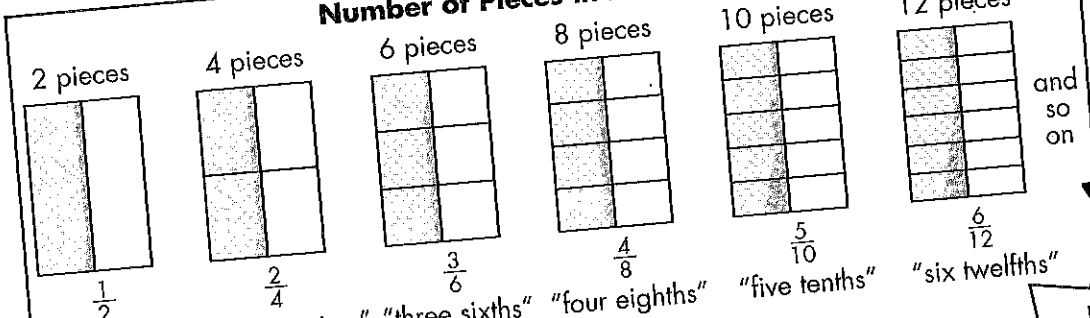
# Number: Equivalent Fractions

Sonia gives Terry half a bar of chocolate. The ***fraction*** of the bar that Terry gets can be written in many different ways, depending on how many pieces there are in the bar.


**Number of Pieces in the Bar**



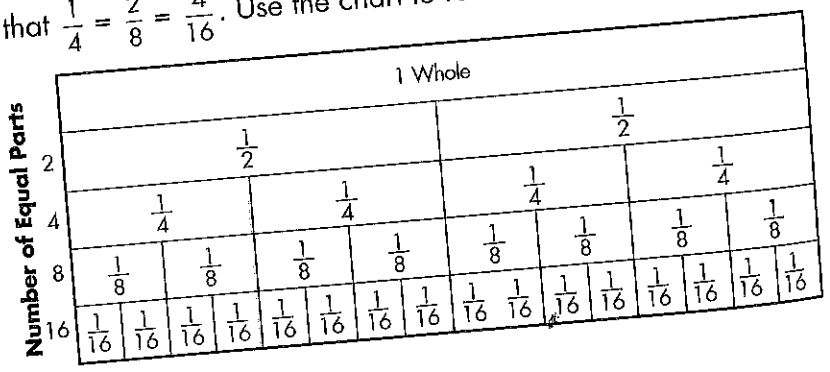

**Terry's Share (shown shaded)**

**Equivalent fractions** are fractions which are equal in size, although they are written differently.

The fractions $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{6}$, $\frac{5}{10}$, and $\frac{6}{12}$ all represent the same amount of chocolate. These six fractions are called ***equivalent fractions***.

## Practising Equivalent Fractions

1. The chart below shows different ways of dividing up 1 whole. For example the shaded part part shows that $\frac{1}{4} = \frac{2}{8} = \frac{4}{16}$. Use the chart to rename the fractions:



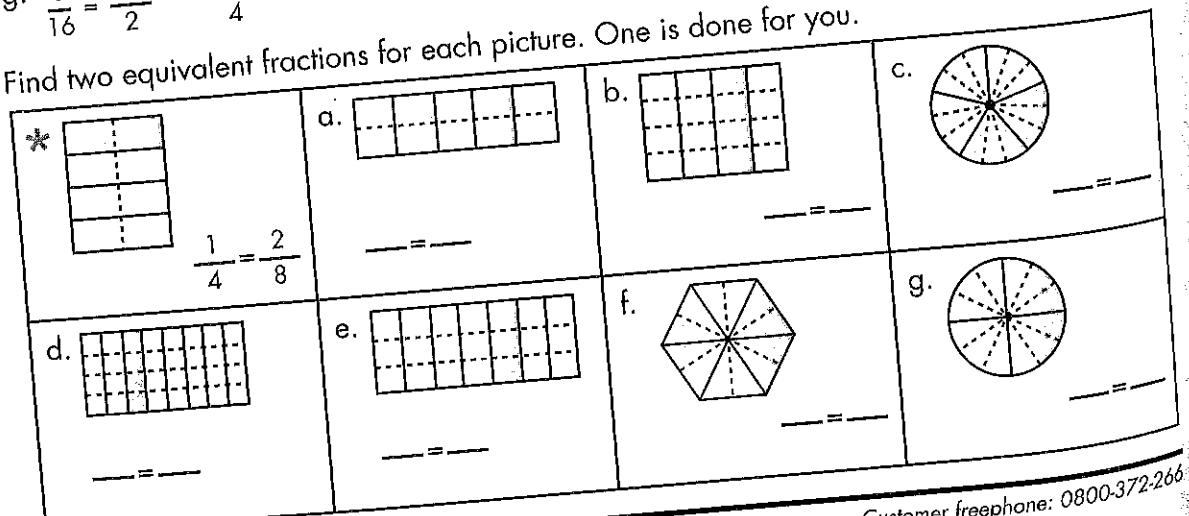

a. $\frac{2}{8} = \frac{\quad}{4}$ b. $\frac{4}{16} = \frac{\quad}{4}$

c. $\frac{4}{8} = \frac{\quad}{16}$ d. $\frac{4}{8} = \frac{2}{\quad}$

e. $\frac{7}{8} = \frac{14}{\quad}$ f. $\frac{3}{8} = \frac{\quad}{16}$

g. $\frac{8}{16} = \frac{\quad}{2}$ h. $\frac{3}{4} = \frac{12}{\quad}$

2. Find two equivalent fractions for each picture. One is done for you.



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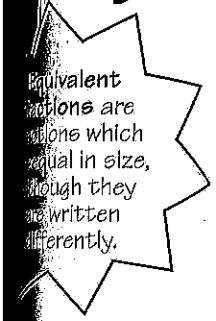
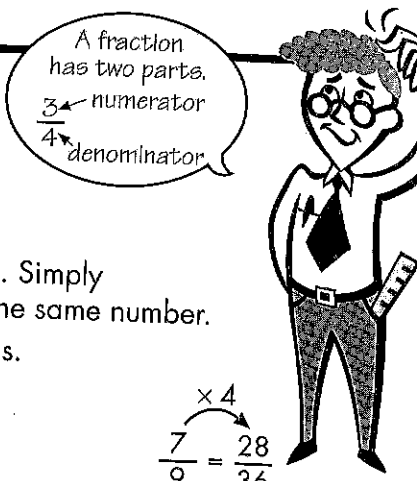
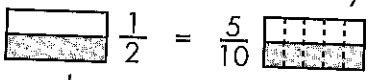
# Number: More Equivalent Fractions




***Equivalent fractions*** can be formed by drawing diagrams.

**Example:**  $\frac{1}{2} = \frac{5}{10}$

2 equal parts
1 part shaded

$5 \times 2 = 10$ equal parts
$5 \times 1 = 5$ parts shaded

The example shows a shorter way of forming equivalent fractions. Simply multiply the ***numerator*** and ***denominator*** of the fraction by the same number.

**Examples:** Fill in the shaded spaces to make equivalent fractions.

$\frac{1}{2} \overset{\times 5}{=} \frac{5}{10}$ (×5)

numerator has been multiplied by 5 so multiply denominator by 5 also

$\frac{3}{4} \overset{\times 3}{=} \frac{9}{12}$ (×3)

numerator has been multiplied by 3 so multiply denominator by 3 also

$\frac{5}{8} \overset{\times 2}{=} \frac{10}{16}$ (×2)

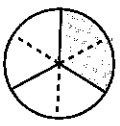
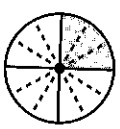
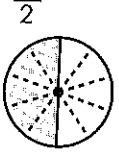
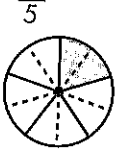
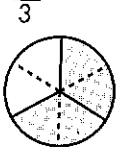
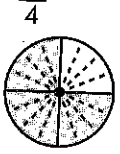
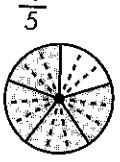
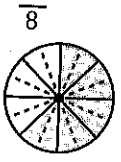
denominator has been doubled so double the numerator also

$\frac{7}{9} \overset{\times 4}{=} \frac{28}{36}$ (×4)

denominator has been multiplied by 4 so multiply numerator by 4 also

## Practising More Equivalent Fractions

1. Match the equivalent fractions by putting the letters A–H in the boxes. Use the diagrams to help you. The first one is done for you.

| $\frac{1}{3}$ D | $\frac{1}{4}$ | $\frac{1}{2}$ | $\frac{1}{5}$ | $\frac{2}{3}$ | $\frac{3}{4}$ | $\frac{4}{5}$ | $\frac{5}{8}$ |
|---|---|---|---|---|---|---|---|
|  |  |  |  |  |  |  |  |
| A $\frac{5}{10}$ | B $\frac{2}{10}$ | C $\frac{12}{16}$ | D $\frac{2}{6}$ | E $\frac{12}{15}$ | F $\frac{10}{16}$ | G $\frac{3}{12}$ | H $\frac{4}{6}$ |

2. Fill in the gaps to make equivalent fractions. The first one is done for you.

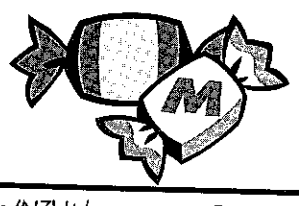
| * $\frac{2}{3} \overset{\times 2}{=} \frac{4}{6}$ (×2) | a. $\frac{4}{7} \overset{\times}{=} \frac{\quad}{28}$ (×) | b. $\frac{4}{5} \overset{\times}{=} \frac{\quad}{15}$ (×) | c. $\frac{1}{10} \overset{\times}{=} \frac{\quad}{100}$ (×) | d. $\frac{6}{7} \overset{\times}{=} \frac{18}{\quad}$ (×) |
|---|---|---|---|---|

3. Fill in the gaps to make equivalent fractions.

| a. $\frac{3}{4} = \frac{15}{\quad}$ | b. $\frac{1}{3} = \frac{3}{\quad}$ | c. $\frac{7}{8} = \frac{\quad}{16}$ | d. $\frac{1}{2} = \frac{\quad}{10}$ | e. $\frac{5}{6} = \frac{10}{\quad}$ |
|---|---|---|---|---|
| f. $\frac{3}{12} = \frac{\quad}{48}$ | g. $\frac{6}{9} = \frac{\quad}{36}$ | h. $\frac{4}{10} = \frac{\quad}{100}$ | i. $\frac{3}{8} = \frac{\quad}{32}$ | j. $\frac{2}{3} = \frac{18}{\quad}$ |

4. Jack got $\frac{5}{12}$ of the sweets. Jill got $\frac{1}{3}$ of the sweets. Express $\frac{1}{3}$ in the form $\frac{\quad}{12}$ to find out who got more.

got more




Working Space

$\frac{1}{3} = \frac{\quad}{12}$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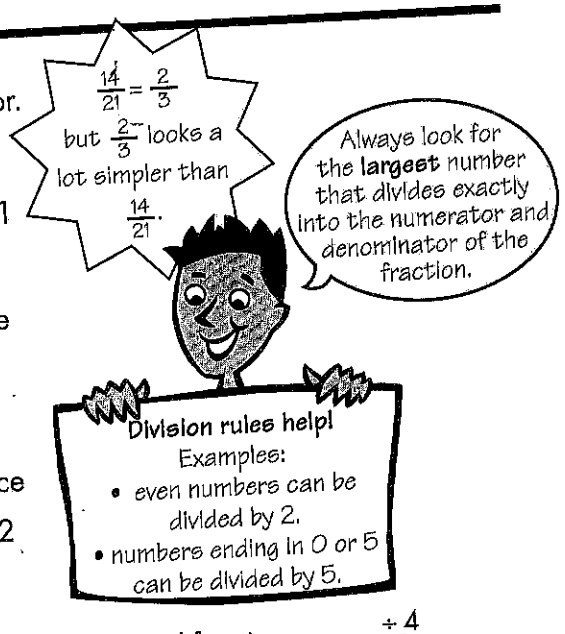
## Number: Simplifying Fractions

A fraction can be **simplified** if there is a number that divides equally into both the numerator and denominator.

**Example:** $\frac{14}{21} \overset{\div 7}{=} \frac{2}{3}$ 7 divides exactly into 14 and 21




When you have simplified check that you cannot divide again. A fraction is in its ***simplest form*** when the numerator and denominator cannot be divided further.

**Example:** $\frac{20}{24} \overset{\div 2}{=} \frac{10}{12}$ $\frac{10}{12}$ is not the simplest form since 2 divides into both 10 and 12

$\frac{10}{12} \overset{\div 2}{=} \frac{5}{6}$ dividing again brings $\frac{20}{24}$ to its simplest form $\frac{5}{6}$

**Note:** This simplification could have been done in one step using divison by 4. $\frac{20}{24} \overset{\div 4}{=} \frac{5}{6}$

### Practising Simplifying Fractions

1. Express each of the following fractions in simplest form. The first is done for you.

| * | a. | b. | c. | d. |
|---|---|---|---|---|
| $\frac{6}{12} \overset{\div 6}{=} \frac{1}{2}$ | $\frac{3}{18} = \frac{\quad}{\quad}$ | $\frac{12}{16} = \frac{\quad}{\quad}$ | $\frac{10}{100} = \frac{\quad}{\quad}$ | $\frac{21}{28} = \frac{\quad}{\quad}$ |

2. Simplify these fractions using the method in question 1.

a. $\frac{40}{50} = \frac{\quad}{\quad}$ b. $\frac{7}{21} = \frac{\quad}{\quad}$ c. $\frac{5}{25} = \frac{\quad}{\quad}$ d. $\frac{9}{12} = \frac{\quad}{\quad}$ e. $\frac{22}{33} = \frac{\quad}{\quad}$

f. $\frac{8}{10} = \frac{\quad}{\quad}$ g. $\frac{4}{40} = \frac{\quad}{\quad}$ h. $\frac{18}{30} = \frac{\quad}{\quad}$ i. $\frac{15}{45} = \frac{\quad}{\quad}$ j. $\frac{20}{40} = \frac{\quad}{\quad}$

3. Tick the fractions in the following list which are in their simplest form. Convert the other fractions to their simplest form.

a. $\frac{1}{2}$ b. $\frac{4}{6}$ c. $\frac{5}{8}$ d. $\frac{4}{7}$ e. $\frac{10}{20}$ f. $\frac{11}{15}$

g. $\frac{6}{8}$ h. $\frac{14}{49}$ i. $\frac{35}{45}$ j. $\frac{28}{35}$ k. $\frac{18}{54}$ l. $\frac{7}{18}$

4. Match the fractions to their simplified forms by putting the letters A–F in the boxes.

a. $\frac{15}{40}$ b. $\frac{10}{35}$ c. $\frac{12}{15}$ d. $\frac{12}{27}$ e. $\frac{14}{28}$ f. $\frac{20}{24}$

A. $\frac{4}{9}$ B. $\frac{5}{6}$ C. $\frac{3}{8}$ D. $\frac{2}{7}$ E. $\frac{4}{5}$ F. $\frac{1}{2}$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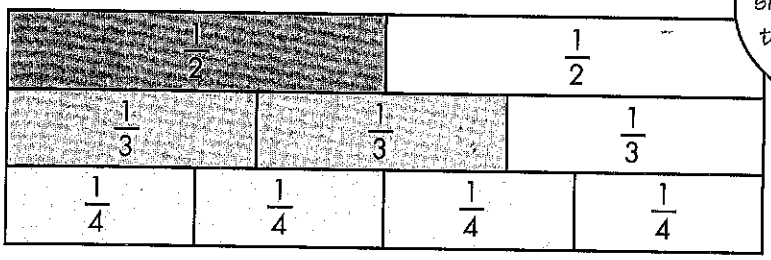
■ *Level 4: Find fractions equivalent to one given.*

## Number: Comparing Fractions

One way of comparing the sizes of fractions is to use diagrams like this.



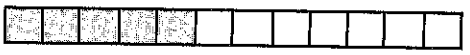


It is easy to compare fractions without using diagrams when their denominators are the same.

**Example:** $\frac{7}{12}$ is bigger than $\frac{5}{12}$, because 7 is bigger than 5.

Using symbols $\frac{7}{12} > \frac{5}{12}$.


You can check this with a diagram.




> means "is larger than".

It is not so easy to compare fractions when their denominators are different.

**Example:** Which is smaller, $\frac{2}{3}$ or $\frac{5}{8}$?

From the diagram $\frac{5}{8}$ is less than $\frac{2}{3}$.

Another way of comparing sizes is to express both fractions as equivalent fractions with the same denominator.

$$\frac{2}{3} \overset{\times 8}{=} \frac{16}{24}$$

multiplication by 8 makes denominator 24

$$\frac{5}{8} \overset{\times 3}{=} \frac{15}{24}$$

multiplication by 3 makes denominator 24


So $\frac{5}{8}$ is less than $\frac{2}{3}$ (write $\frac{5}{8} < \frac{2}{3}$) because $\frac{15}{24}$ is smaller than $\frac{16}{24}$.

### Practising Comparing Fractions

1. Put < (is less than) or > (is greater than) signs to show the relationship between the following pairs of fractions. The diagram at the top of the page may help you.

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| a. | $\frac{5}{8}$ | $\frac{7}{8}$ | b. | $\frac{11}{12}$ | $\frac{7}{12}$ | c. | $\frac{4}{10}$ | $\frac{5}{10}$ | d. | $\frac{1}{3}$ | $\frac{1}{2}$ |
| e. | $\frac{2}{3}$ | $\frac{1}{2}$ | f. | $\frac{1}{4}$ | $\frac{1}{3}$ | g. | $\frac{2}{3}$ | $\frac{3}{4}$ | h. | $\frac{1}{2}$ | $\frac{1}{4}$ |
| i. | $\frac{2}{7}$ | $\frac{3}{14}$ | j. | $\frac{1}{4}$ | $\frac{5}{12}$ | k. | $\frac{1}{5}$ | $\frac{1}{4}$ | l. | $\frac{3}{5}$ | $\frac{1}{2}$ |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## Number: Comparing Fractions

2. Express each fraction as an equivalent fraction with the denominator shown. Then put the original fractions in order of size from smallest to largest. One is done for you.

* $\frac{2}{5} \overset{\times 6}{=} \frac{12}{30}$, $\frac{4}{15} \overset{\times 2}{=} \frac{8}{30}$, $\frac{3}{10} \overset{\times 3}{=} \frac{9}{30}$ smallest $\frac{4}{15}$, $\frac{3}{10}$, $\frac{2}{5}$ largest

a. $\frac{3}{4} = \frac{\quad}{40}$, $\frac{7}{10} = \frac{\quad}{40}$, $\frac{1}{2} = \frac{\quad}{40}$ smallest ___, ___, ___ largest

b. $\frac{1}{2} = \frac{\quad}{16}$, $\frac{7}{16} = \frac{\quad}{16}$, $\frac{3}{8} = \frac{\quad}{16}$ smallest ___, ___, ___ largest

c. $\frac{3}{8} = \frac{\quad}{24}$, $\frac{1}{3} = \frac{\quad}{24}$, $\frac{5}{12} = \frac{\quad}{24}$ smallest ___, ___, ___ largest

d. $\frac{1}{3} = \frac{\quad}{36}$, $\frac{4}{9} = \frac{\quad}{36}$, $\frac{5}{12} = \frac{\quad}{36}$ smallest ___, ___, ___ largest

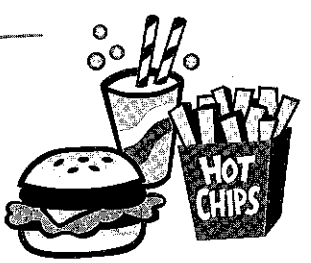
3. Rewrite these groups of fractions as equivalent fractions with the same denominators. Then put the original list in order of size from largest to smallest. One is done for you.

| | | | Largest | Middle | Smallest |
|---|---|---|---|---|---|
| * $\frac{2}{3} = \frac{24}{36}$ | $\frac{5}{6} = \frac{30}{36}$ | $\frac{7}{9} = \frac{28}{36}$ | $\frac{5}{6}$ | $\frac{7}{9}$ | $\frac{2}{3}$ |
| a. $\frac{3}{5} = \frac{\quad}{20}$ | $\frac{1}{2} = \frac{\quad}{\quad}$ | $\frac{3}{4} = \frac{\quad}{\quad}$ | | | |
| b. $\frac{4}{5} = \frac{\quad}{\quad}$ | $\frac{3}{10} = \frac{\quad}{\quad}$ | $\frac{1}{2} = \frac{\quad}{10}$ | | | |
| c. $\frac{7}{12} = \frac{\quad}{\quad}$ | $\frac{3}{4} = \frac{\quad}{24}$ | $\frac{5}{8} = \frac{\quad}{\quad}$ | | | |
| d. $\frac{3}{7} = \frac{\quad}{14}$ | $\frac{1}{2} = \frac{\quad}{\quad}$ | $\frac{9}{14} = \frac{\quad}{\quad}$ | | | |

4. Hepi spent $\frac{1}{3}$ of his money on takeaways and $\frac{2}{5}$ of his money on football cards. Which cost more, the takeaways or the cards? _____

5. Dad painted $\frac{3}{8}$ of the deck on Saturday and $\frac{1}{4}$ of the deck on Sunday. On which day did he paint less? _____

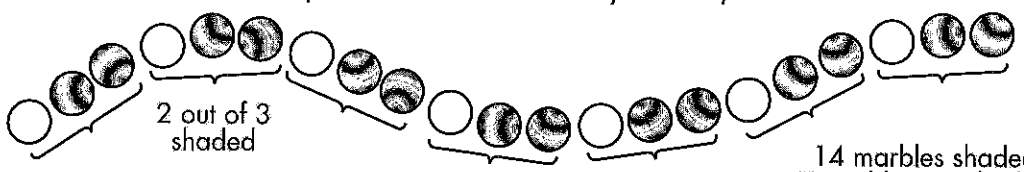
6. Patrick weeded $\frac{1}{4}$ of the garden. Molly weeded $\frac{3}{10}$ [illegible] garden. Who weeded more? _____



## *Number: Fractions of Quantities*

One way of finding fractions of quantities is to draw pictures.


**Example:** Simon wants $\frac{2}{3}$ of a bag of 21 marbles.
He lines up the marbles on the floor and takes 2 out of every 3 marbles.
(The shaded marbles represent marbles taken by Simon.)




Simon takes 14 of the 21 marbles, so $\frac{2}{3}$ of 21 = 14.

One third of the marbles are left behind. Simon has left 7 marbles so $\frac{1}{3}$ of 21 is 7.

### Practising Fractions of Quantities

1. Shade the following diagrams to find these amounts. An example is done for you.

* $\frac{3}{4}$ of 20  $\frac{3}{4}$ of 20 = 15

shade 3 out of every 4

a. $\frac{2}{3}$ of 15  $\frac{2}{3}$ of 15 =

b. $\frac{4}{5}$ of 10  $\frac{4}{5}$ of 10 =

c. $\frac{5}{8}$ of 16  $\frac{5}{8}$ of 16 =

d. $\frac{3}{7}$ of 21  $\frac{3}{7}$ of 21 =

e. $\frac{5}{6}$ of 18  $\frac{5}{6}$ of 18 =

f. $\frac{1}{3}$ of 12  $\frac{1}{3}$ of 12 =

g. $\frac{3}{5}$ of 20  $\frac{3}{5}$ of 20 =

2. Use the unshaded parts of the above diagrams to find these amounts.

a. $\frac{1}{4}$ of 20 =  b. $\frac{1}{3}$ of 15 =  c. $\frac{1}{5}$ of 10 =  d. $\frac{3}{8}$ of 16 =

e. $\frac{4}{7}$ of 21 =  f. $\frac{1}{6}$ of 18 =  g. $\frac{2}{3}$ of 12 =  h. $\frac{2}{5}$ of 20 =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## umber: More Fractions of Quantities

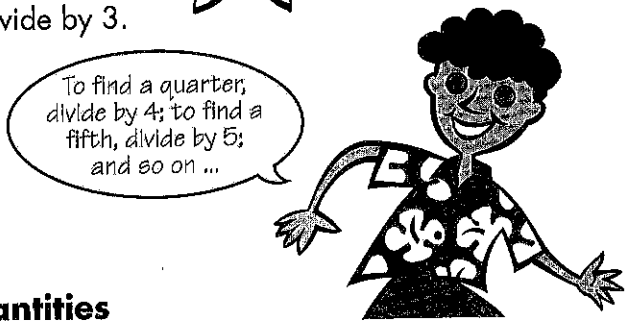
are wants to give her friend half of her bag of cakes. She divides her cakes into equal shares of 5 cakes each.

$\frac{1}{2}$ of 10 = 10 ÷ 2
= 5


find a third of a quantity you need to divide by 3.

**kample:** On the previous page, mon left behind $\frac{1}{3}$ of the 21 marbles.

of 21 = 21 ÷ 3
= 7 marbles.




## ractising More Fractions of Quantities

Complete each fraction of the quantities calculation. Then match the answers in the two lists by putting the letters A–G in the boxes. The first one is done for you

* $\frac{1}{4}$ of 24 = 24 ÷ 4 = 6 C

a. $\frac{1}{4}$ of 28 = 28 ÷ =

b. $\frac{1}{5}$ of 25 = 25 ÷ =

c. $\frac{1}{3}$ of 30 = 30 ÷ =

d. $\frac{1}{2}$ of 26 = 26 ÷ =

e. $\frac{1}{6}$ of 18 = 18 ÷ =

f. $\frac{1}{9}$ of 36 = 36 ÷ =

A. $\frac{1}{8}$ of 56 = 56 ÷ =

B. $\frac{1}{6}$ of 24 = 24 ÷ =

C. $\frac{1}{8}$ of 48 = 48 ÷ 8 = 6

D. $\frac{1}{3}$ of 39 = 39 ÷ =

E. $\frac{1}{5}$ of 50 = 50 ÷ =

F. $\frac{1}{4}$ of 20 = 20 ÷ =

G. $\frac{1}{10}$ of 30 = 30 ÷ =

. Felix lost $\frac{1}{3}$ of the 12 games in his soccer season. How many games did Felix lose? games

. What sort of cake do you *not* want on your birthday? Find these fractions of quantities to find out. The first one is done for you.

A: $\frac{1}{10}$ of 50 = 5 C: $\frac{1}{4}$ of 36 = E: $\frac{1}{5}$ of 35 = H: $\frac{1}{3}$ of 24 =

M: $\frac{1}{50}$ of 100 = O: $\frac{1}{12}$ of 36 = S: $\frac{1}{6}$ of 36 = T: $\frac{1}{7}$ of 28 =

## Number: Further Fractions of Quantities

Ruby's necklace has 24 beads altogether.

3 out of every 4 beads are coloured, which makes a total of 18 coloured beads.

So $\frac{3}{4}$ of 24 = 18.

Without using pictures Ruby worked out that

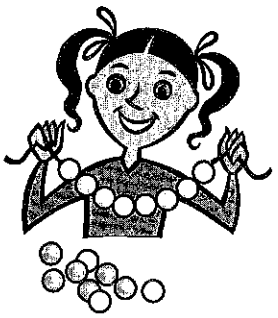
$\frac{1}{4}$ of 24 = 6 because 24 ÷ 4 = 6

So $\frac{3}{4}$ of 24 = 3 × ($\frac{1}{4}$ of 24) since $\frac{3}{4}$ means 3 lots of $\frac{1}{4}$

= 3 × 6

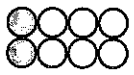
= 18

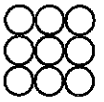
Remember: "Of means multiply".

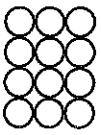


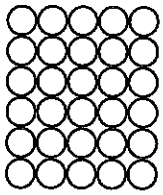
### Practising Further Fractions of Quantities

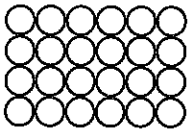
1. Shade the diagrams to find these fractions of quantities. The first is done for you.

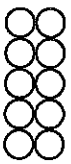
*  $\frac{1}{4}$ of 8 = 2

a.  $\frac{2}{3}$ of 9 =

b.  $\frac{3}{4}$ of 12 =

c.  $\frac{3}{5}$ of 30 =

d.  $\frac{5}{6}$ of 24 =

e.  $\frac{4}{5}$ of 10 =

2. Use division to find these fractions of quantities. The first is done for you.

| * | a. | b. | c. | d. |
|---|---|---|---|---|
| $\frac{1}{4}$ of 8 | $\frac{1}{3}$ of 9 | $\frac{1}{4}$ of 12 | $\frac{1}{5}$ of 20 | $\frac{1}{6}$ of 24 |
| = 8 ÷ 4 | = 9 ÷ | = 12 ÷ | = 20 ÷ | = 24 ÷ |
| = 2 | = | = | = | = |

3. Use the answers to question 2 to find these fractions of quantities. Compare your answers with those in question 1. The first is done for you.

| * | a. | b. | c. |
|---|---|---|---|
| $\frac{2}{3}$ of 9 | $\frac{3}{4}$ of 12 | $\frac{3}{5}$ of 20 | $\frac{5}{6}$ of 24 |
| = 2 × $\frac{1}{3}$ of 9 | = 3 × | = 3 × | = 5 × |
| = 2 × | = 3 × | = 3 × | = 5 × |
| = 6 | = | = | = |

4. Find these fractions of quantities. Set your answers out like those in question 3.

| a. | b. | c. | d. |
|---|---|---|---|
| $\frac{5}{8}$ of 40 | $\frac{2}{7}$ of 21 | $\frac{4}{5}$ of 45 | $\frac{3}{10}$ of 70 |
| = 5 × | = 2 × | = 4 × | = 3 × |
| = 5 × | = 2 × | = 4 × | = 3 × |
| = | = | = | = |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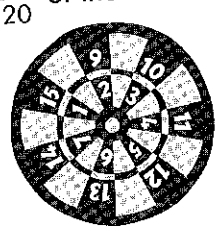
## Number: Expressing a Quantity as a Fraction of the Whole

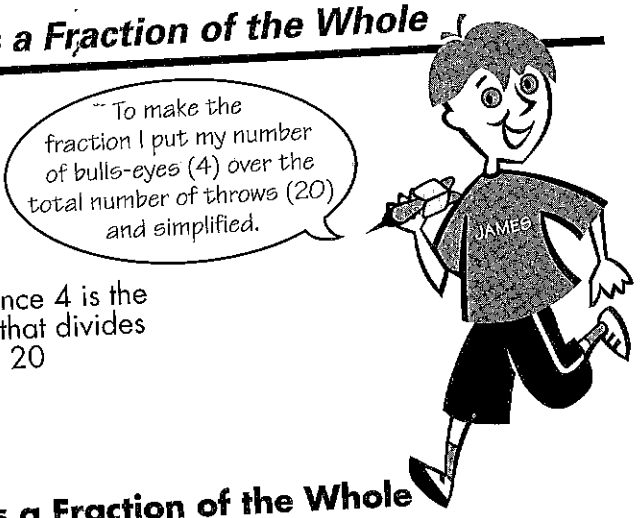
James throws a dart at a board 20 times. He hits the bulls-eye 4 times out of 20 or $\frac{4}{20}$ of the time. Simplifying the fraction:



$\frac{4}{20} = \frac{1}{5}$ (÷4 above, ÷4 below)

dividing by 4 since 4 is the largest number that divides into both 4 and 20

So James hit the bulls-eye $\frac{1}{5}$ of the time.




### Practising Expressing a Quantity as a Fraction of the Whole

1. Match the left-hand side with the correct fraction (simplified) by putting A–H in the boxes.

Working Space

| | | |
|---|---|---|
| * 200 out of 500 | C | A. $\frac{1}{4}$ |
| a. 24 out of 25 | | B. $\frac{1}{7}$ |
| b. 10 out of 50 | | C. $\frac{2}{5}$ |
| c. 9 out of 36 | | D. $\frac{3}{4}$ |
| d. 12 out of 84 | | E. $\frac{1}{5}$ |
| e. 18 out of 27 | | F. $\frac{2}{7}$ |
| f. 10 out of 35 | | G. $\frac{2}{3}$ |
| g. 15 out of 20 | | H. $\frac{24}{25}$ |

$\frac{200}{500} = \frac{2}{5}$ (÷100 above, ÷100 below)

2. 150 Year 7 students recorded their preferred T-shirt colours as shown. Complete the table by calculating and simplifying the fraction of the total for each colour. One is done for you.

**Note:** Here the frequency means the number of students who preferred that colour.

| Colour | Frequency | Fraction | Simplified Fraction |
|---|---|---|---|
| *White | 50 | $\frac{50}{150}$ | $\frac{1}{3}$ |
| Red | 30 | $\frac{30}{150}$ | |
| Grey | 10 | | |
| Blue | 25 | | |
| Black | 20 | | |
| Other | 15 | | |

3. Miro played cricket on 14 days of his 21 day holiday.