For greater accuracy, we can measure angles not only in **degrees** (°) but also in parts of degrees, as decimals or using the units **minutes** (') and **seconds** ('').

1 degree = 60 minutes 1 minute = 60 seconds

We will be measuring angles accurate to the nearest minute. Angle 53 degrees 18 minutes is written 53°18'.

EXAMPLE 1

Find the following correct to 4 decimal places.

a cos 84.3°

b sin 68.7°

c tan 15.5°

- d cos 78°15′
- e sin 11°12′
- f tan 17°58′

	Solve	Think	Apply
a	$\cos 84.3^{\circ} \approx 0.0993$	cos 84.3 =	Make sure your calculator is in degree mode.
b	$\sin 68.7^{\circ} \approx 0.9317$	sin 68.7 =	Some calculators have a viii key instead of a DMS key.
c	$\tan 15.5^{\circ} \approx 0.2773$	tan 15.5 =	instead of a Chib key.

EXAMPLE 1 CONTINUED

	Solve	Think	Apply
d	$\cos 78^{\circ}15' \approx 0.2036$	cos 78 DMS 15 DMS =	On some calculators the second DMS is not
e	$\sin 11^{\circ}12' \approx 0.1942$	sin 11 DMS 12 DMS =	required.
f	$\tan 17^{\circ}58' \approx 0.3243$	tan 17 DMS 58 DMS =	

- 1 Find the following correct to 4 decimal places.
 - a sin 36.8°

- **b** cos 14.23°
- c tan 8.11°

- d cos 65.25°
- e cos 89°21′
- f tan 18°23′

- g tan 68°23′
- h sin 45°21′
- i cos 57°51′

- j cos 33°21′
- k tan 21°33′
- l sin 11°11′

• EXAMPLE 2

Evaluate the following correct to 4 decimal places.

- **a** 12 cos 15.6°
- **b** 5 sin 11°15′
- e $\frac{3 \tan 11^{\circ}51'}{\cos 23^{\circ}15'}$

	Solve	Think	Apply
a	$12 \cos 15.6^{\circ} \approx 11.5580$	12 × cos 15.6 =	Remember that the fraction line is a grouping symbol.
b	$5 \sin 11^{\circ}15' \approx 0.9755$	5 × sin 11 DMS 15 DMS =	Insert brackets if you are not sure of the order of
c	$\frac{3 \tan 11^{\circ}51'}{\cos 23^{\circ}15'} \approx 0.6851$	3 × tan (11 DMS 51 DMS	operations.
		÷ cos (23 DMS 15	
		DMS) =	

2 Evaluate the following correct to 4 decimal places.

a 8 cos 23.1°
 d 23 sin 75°12′
 g 9 sin 11°51′

h $\frac{8 \tan 16^{\circ}16'}{\sin 15^{\circ}}$

e 8.3 tan 58°51′

b 5 tan 16.4°

f 12.3 cos 27°48′ i 12.3 cos 48°

sin 16°15′

c 15 sin 48.18°

 $\sin 31^{\circ}$ $\frac{8.7 \tan 75^{\circ}14'}{13.2}$

 $\frac{4.2 \cos 18.3^{\circ}}{6.8}$

 $\frac{3 \sin 83^{\circ}12'}{16.5}$

 $\mathbf{m} \,\, \frac{4 \sin 18^{\circ} \cos 18^{\circ}}{3}$

 $\frac{11 \tan 16^{\circ} \cos 14^{\circ}}{\sin 12^{\circ}}$

 $\frac{8.3 \cos 11^{\circ}15'}{\sin 11^{\circ}15'}$

Using trigonometric ratios to find angles

You can work backwards on a calculator to find an angle from one of the trigonometric ratios, by using one of the key combinations $\begin{array}{c} \text{SHIFT} \end{array}$ $\begin{array}{c} \text{cos} \end{array}$. These may appear on your calculator display as $\begin{array}{c} \text{tan}^{-1} \text{ or } \text{sin}^{-1} \text{ or } \text{cos}^{-1}. \end{array}$

For example, if $\sin \theta = 0.4369$ then $\theta = \sin^{-1} 0.4369$

where $\sin^{-1} 0.4369$ means 'the angle whose sine is 0.4369'.

Similarly, \cos^{-1} means 'the angle whose cosine is' and \tan^{-1} means 'the angle whose tangent is'.

EXAMPLE 3

Find θ to the nearest:

i degree $\sin \theta = 0.6314$

ii minute.

b $\tan \theta = 3.6$

 $\cos \theta = 0.8$

	Solve	Think	Apply
a i	$\sin \theta = 0.6314$ $\theta = 39.153$ $\approx 39^{\circ}$	SHIFT (sin 0.6314 =	Make sure your calculator is in degree mode.
ii	$\theta = 39^{\circ}9'12.55''$ $\approx 39^{\circ}9'$ to the nearest minute as $12.55 < 30$	As the seconds are less than 30, round the minutes down.	is pressed before the trigonometric ratios so that the answer is an
b i	$\tan \theta = 3.6$ $\theta = 74.475$ $\approx 74^{\circ}$	SHIFT tan 3.6	angle. The half-way point for rounding is 30 seconds. Below 30 seconds round down; 30 seconds or more, round up. Note that some calculators require SHIFT before DMS to convert to minutes and seconds.
ii	$\theta = 74^{\circ}28'33.20''$ $\approx 74^{\circ}29'$ to the nearest minute as $33.2 > 30$	As the seconds are greater than or equal to 30, round the minutes up.	
c i	$\cos \theta = 0.8$ $\theta = 36.869$ $\approx 37^{\circ}$	SHIFT COS 0.8	
ii	$\theta = 36^{\circ}52'11.63''$ $\approx 36^{\circ}52'$ to the nearest minute as $11.63 < 30$	DMS	

3 Write these calculator displays as angles to the nearest minute.

- **a** 43°27′14.2″
- **b** 62°15′58.13′′
- c 14°3′0″

- d 81°53′30′′
- e 21°59′48.72′′
- 10°1′28.42′′

- g 35°28′18.3″
- h 72°51′38.5″
- 27°53′58.1′′

- j 39°35′11.3′′
- k 68°54′41.2″
- 0°3′34.2′′

4 Find the value of θ to the nearest:

- i degree
- **a** $\sin \theta = 0.3625$
- **d** $\cos \theta = 0.6731$
- \mathbf{g} tan $\theta = 0.0371$
- $\sin \theta = 0.0027$
- $\mathbf{m} \cos \theta = 0.6614$

- ii minute.
- ______
- **b** $\cos \theta = 0.1445$ **e** $\tan \theta = 4.1371$
- **h** $\sin \theta = 0.5512$
- k $\tan \theta = 23.7215$ n $\sin \theta = 0.6262$
- c $\tan \theta = 2.1351$
- $\mathbf{f} \quad \sin \theta = 0.1113$
- $\cos \theta = 0.0314$
- $\cos \theta = 0.9811$
- o $\tan \theta = 0.2222$

EXAMPLE 4

Find θ to the nearest:

- i degree
- $\mathbf{a} \quad \sin \theta = \frac{5}{9}$

- ii minute.
- **b** $\cos \theta = \frac{6}{13}$
- c $\tan \theta = \frac{18}{7}$

	Solve	Think	Apply	
a i	$\sin \theta = \frac{5}{9}$ $\theta = 33.74$ $\approx 34^{\circ}$	SHIFT Sin (5 ÷ 9) =	Make sure that the calculator is in degree mode.	
ii	θ = 33°44′56.35'' ≈ 33°45′	As the seconds are greater than 30, round the minutes up.	Press SHIFT first to obtain an angle. Put the fraction in brackets before	
b i	$\cos \theta = \frac{6}{13}$ $\theta = 62.51$ $\approx 63^{\circ}$	SHIFT COS (6 ÷ 13) =	pressing Round accordingly. Note that some	
ii	θ = 62°30′48.86'' ≈ 62°31'	As the seconds are greater than 30, round the minutes up.	calculators require SHIFT DMS to convert to minutes and seconds.	
c i	$\tan \theta = \frac{18}{7}$ $\theta = 68.74$ $\approx 69^{\circ}$	SHIFT (tan) (18 ÷ 7) =		
ii	$\theta = 68^{\circ}44'58.18''$ $\approx 68^{\circ}45'$	As the seconds are greater than 30, round the minutes up.		

5 Find the value of θ to the nearest:

i degree

ii minute.

i $\tan \theta = \frac{11.27}{15}$

a $\tan \theta = \frac{14}{3}$ **b** $\cos \theta = \frac{3}{11}$ **c** $\sin \theta = \frac{11}{18}$ **d** $\sin \theta = \frac{4}{29}$ **e** $\tan \theta = \frac{6}{7}$ **f** $\cos \theta = \frac{14}{17}$ **g** $\sin \theta = \frac{0.013}{0.214}$ **h** $\cos \theta = \frac{6.2}{15}$ $\mathbf{j} \cos \theta = \frac{1}{3}$

 $\mathbf{k} \sin \theta = \frac{3}{4}$

 $1 \quad \tan \theta = \frac{4}{3}$

6 Find angle A to the nearest minute given that:

$$\cos A = 0.7$$

b
$$\sin A = 0.642$$

c
$$\tan A = 3.265$$

EXELCISE OF

2 a 7.3586

d 22.2369

g 3.5884

j 2.5004

m 0.3919

g 35°28′

1 a 0.5990 **d** 0.4187 g 2.5236

e 0.0113 **h** 0.7114 j 0.8353

k 0.3949 b 1.4716e 13.7320

b 0.9693

h 9.0191 k 0.5864 n 14.7202

3 a 43°27′ **b** 62°16′

d 81°54′ e 22°0′ g 35°28′ h 72°52′ i 39°35′ k 68°55′ c 0.1425

f 0.3323 i 0.5321 1 0.1939

> c 11.1786 f 10.8803

i 29.4119

1 0.1805 o 41.7269

f 10°1′ i 27°54′

c 14°3′

1 0°4′

4 a i 21° ii 21°15′ b i 82° ii 81°42′ i 65° ii 64°54′ d i 48° ii 47°42′ ii 76°25′ **f** i 6° i 76° ii 6°23′ i 2° h i 33° ii 2°7′ ii 33°27′ i 88° ii 88°12′ i 0° ii 0°9′ i 88° ii 87°35′ l i 11° ii 11°9′ i 49° n i 39° ii 38°46′ ii 48°36′ m i 13° ii 12°32′ i 78° ii 77°54′ b i 74° ii 74°10′ 5 a ii 7°56′ i 38° ii 37°40′ d i 8° c f i 35° ii 34°34′ i 41° ii 40°36′ e i 3° ii 3°29′ h i 66° ii 65°35′ ii 70°32′ i 37° ii 36°55′ i i 71° 1 i 53° i 49° ii 48°35′ ii 53°8′ 6 a 45°34′ **b** 39°56′ c 72°58′