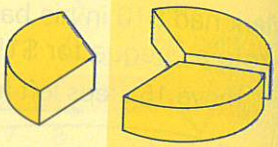
Number/Algebra: Equivalent fractions

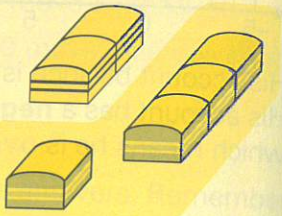
A **fraction** compares part of an object with the whole object. For example, if a cake is divided into 3 equal parts and you have 2 parts then the fraction of the cake that you have is written:

$\frac{2}{3}$ ← **numerator** – the *number* of parts you have

← **denominator** – the *size* of the parts (total number of *equal* parts in the whole)



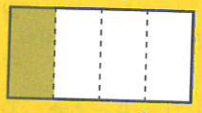
If two cakes are each divided into thirds and you have four pieces then you have four thirds of a cake, which is written $\frac{4}{3}$.



Equivalent fractions

Jan divides her chocolate bar into parts of equal size. Shown below are 3 ways Jan can break off equivalent pieces (shown shaded) of her chocolate bar.

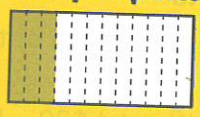
4 equal parts



Each part is $\frac{1}{4}$

Shaded piece is $\frac{1}{4}$

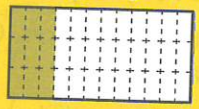
12 equal parts



Each part is $\frac{1}{12}$

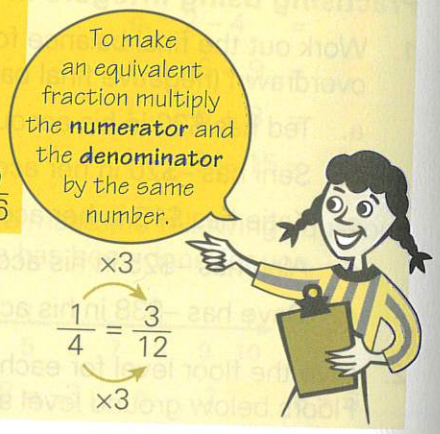
Shaded piece is $\frac{3}{12}$

36 equal parts



Each part is $\frac{1}{36}$

Shaded piece is $\frac{9}{36}$



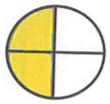
The fractions $\frac{1}{4}$, $\frac{3}{12}$ and $\frac{9}{36}$ all represent the same amount of chocolate in Jan's bar.

They are called **equivalent fractions**.

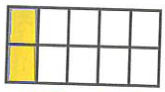
$\frac{1}{4} = \frac{3}{12}$ (×3 above, ×3 below)

Practising equivalent fractions

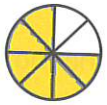
1. Name each diagram with two equivalent fractions.

* 

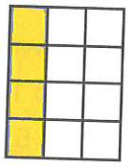
$\frac{1}{2} = \frac{2}{4}$

a. 

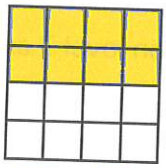
$\frac{\square}{5} = \frac{\square}{\square}$

b. 

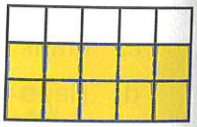
$\frac{\square}{4} = \frac{\square}{\square}$

c. 

$\frac{\square}{3} = \frac{\square}{\square}$

d. 

$\frac{\square}{4} = \frac{\square}{\square}$

e. 

$\frac{\square}{3} = \frac{\square}{\square}$

2. Make equivalent fractions by multiplying numerator and denominator by the same number. One has been done for you.

* $\frac{3}{4} \overset{\times 2}{=} \frac{6}{8}$ ($\times 2$ below)

a. $\frac{2}{3} \overset{\times\square}{=} \frac{\square}{9}$ ($\times\square$ below)

b. $\frac{7}{8} \overset{\times\square}{=} \frac{\square}{40}$ ($\times\square$ below)

c. $\frac{5}{12} \overset{\times\square}{=} \frac{20}{\square}$ ($\times\square$ below)

d. $\frac{7}{10} \overset{\times\square}{=} \frac{\square}{100}$ ($\times\square$ below)

e. $\frac{4}{5} \overset{\times\square}{=} \frac{12}{\square}$ ($\times\square$ below)

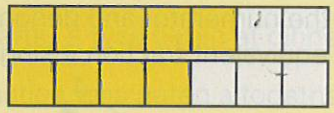
3. Rewrite each fraction as an equivalent fraction over 10 or 100.

a. $\frac{4}{5} = \frac{\square}{10}$ b. $\frac{7}{20} = \frac{\square}{100}$ c. $\frac{3}{4} = \frac{\square}{100}$ d. $\frac{13}{20} = \frac{\square}{100}$ e. $\frac{11}{25} = \frac{\square}{100}$

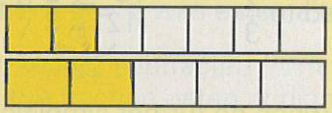
Number/Algebra: Comparing fractions

The fraction $\frac{5}{7}$ is bigger than $\frac{4}{7}$ because 5 is bigger than 4.

If the numerators of two fractions are the same, then the fraction with the smaller denominator is larger, e.g. $\frac{2}{5}$ is bigger than $\frac{2}{7}$.



$\frac{5}{7} > \frac{4}{7}$



$\frac{2}{5} > \frac{2}{7}$

Comparing fractions with **equal denominators** like $\frac{5}{7}$ and $\frac{4}{7}$ is easy.

Benchmark fractions like $\frac{1}{2}$ are useful for comparing fractions, e.g. $\frac{7}{10} > \frac{1}{2}$ and $\frac{4}{9} < \frac{1}{2}$ so $\frac{4}{9} < \frac{3}{5}$.

A useful strategy for comparing fractions is to use **equivalent fractions**.

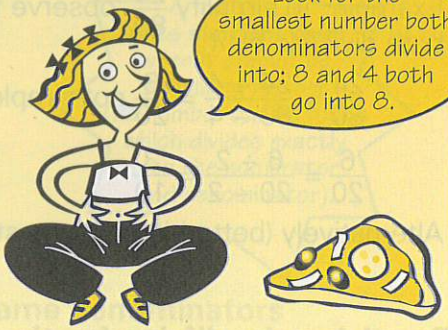
Example: Joe and Felicity both ordered large pizzas.

Joe ate $\frac{5}{8}$ of his pizza. Felicity ate $\frac{3}{4}$ of her pizza.

Who ate more pizza?

$\frac{3}{4} \overset{\times 2}{=} \frac{6}{8}$ change $\frac{3}{4}$ to the equivalent fraction $\frac{6}{8}$

Look for the smallest number both denominators divide into; 8 and 4 both go into 8.



$\frac{3}{4}$ is bigger than $\frac{5}{8}$ because $\frac{6}{8}$ is bigger than $\frac{5}{8}$. So Felicity ate more pizza.

Practising comparing fractions

1. Put < (less than) or > (greater than) between the following pairs of fractions:

   a. $\frac{3}{8} \square \frac{5}{8}$ b. $\frac{3}{4} \square \frac{3}{8}$ c. $\frac{7}{15} \square \frac{4}{15}$ d. $\frac{6}{7} \square \frac{5}{4}$ e. $\frac{3}{8} \square \frac{4}{5}$ f. $\frac{1}{5} \square \frac{1}{10}$

2. Write equivalent fractions with equal denominators below each pair of fractions. Then put > (is greater than) or < (is less than) between them. One has been done for you.

   * $\frac{3}{8} < \frac{2}{5}$ because $\frac{15}{40} < \frac{16}{40}$

   a. $\frac{2}{3} \square \frac{5}{6}$ because $\frac{\square}{6} \square \frac{\square}{6}$

   b. $\frac{3}{4} \square \frac{5}{8}$ because $\frac{\square}{8} \square \frac{\square}{8}$

   c. $\frac{2}{3} \square \frac{3}{5}$ because $\frac{\square}{15} \square \frac{\square}{15}$

   d. $\frac{5}{12} \square \frac{2}{5}$ because $\frac{\square}{\square} \square \frac{\square}{\square}$

3. Three fractions are given. Rename each fraction with the same denominator, then list these fractions in order of size from smallest to biggest. The first has been done for you.

   * $\frac{1}{2} = \frac{10}{20}$ $\frac{2}{5} = \frac{8}{20}$ $\frac{11}{20} = \frac{11}{20}$ order is $\frac{2}{5} < \frac{1}{2} < \frac{11}{20}$

   a. $\frac{3}{5} = \frac{\square}{30}$ $\frac{2}{3} = \frac{\square}{30}$ $\frac{1}{2} = \frac{\square}{30}$ order is $\frac{\square}{\square} < \frac{\square}{\square} < \frac{\square}{\square}$

   b. $\frac{1}{2} = \frac{\square}{\square}$ $\frac{2}{5} = \frac{\square}{\square}$ $\frac{1}{3} = \frac{\square}{\square}$ order is $\frac{\square}{\square} < \frac{\square}{\square} < \frac{\square}{\square}$

   c. $\frac{1}{4} = \frac{\square}{\square}$ $\frac{1}{5} = \frac{\square}{\square}$ $\frac{3}{8} = \frac{\square}{\square}$ order is $\frac{\square}{\square} < \frac{\square}{\square} < \frac{\square}{\square}$

   d. $\frac{1}{2} = \frac{\square}{\square}$ $\frac{2}{5} = \frac{\square}{\square}$ $\frac{1}{4} = \frac{\square}{\square}$ order is $\frac{\square}{\square} < \frac{\square}{\square} < \frac{\square}{\square}$

Number/Algebra: Simplifying fractions

A fraction can be **simplified** if the numerator and denominator can *both* be divided by the *same* number. This creates an equivalent fraction, using smaller numbers.

Example: $\frac{8}{12}$ can be simplified to $\frac{2}{3}$ $\quad \frac{8}{12} \overset{\div 4}{=} \frac{2}{3}$

A fraction is in **simplest form** when no further simplification is possible.

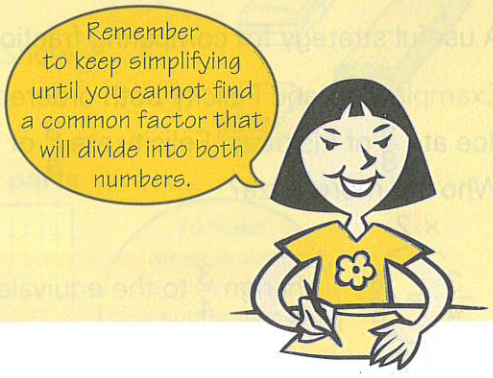
To save extra steps of working, always look for the *largest possible factor* that divides both numbers in the fraction.

Example: To simplify $\frac{24}{80}$, observe that 24 and 80 are both divisible by 4.

$\frac{24}{80} = \frac{24 \div 4}{80 \div 4} = \frac{6}{20}$ not simplest form

$\frac{6}{20} = \frac{6 \div 2}{20 \div 2} = \frac{3}{10}$

Alternatively (better) in only one step: $\frac{24}{80} \overset{\div 8}{=} \frac{3}{10}$



Practising simplifying fractions

1. Complete the working to simplify the following fractions. The first one has been done for you.

* $\frac{15}{25} \overset{\div 5}{=} \frac{3}{5}$ $\quad$ a. $\frac{18}{27} \overset{\div 9}{=} \frac{\square}{\square}$ $\quad$ b. $\frac{40}{30} \overset{\div 10}{=} \frac{\square}{\square}$ $\quad$ c. $\frac{28}{32} \overset{\div 4}{=} \frac{\square}{\square}$

d. $\frac{24}{30} = \frac{\square}{5}$ $\quad$ e. $\frac{49}{56} = \frac{7}{\square}$ $\quad$ f. $\frac{125}{150} = \frac{\square}{6}$ $\quad$ g. $\frac{48}{88} = \frac{\square}{11}$

2. Express each of the following fractions in simplest form.

a. $\frac{80}{100}$ _____ $\quad$ b. $\frac{42}{66}$ _____ $\quad$ c. $\frac{32}{64}$ _____ $\quad$ d. $\frac{45}{63}$ _____

e. $\frac{56}{80}$ _____ $\quad$ f. $\frac{25}{80}$ _____ $\quad$ g. $\frac{48}{36}$ _____ $\quad$ h. $\frac{8}{28}$ _____

3. Simplify fractions where necessary, then insert an equal sign (=) or a not equal sign (≠) between the following pairs of fractions.

a. $\frac{5}{8} \square \frac{12}{16}$ _____ $\quad$ b. $\frac{6}{9} \square \frac{16}{24}$ _____

c. $\frac{8}{10} \square \frac{12}{15}$ _____ $\quad$ d. $\frac{7}{4} \square \frac{9}{6}$ _____

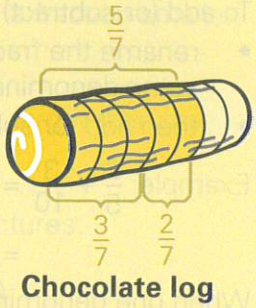
e. $\frac{45}{60} \square \frac{4}{6}$ _____ $\quad$ f. $\frac{16}{6} \square \frac{30}{9}$ _____

g. $\frac{20}{32} \square \frac{3}{4}$ _____ $\quad$ h. $\frac{2}{5} \square \frac{18}{40}$ _____

Number/Algebra: Adding and subtracting fractions with same denominators

A chocolate log was divided into 7 parts. $\frac{3}{7}$ was eaten at dinner and $\frac{2}{7}$ was eaten at supper. What fraction was eaten altogether?

From the diagram it can be seen that $\frac{3}{7} + \frac{2}{7} = \frac{5}{7}$ was eaten altogether.



Chocolate log

Also $\frac{5}{7} - \frac{2}{7} = \frac{3}{7}$ the fraction eaten at supper subtracted from the total fraction eaten gives the fraction eaten at dinner

Example: 1. $\frac{5}{15} + \frac{2}{15} = \frac{7}{15}$ (add numerators; denominator stays same)

2. $\frac{8}{10} - \frac{5}{10} = \frac{3}{10}$ (subtract numerators)

Simplify the fractions in your answers where you can (divide numerator and denominator by the same number).

For example $\frac{3}{4} - \frac{1}{4} = \frac{2}{4}$ which simplifies to $\frac{1}{2}$.

A fraction is in its **simplest form** when it cannot be simplified further (no number can be found which divides exactly into the numerator and denominator).

Practising adding and subtracting fractions with same denominators

1. Add or subtract these fractions with equal denominators. Shade the diagrams to help you.

a. $\frac{7}{10} + \frac{2}{10} = \frac{\quad}{10}$	b. $\frac{9}{15} - \frac{7}{15} = \frac{\quad}{15}$	c. $\frac{4}{5} - \frac{2}{5} = \frac{\quad}{\quad}$	d. $\frac{2}{9} + \frac{2}{9} = \frac{\quad}{\quad}$

2. Simplify your answers to these addition and subtraction sums (divide the numerator and denominator by the same number). The first one has been done for you.

* $\frac{5}{12} + \frac{1}{12}$ $= \frac{5+1}{12}$ $= \frac{6}{12}$ $= \frac{1}{2}$	a. $\frac{6}{15} + \frac{4}{15}$ $= \frac{\quad}{\quad}$ $= \frac{\quad}{\quad}$ $= \frac{\quad}{\quad}$	b. $\frac{5}{9} - \frac{2}{9}$ $= \frac{\quad}{\quad}$ $= \frac{\quad}{\quad}$ $= \frac{\quad}{\quad}$	c. $\frac{7}{20} - \frac{3}{20}$ $= \frac{\quad}{\quad}$ $= \frac{\quad}{\quad}$ $= \frac{\quad}{\quad}$	d. $\frac{5}{8} + \frac{1}{8}$ $= \frac{\quad}{\quad}$ $= \frac{\quad}{\quad}$ $= \frac{\quad}{\quad}$	e. $\frac{19}{25} - \frac{14}{25}$ $= \frac{\quad}{\quad}$ $= \frac{\quad}{\quad}$ $= \frac{\quad}{\quad}$

3. Kester spent $\frac{1}{5}$ of his money on food and $\frac{2}{5}$ of his money on a book.

   a. What fraction did he spend altogether? $\frac{\quad}{\quad}$

   b. What fraction of his money was not spent? $\frac{\quad}{\quad}$

   c. True or false?
   The book cost twice as much as the food. ____

4. Henry and Dan moved $\frac{7}{8}$ of the books in a warehouse. Henry moved $\frac{5}{8}$ of the books. What fraction did Dan move?

   $\frac{\quad}{\quad} = \frac{\quad}{\quad}$

Number/Algebra: Adding and subtracting fractions with different denominators

To add (or subtract) fractions with *different* denominators:

- rename the fractions as equivalent fractions with the *same* denominators (**common denominators**)
- then add (or subtract) the numerators.

Rename two fifths as four tenths so denominators are the same.

Example: $\frac{2}{5} + \frac{3}{10} = \frac{4}{10} + \frac{3}{10}$

$= \frac{7}{10}$

$\frac{2}{5} = \frac{2 \times 2}{5 \times 2} = \frac{4}{10}$

When one denominator cannot be divided evenly into the other, you need to rename **both** denominators.

Example: $\frac{2}{3} - \frac{1}{8} = \frac{16}{24} - \frac{3}{24}$

$= \frac{13}{24}$

$\frac{2}{3} = \frac{16}{24}$ ($\times 8$ top and bottom)

$\frac{1}{8} = \frac{3}{24}$ ($\times 3$ top and bottom)

subtracting numerators (denominator same)

Simplify your answer whenever possible.

Example: $\frac{4}{5} - \frac{3}{10} = \frac{8}{10} - \frac{3}{10}$

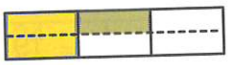
$= \frac{5}{10}$ or $\frac{1}{2}$

$\frac{4}{5} = \frac{4 \times 2}{5 \times 2} = \frac{8}{10}$

simplifying (divide numerator and denominator by 5)

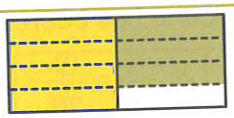
Practising adding and subtracting fractions with different denominators

1. Use the diagrams to add these fractions (with different denominators). One has been done for you.

* 

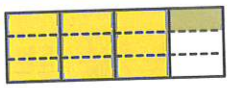
$\frac{1}{3} + \frac{1}{6} = \frac{2}{6} + \frac{1}{6}$

$= \frac{3}{6}$ or $\frac{1}{2}$

a. 

$\frac{1}{2} + \frac{3}{8} = \frac{\quad}{\quad} + \frac{\quad}{\quad}$

$= \frac{\quad}{\quad}$

b. 

$\frac{3}{4} + \frac{1}{12} = \frac{\quad}{\quad} + \frac{\quad}{\quad}$

$= \frac{\quad}{\quad}$ or $\frac{\quad}{\quad}$

c. 

$\frac{1}{2} + \frac{1}{4} = \frac{\quad}{\quad} + \frac{\quad}{\quad}$

$= \frac{\quad}{\quad}$

2. Rename these fractions with equal denominators before adding or subtracting. Simplify where possible. One has been done for you.

* $\frac{5}{8} - \frac{1}{4} = \frac{5}{8} - \frac{2}{8}$

$= \frac{3}{8}$

a. $\frac{5}{6} - \frac{1}{3} = \frac{5}{6} - \frac{\quad}{6}$

$= \frac{\quad}{6}$

$= \frac{\quad}{\quad}$

b. $\frac{3}{5} - \frac{1}{10} = \frac{\quad}{10} - \frac{1}{10}$

$= \frac{\quad}{10}$

$= \frac{\quad}{\quad}$

c. $\frac{3}{5} + \frac{1}{3} = \frac{\quad}{15} + \frac{\quad}{15}$

$= \frac{\quad}{\quad}$

d. $\frac{5}{6} - \frac{1}{4} = \frac{\quad}{12} - \frac{\quad}{12}$

$= \frac{\quad}{\quad}$

e. $\frac{7}{8} - \frac{2}{3} = \frac{\quad}{24} - \frac{\quad}{24}$

$= \frac{\quad}{\quad}$

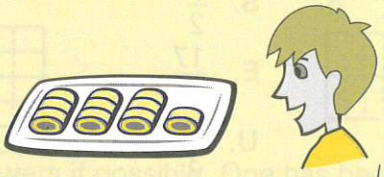
Number/Algebra: Mixed numbers and improper fractions

A **mixed number** is made up of a whole number and a **proper fraction** (a fraction smaller than one whole), e.g. $3\frac{1}{3}$ which means $3 + \frac{1}{3}$

An **improper fraction** (a fraction larger than one whole) has a numerator larger than the denominator, e.g. $\frac{10}{3}$ or 10 thirds.

One way of converting between these two types of number is to use pictures.

Example: I have 10 pieces of sausage roll.
Each piece is $\frac{1}{3}$ of a sausage roll.
How much do I have?

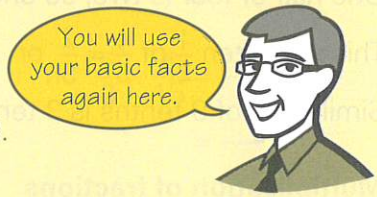


Solution: I have ten thirds $\left(\frac{10}{3}\right)$ or 3 whole sausage rolls and 1 extra third $\left(3\frac{1}{3}\right)$.

The improper fraction $\frac{10}{3}$ and the mixed number $3\frac{1}{3}$ mean the same amount.

Converting improper fractions to and from mixed numbers

To change an improper fraction (e.g. $\frac{23}{4}$) to a mixed numeral follow the steps:



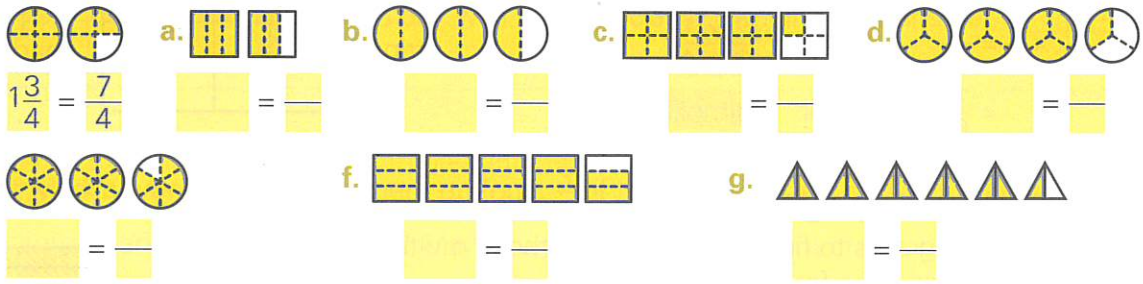
1. **Whole number part:** divide the numerator by the denominator. e.g. 4 goes into 23 **5** times with remainder 3.
2. **Fraction part:** put the remainder over the denominator. e.g. the remainder is the fraction $\frac{\mathbf{3}}{\mathbf{4}}$. So $\frac{23}{4}$ converts to the mixed number $5\frac{3}{4}$.

To change a mixed numeral (e.g. $5\frac{3}{4}$) to an improper fraction follow the steps:

1. **Numerator:** multiply the whole number by the denominator then add on the numerator, e.g. the numerator of $5\frac{3}{4}$ is $5 \times 4 + 3 = 23$
2. **Denominator:** The denominator is unchanged, e.g. $5\frac{3}{4}$ equals the improper fraction $\frac{23}{4}$

Practising mixed numbers and improper fractions

1. Use the diagrams to write each shaded amount as a mixed number and as an improper fraction. The first one has been done for you.



Number/Algebra: Mixed numbers and improper fractions

2. Match equal mixed numbers and improper fractions.

*	$3\frac{1}{2}$	S	P. $\frac{29}{8}$
a.	$2\frac{2}{3}$		R. $\frac{16}{3}$
b.	$3\frac{5}{8}$		S. $\frac{7}{2}$
c.	$2\frac{1}{8}$		E. $\frac{17}{8}$
d.	$5\frac{1}{3}$		U. $\frac{8}{3}$

3. In these additions, simplify improper fractions to mixed numbers.

* $\frac{5}{8} + \frac{5}{8} = \frac{10}{8} = 1\frac{2}{8} = 1\frac{1}{4}$

a. $\frac{4}{5} + \frac{3}{5} = __ = $

b. $\frac{7}{12} + \frac{11}{12} = __ = __ = $

c. $\frac{4}{9} + \frac{8}{9} = __ = __ = $

d. $\frac{3}{4} + \frac{3}{4} = __ = __ = $

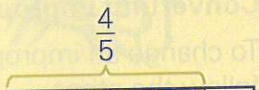
Number/Algebra: Multiplying fractions

Finding a fraction of a quantity

One half of four is two, so one half of four fifths is two fifths.

This is written $\frac{1}{2}$ of $\frac{4}{5} = \frac{2}{5}$ or $\frac{1}{2} \times \frac{4}{5} = \frac{2}{5}$ the word 'of' means multiply.

Similarly $\frac{1}{3}$ of 9 tenths is 3 tenths ($\frac{1}{3} \times \frac{9}{10} = \frac{3}{10}$) and so on.

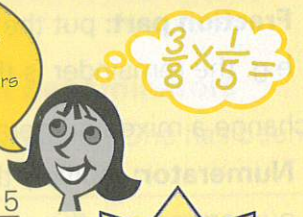


$\frac{1}{2}$ of $\frac{4}{5} = \frac{2}{5}$

Multiplication of fractions

The general rule for multiplying fractions is shown in the next example.

Rules to remember for multiplying fractions:
1. Multiply the top numbers (numerators) together.
2. Multiply the bottom numbers (denominators) together.
3. Simplify your answer as far as possible.



Example:

1. $\frac{3}{8} \times \frac{1}{5} = \frac{3 \times 1}{8 \times 5} = \frac{3}{40}$

2. $\frac{2}{5} \times \frac{7}{8} = \frac{2 \times 7}{5 \times 8} = \frac{14}{40} = \frac{7}{20}$

3. $\frac{4}{5} \times 15 = \frac{4}{5} \times \frac{15}{1} = \frac{60}{5} = 12$

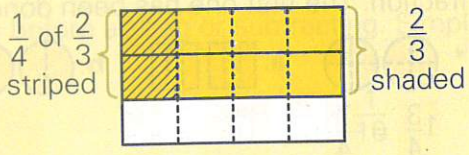
Remember: You can write whole numbers as fractions, e.g. $6 = \frac{6}{1}$

4. $\frac{2}{3}$ of the local swimming pool can be hired.

A lifesaving club wants to use $\frac{1}{4}$ of the available space. The club will use $\frac{1}{4}$ of $\frac{2}{3}$ of the pool:

$\frac{1}{4} \times \frac{2}{3} = \frac{2}{12}$ $\frac{2 \times 1}{3 \times 4}$

$= \frac{1}{6}$ of the pool simplifying

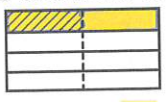


Practising multiplying fractions

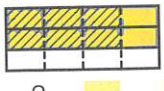
1. Use the diagrams to find the answers to these multiplications. One has been done for you.

* 

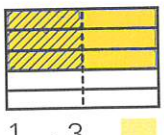
$\frac{1}{3}$ of $\frac{1}{2} = \frac{1}{6}$

a. 

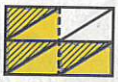
$\frac{1}{2}$ of $\frac{1}{4} =$ __

b. 

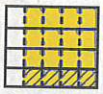
$\frac{3}{4}$ of $\frac{2}{3} =$ __ or __

c. 

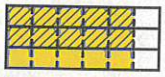
$\frac{1}{2}$ of $\frac{3}{5} =$ __

d. 

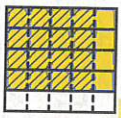
$\frac{1}{2}$ of $\frac{3}{4}$ = ___

e. 

$\frac{1}{4}$ of $\frac{4}{5}$ = ___

f. 

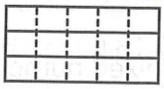
$\frac{2}{3}$ of $\frac{5}{6}$ = ___ or ___

g. 

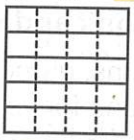
$\frac{4}{5}$ of $\frac{4}{5}$ = ___

2. Use the diagrams to work out the following products.

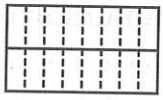
a. $\frac{1}{5} \times \frac{2}{3}$ = ___



b. $\frac{3}{4} \times \frac{4}{5}$ = ___ = ___



c. $\frac{5}{8} \times \frac{1}{2}$ = ___



3. Multiply these fractions. Simplify the answers if possible. One has been done for you.

* $\frac{2}{3} \times \frac{6}{7} = \frac{12}{21} = \frac{4}{7}$

a. $\frac{3}{5} \times \frac{7}{10}$ = ___

b. $\frac{6}{7} \times \frac{2}{5}$ = ___

c. $\frac{5}{8} \times \frac{2}{5}$ = ___ = ___

d. $\frac{2}{5} \times \frac{3}{10}$ = ___ = ___

e. $\frac{2}{3} \times 9$ = ___ = ___

f. $8 \times \frac{3}{4}$ = ___ = ___

g. $\frac{4}{9} \times \frac{6}{7}$ = ___ = ___

h. $\frac{5}{8} \times \frac{4}{15}$ = ___ = ___

4. James has $\frac{1}{2}$ a bar of chocolate. He eats $\frac{2}{3}$ of it. What fraction did James eat? ___

5. Henny spent $\frac{3}{4}$ of an hour doing homework. She spent $\frac{1}{3}$ of the time doing maths. What fraction of an hour was spent on maths? ___

Working space

___ × ___ = ___ or ___

___ × ___ = ___ or ___

6. Aroha spent $\frac{2}{3}$ of an hour exercising. For $\frac{1}{4}$ of that time she jogged around the athletics field. What amount of time did she spend jogging? _____

_____

7. Yoshi spent one quarter of his day at school. Four fifths of the time at school was spent in class. What fraction of Yoshi's day was spent in class? Simplify your answer. _____

_____

_____

8. $\frac{1}{3}$ of Mark's weekly budget is spent on food. $\frac{3}{4}$ of the money spent on food is spent at the supermarket. What fraction of Mark's weekly budget is spent at the supermarket?

_____

_____

Number/Algebra: Multiplying improper fractions and mixed numbers

Example: **1.** $4\frac{1}{3} \times \frac{2}{5} = \frac{13}{3} \times \frac{2}{5}$

$= \frac{26}{15}$

$= 1\frac{11}{15}$

2. $1\frac{2}{3} \times 2\frac{1}{5} = \frac{5}{3} \times \frac{11}{5}$

$= \frac{55}{15}$

$= \frac{11}{3}$ or $3\frac{2}{3}$

Practising multiplying improper fractions and mixed numbers

1. Solve the following multiplication problems, giving your answers as mixed numbers in simplest form. The first one has been done for you.

* $1\frac{2}{5} \times \frac{3}{4} = \frac{7}{5} \times \frac{3}{4}$

$= \frac{21}{20}$

$= 1\frac{1}{20}$

a. $2\frac{3}{4} \times \frac{3}{5} = \_\_ \times \_\_$

$= \_\_$

$=$

b. $\frac{2}{5} \times 4\frac{1}{2} = \_\_ \times \_\_$

$= \_\_$

$=$

c. $1\frac{1}{3} \times \frac{3}{2} = \_\_ \times \_\_$

$= \_\_$

$=$

d. $2\frac{1}{3} \times 2\frac{1}{2} = \_\_ \times \_\_$

$= \_\_$

$=$

e. $8 \times 1\frac{4}{5} = \_\_ \times \_\_$

$= \_\_$

$=$

Remember
your fraction rules.
To add or subtract:
denominators must be the same (change to equivalent fractions if necessary).
To multiply:
multiply numerators together, then denominators.
Simplify answers where possible!

Choose addition, subtraction or multiplication to solve the following word problems.

2. A teacher noted that $\frac{3}{4}$ of the class brought their money on Monday and a further $\frac{1}{8}$ of the class brought their money on Tuesday.

a. What fraction of the class has now brought their money? __

b. What fraction of the class still needs to bring their money? __

$\frac{3}{4} + \frac{1}{8} = \_\_ + \_\_$

$= \_\_$

3. Jack read $\frac{7}{10}$ of his book during one wet day of his holidays. He read $\frac{1}{2}$ of the book in the morning.

a. What fraction did he read in the afternoon? __

b. Jack plans to finish reading his book in the evening. What fraction does he still have left to read? __

__ − __ = __ − __

= __ or __

4. Beth was given $1\frac{4}{5}$ cans of paint to paint the walls of her room. She used $\frac{2}{7}$ of the paint. How much paint does Beth have left? _____

Number/Algebra: Decimal fractions

Fractions like 0.4, 0.07, 0.003 are called **decimal fractions**.

Each digit in a **decimal fraction** has its own place value.

In words	O Ones	•	t tenths	h hundredths	th thousandths	
4 tenths	0	•	4			0.4 has 4 in the tenths place
7 hundredths	0	•	0	7		0.07 has 7 in the hundredths place
3 thousandths	0	•	0	0	3	0.003 has 3 in the thousandths place

A number such as 0.27 is made up of 2 tenths ■ and 7 hundredths ■ (see diagram).

$0.27 = \frac{2}{10} + \frac{7}{100}$ writing as fractions

$= \frac{20}{100} + \frac{7}{100}$ renaming $\frac{2}{10} = \frac{20}{100}$ (×10)

$= \frac{27}{100}$ which is 27 hundredths

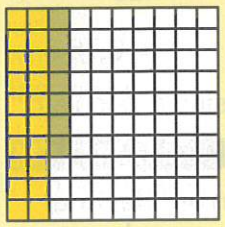


Diagram of 0.27

Count the number of places after the decimal point – that's how many zeros you need after the 1 in the denominator
e.g. $0.24 = \frac{24}{100}$
2 places, 2 zeros

Examples:

1. 0.126 is $\frac{126}{1\,000}$ or 126 thousandths — 3 decimal places and 3 zeros
2. 0.05 is $\frac{5}{100}$ or 5 hundredths — 2 decimal places and 2 zeros
3. 0.047 is $\frac{47}{1\,000}$ or 47 thousandths — 3 decimal places and 3 zeros

Practising decimal fractions

1. Write these decimals in words and as fractions.

*	0.072	72 thousandths	$\frac{72}{1\,000}$
a.	0.7		
b.	0.83		
c.	0.051		
d.	0.167		
e.	0.019		

2. Change these fractions to decimals.

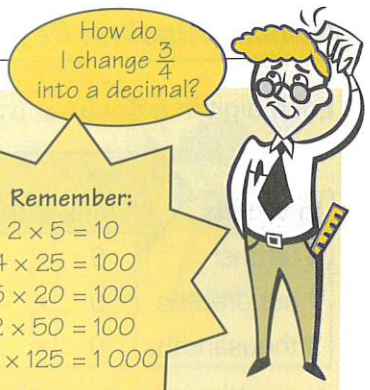
		O	•	t	h	th
*	$\frac{9}{1\,000}$	0	•	0	0	9
a.	$\frac{3}{10}$					
b.	$\frac{26}{100}$					
c.	$\frac{384}{1\,000}$					
d.	$\frac{1}{100}$					
e.	$\frac{29}{1\,000}$					

3. Change these decimals to fractions.

* $0.91 = \frac{91}{100}$ a. $0.8 =$ ___ b. $0.07 =$ ___ c. $0.03 =$ ___ d. $0.123 =$ ___

e. $0.71 =$ ___ f. $0.011 =$ ___ g. $0.177 =$ ___ h. $0.109 =$ ___ i. $0.037 =$ ___

Number/Algebra: Changing fractions to decimals



Fractions whose denominators are not 10,100,1 000, etc. may need to be changed into decimals. **Rename** these fractions first (where possible) as **equivalent fractions** with one of these denominators, before changing them into decimals.

Remember:
$2 \times 5 = 10$
$4 \times 25 = 100$
$5 \times 20 = 100$
$2 \times 50 = 100$
$8 \times 125 = 1\,000$

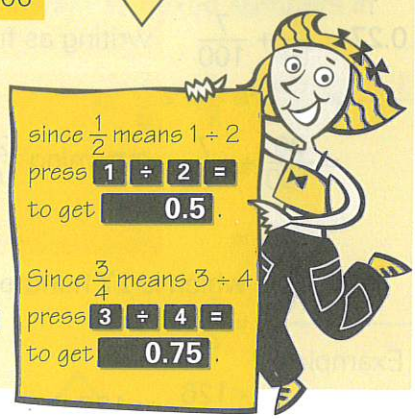
Examples:

1. $\frac{1}{2} \overset{\times 5}{=} \frac{5}{10}$ rename $\frac{1}{2}$ with denominator 10
$\overset{\times 5}{=}$ 0.5

2. $\frac{3}{4} \overset{\times 25}{=} \frac{75}{100}$ rename $\frac{3}{4}$ with denominator 100
$\overset{\times 25}{=}$ 0.75

To change $\frac{4}{9}$ to a decimal using a calculator press **4** **÷** **9** **=** to get **0.4444444444**.

0.4444444444... is called a **recurring decimal** (a repeating decimal) which is written $0.\dot{4}$. The dot above the 4 shows that the 4 repeats forever. Similarly, the decimal $0.\dot{4}\dot{5}$ means that the 45 repeats, so $0.\dot{4}\dot{5} = 0.45454545...$



Practising changing fractions to decimals

1. Complete the working to change these fractions to decimals. One has been done for you.

* $\frac{2}{5} \overset{\times 2}{=} \frac{4}{10} = 0.4$	a. $\frac{4}{5} \overset{\times \square}{=} \frac{\square}{10} = \square$	b. $\frac{9}{20} \overset{\times \square}{=} \frac{\square}{100} = \square$
c. $\frac{3}{8} \overset{\times 125}{=} \frac{\square}{\square} = \square$	d. $\frac{7}{25} \overset{\times \square}{=} \frac{\square}{100} = \square$	e. $\frac{1}{4} \overset{\times \square}{=} \frac{\square}{\square} = \square$

Use a calculator to check your answers.

2. Change these fractions to equivalent fractions over 10,100, etc then write them as decimals.

* $\frac{3}{5} = \frac{6}{10} = 0.6$	a. $\frac{1}{20} = \frac{\square}{100} = \square$	b. $\frac{11}{20} = \frac{\square}{100} = \square$
c. $\frac{13}{50} = \frac{\square}{100} = \square$	d. $\frac{2}{25} = \frac{\square}{100} = \square$	e. $\frac{7}{8} = \frac{\square}{1000} = \square$

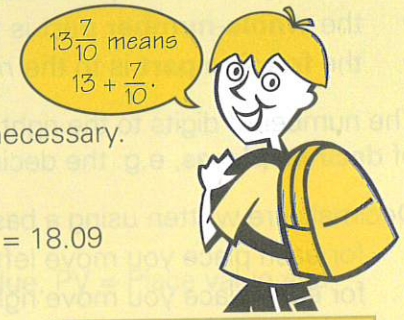
3. Use a calculator to match the fractions to the recurring decimals. Put the letter in the box.

a. $\frac{1}{3}$ ☐	b. $\frac{5}{9}$ ☐	c. $\frac{3}{11}$ ☐	d. $\frac{2}{3}$ ☐	e. $\frac{1}{9}$ ☐	f. $\frac{5}{18}$ ☐
A $0.2\dot{7}$	B $0.\dot{6}$	C $0.\dot{3}$	D $0.\dot{1}$	E $0.\dot{5}$	F $0.\dot{2}\dot{7}$

Number/Algebra: Mixed numbers and decimals

To change **mixed numbers** like $13\frac{7}{10}$ to **decimals**, write the whole-number part in the usual way to the left of the decimal point.

Example: $13\frac{7}{10} = 13.7$ 1 ten + 3 ones + 7 tenths



Rename the fraction with denominator 10, 100, etc. first if necessary.

Examples:

1. $12\frac{4}{5} = 12\frac{8}{10}$
   $= 12.8$
2. $3\frac{17}{50} = 3\frac{34}{100}$
   $= 3.34$
3. $18\frac{9}{100} = 18.09$

To change a decimal to a mixed numeral, write the whole-number part of the decimal beside the fraction. Simplify the fraction where possible.

Examples:
1. $15.3 = 15\frac{3}{10}$
2. $6.85 = 6\frac{85}{100}$
   $= 6\frac{17}{20}$
3. $126.001 = 126\frac{1}{1\,000}$

Some common conversions to learn:

$\frac{1}{4} = 0.25$	$\frac{1}{10} = 0.1$
$\frac{1}{2} = 0.5$	$\frac{1}{5} = 0.2$
$\frac{3}{4} = 0.75$	$\frac{1}{20} = 0.05$

Practising mixed numbers and decimals

1. Match the mixed numerals with their decimal equivalents by putting the letters in the boxes.

$17\frac{9}{10}$	$17\frac{9}{1\,000}$	$25\frac{15}{100}$	$17\frac{9}{100}$	$25\frac{15}{1\,000}$
E 25.15	A 17.09	G 17.9	T 25.015	R 17.009

2. Change these decimals to mixed numerals and simplify where possible.

* $56.8 = 56\frac{8}{10} = 56\frac{4}{5}$

a. $35.2 =$ ____ $=$ ____

b. $129.5 =$ ____ $=$ ____

c. $5.45 =$ ____ $=$ ____

d. $13.05 =$ ____ $=$ ____

e. $8.025 =$ ____ $=$ ____

3. Change these mixed numerals to decimals. Rename the fraction part first with denominator 10, 100, etc. One has been done for you.

* $8\frac{2}{5} = 8\frac{4}{10} = 8.4$

a. $3\frac{4}{5} =$ ____ $=$ ____

b. $17\frac{7}{20} =$ ____ $=$ ____

c. $28\frac{13}{50} =$ ____ $=$ ____

d. $35\frac{2}{25} =$ ____ $=$ ____

e. $46\frac{3}{4} =$ ____ $=$ ____

4. Use your calculator to change these mixed numerals to decimals. (Some may have recurring decimals.) Change the fraction to a decimal first, then add on the whole number.

* $17\frac{5}{8}$ Do the fraction first: press 5 ÷ 8 = to get 0.625. So $17\frac{5}{8} = 17.625$.

a. $28\frac{3}{8} =$ ____

b. $6\frac{3}{4} =$ ____

c. $115\frac{8}{9} =$ ____

d. $12\frac{4}{5} =$ ____