\cos 60^\circ = 0.5$

d $\tan 20^\circ = 0.364$ (3 d.p.)

e $\tan 50^\circ = 1.192$ (3 d.p.)

f $\sin 85^\circ = 0.996$ (3 d.p.)

g $3\cos 45^\circ = 2.121$ (3 d.p.)

h $\sqrt{2}\sin 45^\circ = 1$

i $\sqrt{3}\tan 30^\circ = 1$

j $4\sin 73^\circ = 3.825$ (3 d.p.)

k $\dfrac{\cos 23^\circ}{2} = 0.460$ (3 d.p.)

l $\dfrac{3\tan 80^\circ}{4} = 4.253$ (3 d.p.)

5. Find the value of θ (to the nearest degree) if:

a $\cos\theta = 0.5$

$\theta = \cos^{-1}(0.5)$

$\theta = 60^\circ$

b $\sin\theta = 0.25$

$\theta = \sin^{-1}(0.25)$

$\theta = 14^\circ$ (nearest degree)

c $\tan\theta = \sqrt{3}$

$\theta = \tan^{-1}(\sqrt{3})$

$\theta = 60^\circ$

d $\tan\theta = 4.5$

$\theta = \tan^{-1}(4.5)$

$\theta = 77^\circ$ (nearest degree)

e $\cos\theta = 0.81$

$\theta = \cos^{-1}(0.81)$

$\theta = 36^\circ$ (nearest degree)

f $\sin\theta = \dfrac{\sqrt{2}}{2}$

$\theta = \sin^{-1}\left(\dfrac{\sqrt{2}}{2}\right)$

$\theta = 45^\circ$

g $\cos\theta = \dfrac{\sqrt{3}}{2}$

$\theta = \cos^{-1}\left(\dfrac{\sqrt{3}}{2}\right)$

$\theta = 30^\circ$

h $\tan\left(\dfrac{\theta}{2}\right) = 3.1$

$\left(\dfrac{\theta}{2}\right) = \tan^{-1}(3.1)$

$\dfrac{\theta}{2} = 72^\circ$

$\theta = 144^\circ$ (nearest degree)

i $\sin(2\theta) = 1$

$2\theta = \sin^{-1}(1)$

$2\theta = 90^\circ$

$= 45^\circ$

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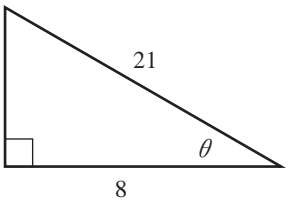
1. Find θ in each triangle to the nearest degree:

a



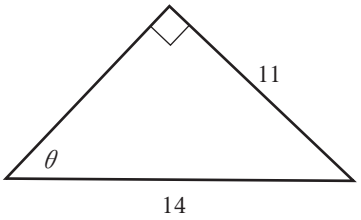
$\cos\theta = \frac{A}{H}$

$\cos\theta = \frac{8}{21}$

$\theta = \cos^{-1}\left(\frac{8}{21}\right)$

$\theta = 68^\circ$

b



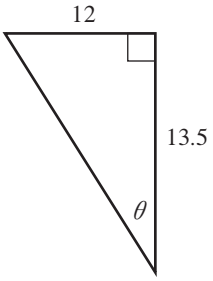
$\sin\theta = \frac{O}{H}$

$\sin\theta = \frac{11}{14}$

$\theta = \sin^{-1}\left(\frac{11}{14}\right)$

$\theta = 52^\circ$

c



$\tan\theta = \frac{O}{A}$

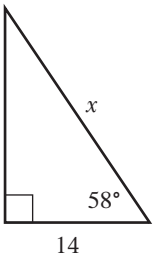
$\tan\theta = \frac{12}{13.5}$

$\theta = \tan^{-1}\left(\frac{12}{13.5}\right)$

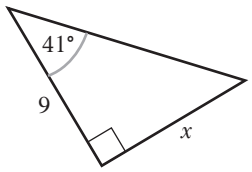
$\theta = 42^\circ$

2. Complete the table below if you are solving side labeled x:

a



b



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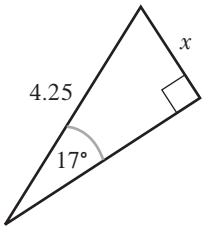
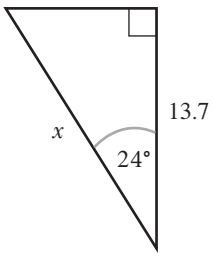


d



Triangle	Given side for angle	Missing side for angle (x)	Correct ratio to use (sin, cos, tan)
a	Adjacent	Hypotenuse	cos
b	Adjacent	Opposite	tan
c	Hypotenuse	Opposite	sin
d	Adjacent	Hypotenuse	cos

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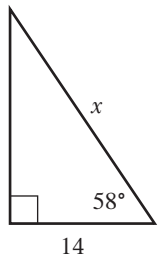
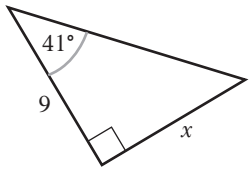
3. Find the value of x in each of the triangles from the previous question:

a



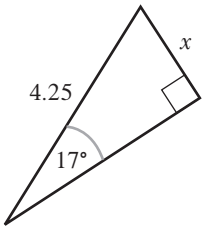
$$\cos\theta = \frac{A}{H}$$
$$\cos 58^\circ = \frac{14}{x}$$
$$x = \frac{14}{\cos 58^\circ}$$
$$x = 26.4 \text{ (1 d.p.)}$$

b



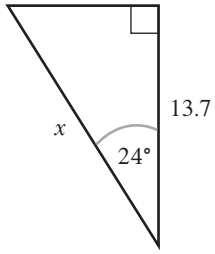
$$\tan\theta = \frac{O}{A}$$
$$\tan 41^\circ = \frac{x}{9}$$
$$x = 9\tan 41^\circ$$
$$x = 7.8 \text{ (1 d.p.)}$$

c



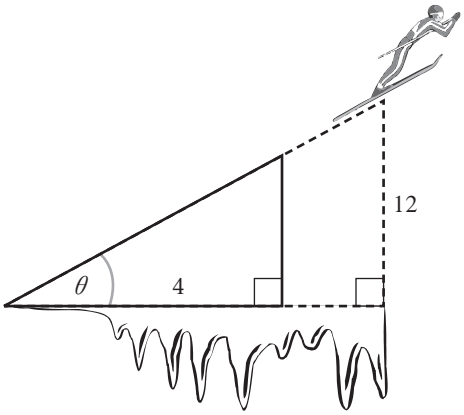
$$\sin\theta = \frac{O}{H}$$
$$\sin 17^\circ = \frac{x}{4.25}$$
$$x = 4.25\sin 17^\circ$$
$$x = 1.2 \text{ (1 d.p.)}$$

d



$$\cos\theta = \frac{A}{H}$$
$$\cos 24^\circ = \frac{13.7}{x}$$
$$x = \frac{13.7}{\cos 24^\circ}$$
$$x = 15.0 \text{ (1 d.p.)}$$

4. A skier jumps a 4m ramp. 2m after the jump the skier's height is 12m. What is the angle of the ramp?

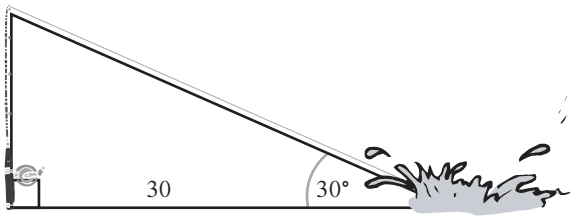


Note: 2 metres is added to 4 since the question states "2 metres after the ramp" therefore making the adjacent side 6 metres.

$$\tan\theta = \frac{O}{A}$$
$$\tan\theta = \frac{12}{6}$$
$$\theta = \tan^{-1}\left(\frac{12}{6}\right)$$
$$\theta = 63^\circ \text{ (nearest degree)}$$

Page 17 questions

5. A fisherman casts his line out and keeps his fishing rod pointing straight upwards. If the line touches the water 30 m from the shore at an angle of $30°$, then how long is the fishing line to the nearest metre?



The length wanted is the hypotenuse.

$$\cos\theta = \frac{A}{H}$$

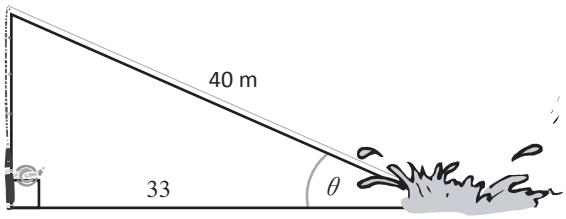
$$\cos 30^\circ = \frac{30}{H}$$

$$H = \frac{30}{\cos 30^\circ}$$

$$H = 34.6\text{m} \ (1\text{ d.p.})$$

Therefore, the fishing line is 35 m.

6. If the fishing line is 40 m long and touches the water 33 m from the shore, at what angle will the line touch the water?



$$\cos\theta = \frac{A}{H}$$

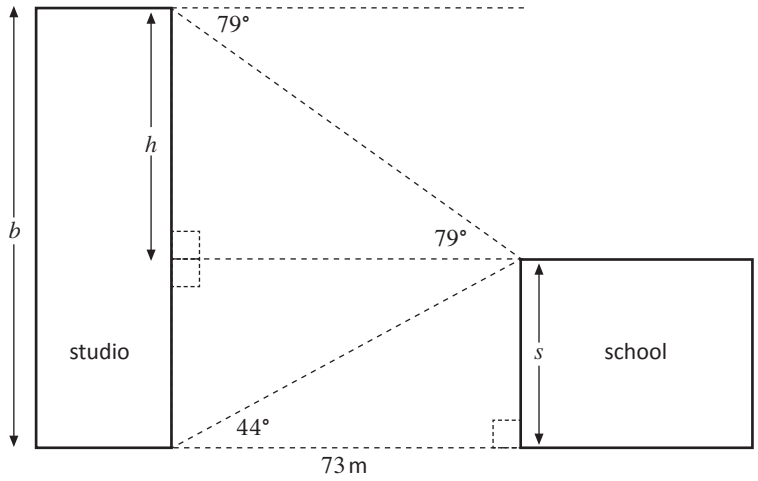
$$\cos\theta = \frac{33}{40}$$

$$\theta = \cos^{-1}\left(\frac{33}{40}\right)$$

$$\theta = 34^\circ \text{ (nearest degree)}$$

Page 20 questions

1. A studio is 73 m to the left of a school. The angle of elevation from the base of the studio to the roof of the school is 44°. The angle of depression from the roof of the studio to the roof of the school is 79°.



a Find the height of the school to 3 decimal places.

$$\tan\theta = \frac{O}{A}$$

$$\tan 44^\circ = \frac{s}{73}$$

$$s = 73 \times \tan 44^\circ$$

$$s = 70.495\text{m} \ (3\text{ d.p.})$$

b How much higher is the studio than the school to 3 decimal places?

$$\tan\theta = \frac{O}{A}$$

$$\tan 79^\circ = \frac{h}{73}$$

$$h = 73 \times \tan 79^\circ$$

$$h = 375.552\text{m} \ (3\text{ d.p.})$$

c What is the total height of the studio to 1 decimal place?

$$b = s + h$$

$$b = 70.459 + 375.552$$

$$b = 446.0\text{m} \ (1\text{ d.p.})$$

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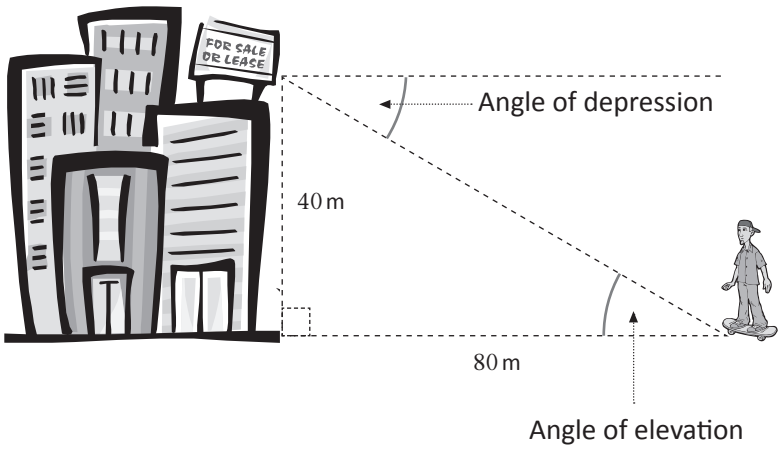
Page 21 questions

2. A skateboarder reads a sign on top of a 40 m building.

a Identify the angle of elevation and the angle of depression in the following diagram:



b If he sees the sign when he is 80 m away from the building, what is the angle of elevation from the skater to the sign?

$$\tan\theta = \frac{O}{A}$$

$$\tan\theta = \frac{40}{80}$$

$$\theta = \tan^{-1}\left(\frac{40}{80}\right)$$

$$\theta = 26.6^\circ \text{ (1 d.p.)}$$

c If the skater continues skating until he is 30 m from the building, will the angle of elevation increase or decrease? By how much?

$$\tan\theta = \frac{O}{A}$$

$$\tan\theta = \frac{40}{30}$$

$$\theta = \tan^{-1}\left(\frac{40}{30}\right)$$

$$\theta = 53.1^\circ \text{ (1 d.p.)}$$

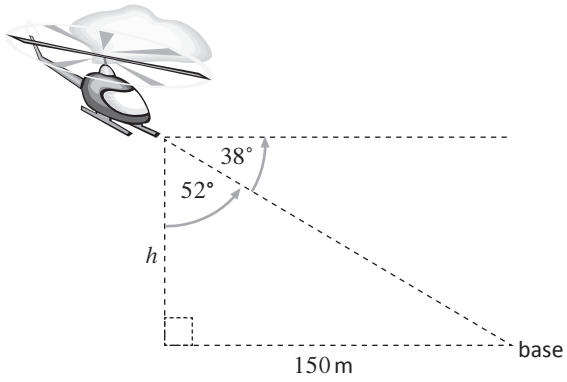
The angle of elevation will increase. It will increase by 26.5°.

Page 22 questions

3. Aiden answered the following question incorrectly. Can you spot his mistake?

The angle of depression from a helicopter to its landing base is $52°$. If the horizontal distance between the helicopter and the landing base is $150\,\text{m}$, then how high is the helicopter (1 decimal place) at this point?

AIDEN'S SOLUTION



$$\tan 52^\circ = \frac{150}{h}$$

$$h = \frac{150}{\tan 52^\circ}$$

$$h = \frac{150}{1.279}$$

$$h = 117.192... \approx 117.2\,\text{m} \text{ (1 decimal place)}$$

a What was Aiden's mistake?

Aiden incorrectly labelled the angle of depression. The angle of depression is formed between the upper horizontal and hypotenuse not h and the hypotenuse.

b Find the correct height of the helicopter at this point.

If the angle of depression is 52° then the angle between the hypotenuse and h is $90^\circ - 52^\circ$ which is 38°.

$$\tan\theta = \frac{O}{A}$$

$$\tan 38^\circ = \frac{150}{h}$$

$$h = \frac{150}{\tan 38^\circ}$$

$$h = 192.0\,\text{m} \text{ (1 d.p.)}$$

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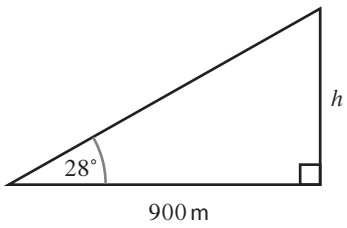
Page 23 questions

4. An aeroplane takes off at an angle of $28°$ to the ground. It flies over a house 900 m from the airport.

a How high is the aeroplane at that point, to 3 decimal places?



$$\tan\theta = \frac{O}{A}$$

$$\tan 28° = \frac{h}{900}$$

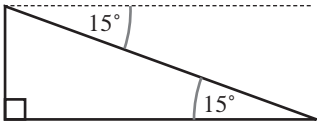
$$h = 900 \times \tan 28°$$

$$h = 478.538 \text{ m} \ (3 \text{ d.p.})$$

b What is the angle of depression at this point?

The angle of depression is $28°$.

c After continuing to fly at the same height, the pilot notices that as they are flying over a lake, the airport has a $15°$ angle of depression. How far is the lake away from the airport, to 2 decimal places?



$$\tan\theta = \frac{O}{A}$$

$$\tan 15° = \frac{478.538}{x}$$

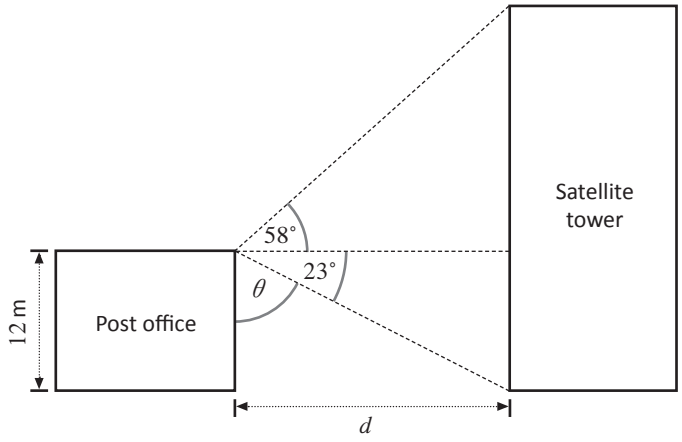
$$x = \frac{478.538}{\tan 15°}$$

$$x = 1785.93 \text{ m} \ (2 \text{ d.p.})$$

Page 24 questions

5. A satellite tower is on the right of a post office and they are separated by a distance, d. The post office has a height of 12 m. The angle of depression from the roof of the post office to the base of the tower is 23°. The angle of elevation from the roof of the post office to the roof of the tower is 58°.

a Draw a diagram to represent this situation.



b Find d, the distance between the buildings to 1 decimal place:

Find θ,

$$\theta = 90^\circ - 23^\circ$$

$$\theta = 67^\circ$$

$$\tan\theta = \frac{O}{A}$$

$$\tan 67^\circ = \frac{d}{12}$$

$$d = 12 \times \tan 67^\circ$$

$$d = 28.3\text{ m} \ (1\text{ d.p.})$$

c Find the total height of the tower to 1 decimal place:

$$\tan\theta = \frac{O}{A}$$

$$\tan 58^\circ = \frac{O}{28.3}$$

$$h = 28.3 \times \tan 58^\circ$$

$$h = 45.3\text{ m} \ (1\text{ d.p.})$$

Therefore, the total of height is equal to $45.3\text{ m} + 12\text{ m} = 57.3\text{ m}$

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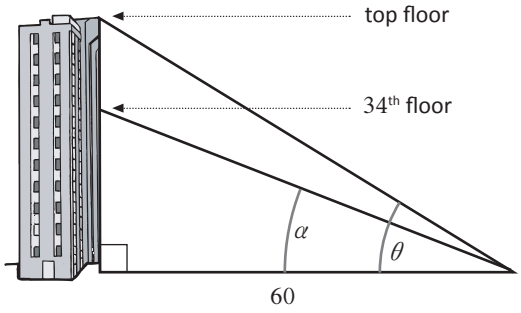
Page 27 questions

1. You and your friend stand in a building with 50 floors; each floor is 2 m high. You are on the 34^{th} floor and your friend is on the top floor. Find the difference between your angles of elevation 60 m away.



Find θ,

To find the opposite: $2 \times 50 = 100$ m since each floor is 2 m and here are 50 floors.

$$\tan\theta = \frac{O}{A}$$
$$\tan\theta = \frac{100}{60}$$
$$\theta = \tan^{-1}\left(\frac{100}{60}\right)$$
$$\theta = 59^\circ$$

To find the opposite: $2 \times 34 = 68$ m since each floor is 2 m and here are 34 floors.

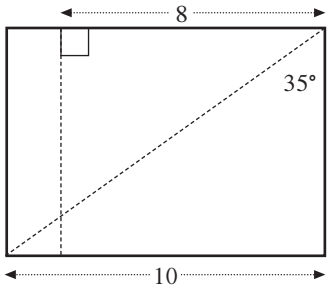
$$\tan\alpha = \frac{68}{60}$$
$$\tan\alpha = \tan^{-1}\frac{68}{60}$$
$$\alpha = 48.6^\circ$$

The difference is:

$$\theta - \alpha = 59^\circ - 48.6^\circ$$
$$= 10.4^\circ$$

2. As a technician you need to tie rope along the dotted line in this rectangle.

a How many right angled triangles are involved in this problem?



There are 3 right angled triangles. However, it is only necessary to use one right angled triangle with the information given, to find all necessary lengths.

Page 27 questions

b Find the total length of rope needed if all measurements are in m (nearest m):

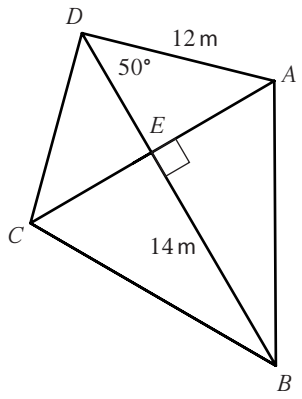
$$\sin\theta = \frac{O}{H}$$
$$\sin 35^\circ = \frac{10}{H}$$
$$H = \frac{10}{\sin 35^\circ}$$
$$H = 17.4\text{ m}\ \ (1\text{ d.p.})$$

$$\tan\theta = \frac{O}{A}$$
$$\tan 35^\circ = \frac{10}{A}$$
$$A = \frac{10}{\tan 35^\circ}$$
$$A = 14.3\text{ m}\ \ (1\text{ d.p.})$$

Therefore the total length of rope needed is $17.4\text{ m} + 14.3\text{ m} = 31.7\text{ m}$ which rounds to 32 m

Page 28 questions

3. In order to for a certain kite to fly it needs to look like this. Find the length of AB and angle $\angle ABE$ each to 1 decimal place.



$$\sin\theta = \frac{O}{H}$$
$$\sin 50^\circ = \frac{AE}{12}$$
$$AE = 12 \times \sin 50^\circ$$
$$AE = 9.19\text{ m}\ \ (2\text{ d.p.})$$

Using Pythagoras,

$$AB^2 = AE^2 + EB^2$$
$$AB^2 = 9.19^2 + 14^2$$
$$AB^2 = 280.46\ \ (2\text{ d.p.})$$
$$AB = \sqrt{280.46}$$
$$AB = 16.7\text{ m}\ \ (1\text{ d.p.})$$

$$\tan\angle B = \frac{9.19}{14}$$
$$\angle ABE = \tan^{-1}(0.6564)$$
$$\angle ABE = 33.3^\circ\ \ (1\text{ d.p.})$$

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