Trigonometry

TRIGONOMETRY

SERIES J

EVERYTHING YOU NEED TO KNOW
100%
AND NOTHING MORE

MATHLETICS

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TRIGONOMETRY

Basically, many situations in the real world can be related to a right angled triangle. 'Trigonometry' sounds difficult, but it's really just methods to find the side lengths and angle sizes in these triangles.

Answer these questions, *before* working through the chapter.

I used to think:

What is the longest side of a triangle called? What are the other sides called?

Each angle has 3 main trigonometric ratios? What is a trigonometric ratio? What are the 3 main ratios?

When would you use the inverse of a ratio?

Answer these questions, *after* working through the chapter.

But now I think:

What is the longest side of a triangle called? What are the other sides called?

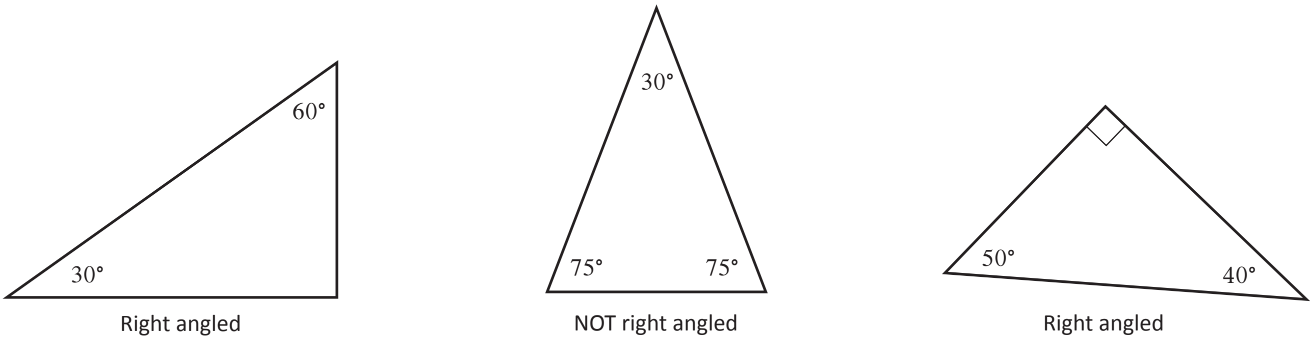
Each angle has 3 main trigonometric ratios? What is a trigonometric ratio? What are the 3 main ratios?

When would you use the inverse of a ratio?

What do I know now that I didn't know before?

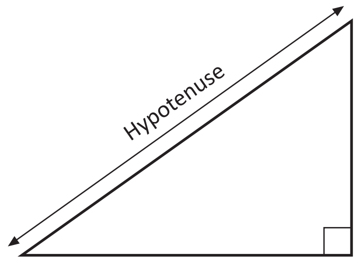
Right Angled Triangle

A right angled triangle is a triangle which has an angle of $90°$.



Do you remember the sum of the interior angles of a triangle?

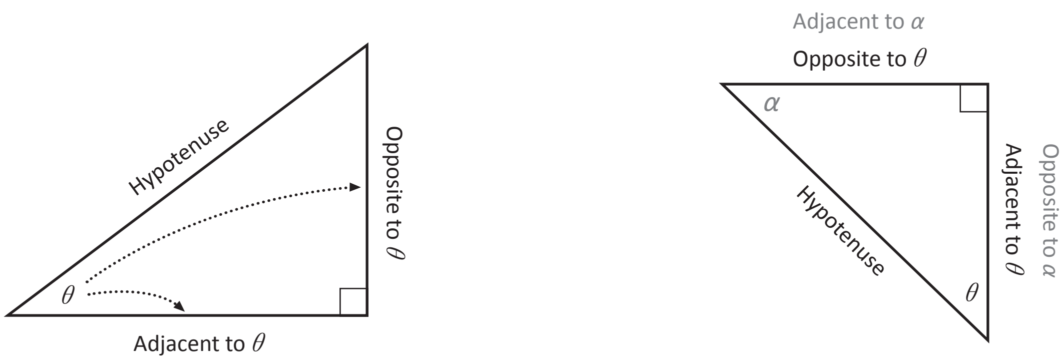
The longest side of a right angled triangle is called the hypotenuse. It is always the side opposite the right angle.



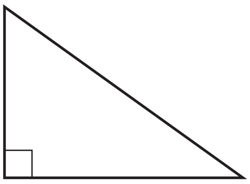
Opposite and Adjacent sides to θ

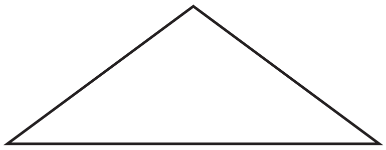
The Greek letters θ (theta) and α (alpha) are used to label angles.

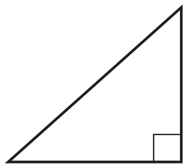
The sides of a triangle are labeled as either adjacent (next to) or opposite these angles.

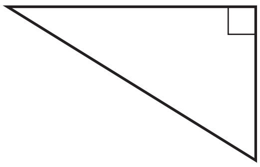


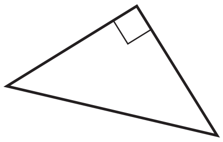
1. Identify if the following triangles are right angled or not?

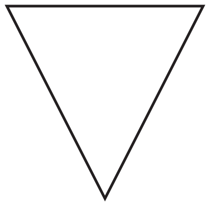
a


b


c


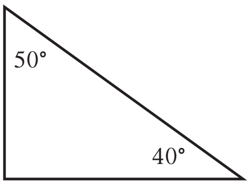
d


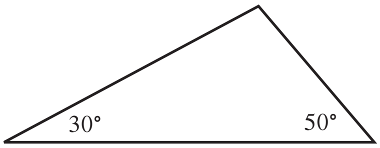
e


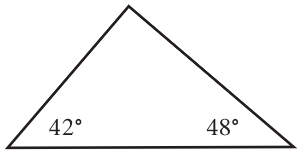
f


2. What is the sum of the interior angles of a triangle?

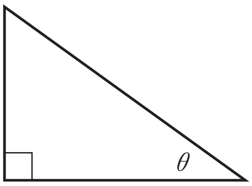
3. Identify if the following triangles are right angled or not?

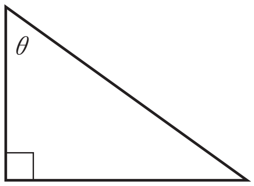
a


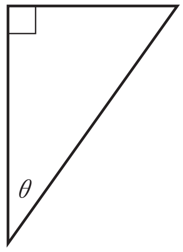
b


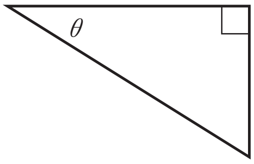
c


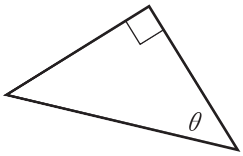
4. Label the opposite, adjacent and hypotenuse in each of the following triangles.

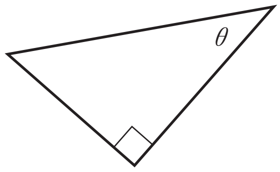
a


b


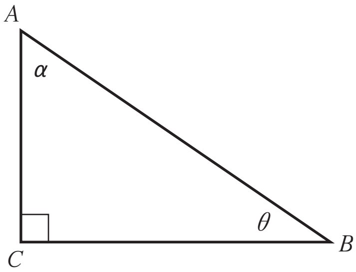
c


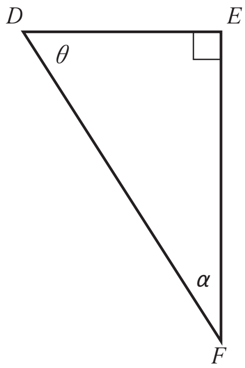
d


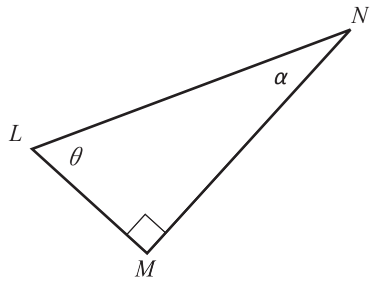
e


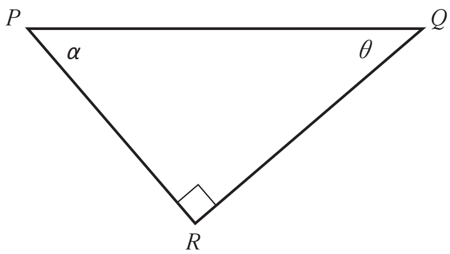
f


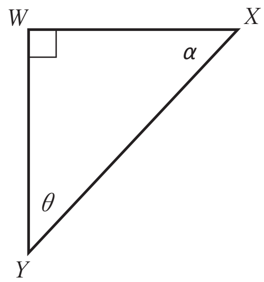
5. Use the following 5 triangles to fill in the correct sides in the table below:







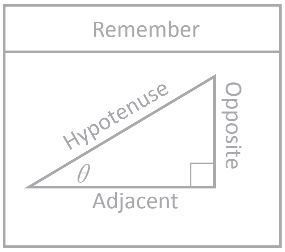


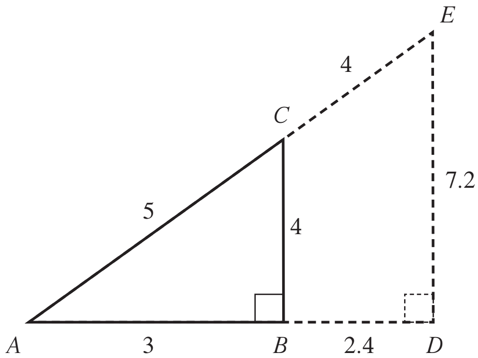


Triangle	Opposite to θ	Adjacent to θ	Opposite to α	Adjacent to α	Hypotenuse
ΔABC	AC	BC			AB
ΔDEF			DE	EF	
ΔLMN		LM			LN
ΔPQR			QR		
ΔWXY					

Sinθ

ΔABC is drawn inside ΔADE. The two triangles are similar. This means they have equal angles (can you show this?)





We're going to find the value of the ratio $\frac{\text{opposite side to } \angle A}{\text{hypotenuse}}$ for both the small and big triangles.

In ΔABC (small triangle):

$$\frac{\text{opposite side to } \angle A}{\text{hypotenuse}} = \frac{BC}{AC} = \frac{4}{5} = 0.8$$

In ΔADE (big triangle):

$$\frac{\text{opposite side to } \angle A}{\text{hypotenuse}} = \frac{DE}{AE} = \frac{7.2}{9} = 0.8$$

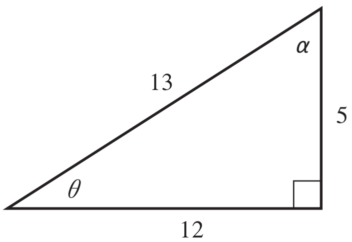
This means the ratio does not depend on the size of the triangle – since the ratio has the same value for different triangles.

The ratio only **depends on the angle.** This ratio is called $\sin\theta$ (pronounced: 'sine' theta). The formula for $\sin\theta$ is:

$$\sin\theta = \frac{\text{opposite side to } \theta}{\text{hypotenuse}}$$

Look at the following examples:

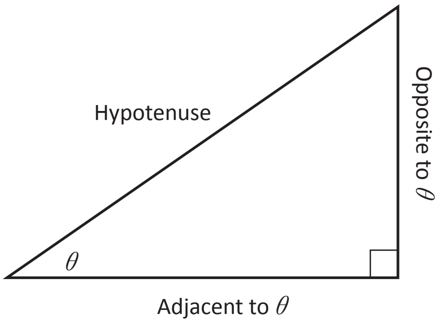
Find $\sin\theta$ *and* $\sin\alpha$



$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{5}{13} = 0.3846...$$

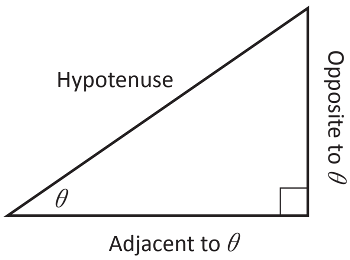
$$\sin\alpha = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{12}{13} = 0.923...$$

Sinθ, Cosθ and Tanθ (Trigonometric Ratios)



We already know that for any right angled triangle, the **sine ratio** of an acute angle θ is:

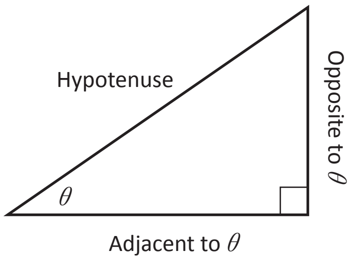
$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$



$\sin\theta$ will be the same for any size right angled triangle since the sine ratio only depends on the angle.

A second ratio is called the **cosine** ratio. We write this as $\cos\theta$ and it is given by:

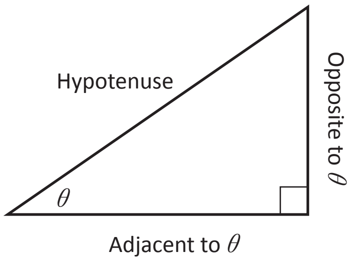
$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$



$\cos\theta$ will be the same for any size right angled triangle since the sine ratio only depends on the angle.

The third ratio is called the **tangent** ratio. We write this as $\tan\theta$ and it is given by:

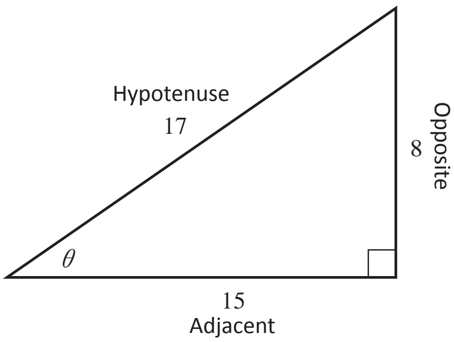
$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$



$\tan\theta$ will be the same for any size right angled triangle since the sine ratio only depends on the angle.

Look at these examples:

Find $\sin\theta$, $\cos\theta$ and $\tan\theta$ in the following triangle:

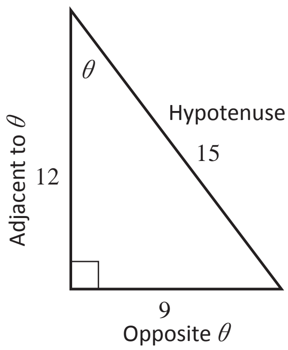


$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{8}{17} = 0.4705882...$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{15}{17} = 0.882352...$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{8}{15} = 0.5333...$$

Find $\sin\theta$, $\cos\theta$ and $\tan\theta$ in the following triangle:



$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{9}{15} = 0.6$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{12}{15} = 0.8$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{9}{12} = 0.75$$

Calculator Tricks

Your calculator has a buttons called '$\sin$', '$\cos$' and '$\tan$.' Remember, all of these only depend on the angle.

Make sure your calculator is set to ***degree*** mode.

To find the value of any ratio just press the buttons: [Ratio] Angle [=]

Find the following ratios:

a) $\sin 75°$

[sin] 75 [=]

[0.965925826]

b) $\cos 67°$

[cos] 67 [=]

[0.390731128]

c) $\tan 18°$

[tan] 18 [=]

[0.324919696]

d) $2\cos 67°$

2 [x] [cos] 67 [=]

[0.781462257]

Above is the method to find the value of the trigonometric ratio from the angle.
A calculator is also used to **find the angle from the trigonometric ratio.**

To find the value of the angle, just press the [shift] key. Here are some examples:

Find θ if:

a) $\cos\theta = \frac{1}{2}$

$\theta =$ [shift] [cos] 0.5

$\theta =$ [60]

$\theta = 30°$

b) $\tan\theta = 1$

$\theta =$ [shift] [tan] 1

$\theta =$ [45]

$\theta = 45°$

c) $\sin\theta = 0.7$

$\theta =$ [shift] [sin] 0.7

$\theta =$ [44.427004]

$\theta = 44°$ (nearest degree)

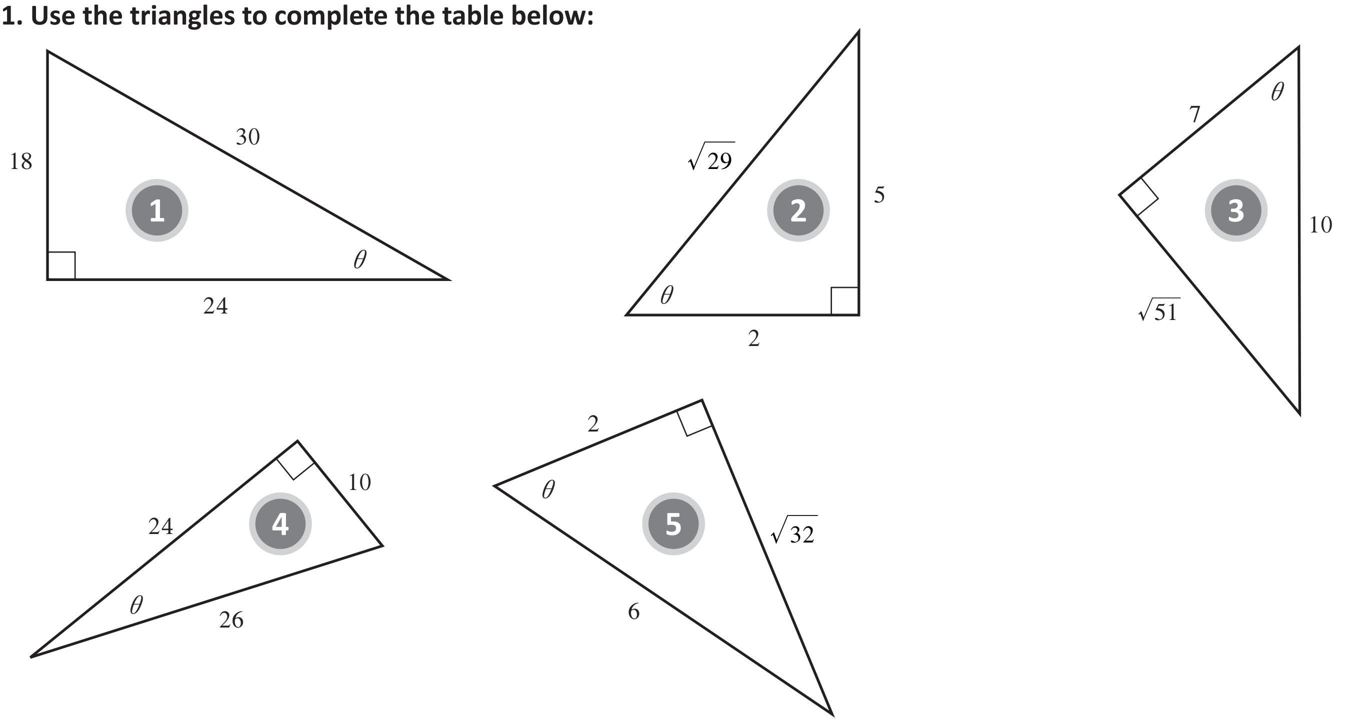
d) $\cos\theta = \frac{4}{5}$

$\theta =$ [shift] [cos] 0.8

$\theta =$ [36.86989765]

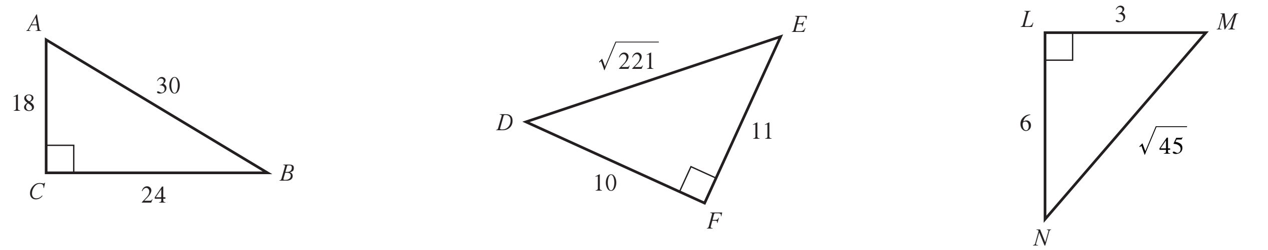
$\theta = 37°$ (nearest degree)

1. Use the triangles to complete the table below:



Triangle	Opposite to θ	Adjacent to θ	Hypotenuse	$\sin\theta$	$\cos\theta$	$\tan\theta$
1	18	24		$\frac{18}{30}$		$\frac{18}{24}$
2			$\sqrt{29}$		$\frac{2}{\sqrt{29}}$	
3	$\sqrt{51}$					
4						
5						

2. Complete the following for each triangle:



$\sin\angle A =$ __________

$\cos$ __________ $= \frac{4}{5}$

$\tan\angle B =$ __________

$\cos$ __________ $= \frac{11}{\sqrt{221}}$

$\tan\angle D$ __________

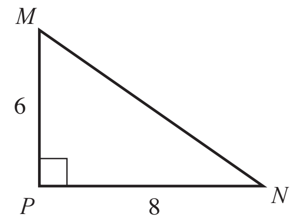
$\sin$ __________ $= \frac{11}{\sqrt{221}}$

$\tan\angle N =$ __________

$\sin$ __________ $= \frac{3}{\sqrt{45}}$

$\cos$ __________ $= \frac{6}{\sqrt{45}}$

3. Find the missing side in each right angled triangle, and then find the ratios that follow:

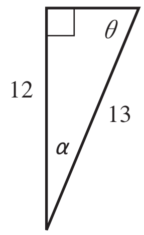
a


$\sin \angle N =$ ___________

$\tan \angle M =$ ___________

$\cos \angle M =$ ___________

$\tan \angle N =$ ___________

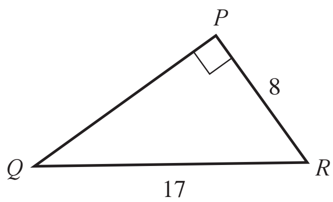
b


$\sin \theta =$ ___________

$\tan \alpha =$ ___________

$\sin \alpha =$ ___________

$\cos \theta =$ ___________

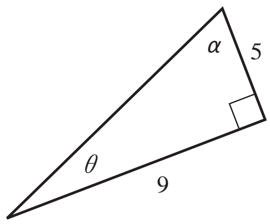
c


$\sin \angle Q =$ ___________

$\cos \angle Q =$ ___________

$\cos \angle R =$ ___________

$\tan \angle R =$ ___________

d


$\tan \theta =$ ___________

$\cos \alpha =$ ___________

$\sin \theta =$ ___________

$\tan \alpha =$ ___________

4. Evaluate the following, to 3 decimal places:

a $\sin 40°$

b $\cos 30°$

c $\cos 60°$

d $\tan 20°$

e $\tan 50°$

f $\sin 85°$

g $3\cos 45°$

h $\sqrt{2}\sin 45°$

i $\sqrt{3}\tan 30°$

j $4\sin 73°$

k $\dfrac{\cos 23^\circ}{2}$

l $\dfrac{3\tan 80^\circ}{4}$

5. Find the value of θ (to the nearest degree) if:

a $\cos\theta = 0.5$

b $\sin\theta = 0.25$

c $\tan\theta = \sqrt{3}$

d $\tan\theta = 4.5$

e $\cos\theta = 0.81$

f $\sin\theta = \dfrac{\sqrt{2}}{2}$

g $\cos\theta = \dfrac{\sqrt{3}}{2}$

h $\tan\left(\dfrac{\theta}{2}\right) = 3.1$

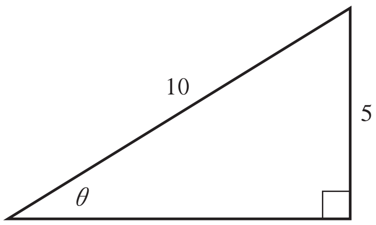
i $\sin(2\theta) = 1$

Finding Angles in Right Triangles

We use the shift button to find angles in triangles. To write the answers easily, use the formulas:

$\sin^{-1} =$ shift sin
$\cos^{-1} =$ shift cos
$\tan^{-1} =$ shift tan

Find θ in the following triangle:



Since the **opposite** side and the **hypotenuse** are given, it makes sense to use $\sin\theta$.

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{5}{10}$$

$$\sin\theta = \frac{1}{2} = 0.5$$

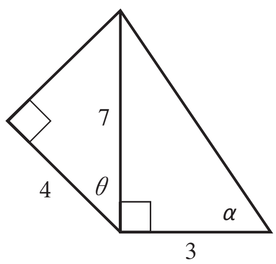
As before: $\theta =$ shift sin 0.5

$$= \sin^{-1}0.5$$

$$= 30^\circ$$

The first step is always to figure out which trigonometric ratio makes the most sense for the angle, based on what's given.

Find θ and α in the following diagram to the nearest degree:



$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{4}{7}$$

As before:

$$\theta = \cos^{-1}\left(\frac{4}{7}\right)$$

$$\theta = 55.15^\circ$$

$$\approx 55^\circ \text{ (nearest degree)}$$

$$\tan\alpha = \frac{\text{opposite}}{\text{adjacent}} = \frac{7}{3}$$

As before:

$$\alpha = \tan^{-1}\left(\frac{7}{3}\right)$$

$$\alpha = 66.801^\circ$$

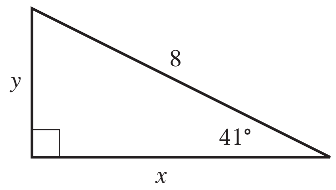
$$\approx 67^\circ \text{ (nearest degree)}$$

For θ, in the above example, it made sense to use $\cos\theta$ since the **adjacent** and **hypotenuse** were given for θ. It made sense to use $\tan\alpha$ since the **opposite** and **adjacent** were given for α.

Finding Sides in Right Triangles

If we're given a side and an acute angle of a right triangle, then we can find missing sides using trigonometry ratios. In each triangle we will be given an angle and be given either the **hypotenuse, opposite** side or **adjacent** side.

Find the lengths of x and y in the following triangle (to 2 decimal places)



Use $\sin 41°$ to find y since it is the **opposite** side and we know the **hypotenuse.**

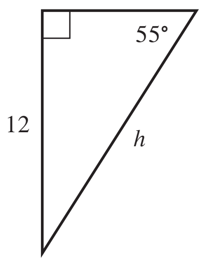
$$\frac{y}{8} = \sin 41°$$
$$y = 8 \times \sin 41°$$
$$= 8 \times (0.656...)$$
$$= 5.248472232$$
$$\approx 5.25 \text{ (2 decimal places)}$$

Use $\cos 41°$ to find x since it is the **adjacent** side and we know the **hypotenuse.**

$$\frac{x}{8} = \cos 41°$$
$$x = 8 \times \cos 41°$$
$$= 8 \times (0.754...)$$
$$= 6.037676642$$
$$\approx 6.04 \text{ (2 decimal places)}$$

In these type of questions, choose the ratio which involves the given side and the missing side. Above $\sin\theta$ was chosen to solve for y because it involves the **opposite side (needed)** and the **hypotenuse (given).**

Find the length of the hypotenuse below (to 2 decimal places)



Choose $\sin\theta$ since it involves the opposite side (given) and the hypotenuse (needed).

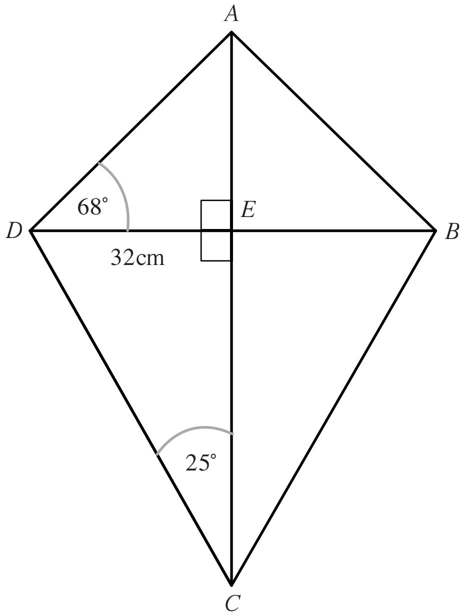
$$\sin 55° = \frac{12}{h}$$
$$h = \frac{12}{\sin 55°}$$
$$= \frac{12}{0.819...}$$
$$= 14.6492...$$
$$\approx 14.65 \text{ (2 decimal places)}$$

In order for a kite to fly, it needs to have the angles in the diagram. DE is 32 cm. Find the lengths of AD & EC:

For AD, use ΔADE.

Since we are missing the **hypotenuse** AD and the **adjacent** side to $\angle ADE$ has been given as 32cm, we use $\cos\theta$.

$\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$



$$\frac{DE}{AD} = \cos\angle ADE$$

$$\frac{32}{AD} = \cos 68^\circ$$

$$AD = \frac{32}{\cos 68^\circ}$$ ← Make AD the subject

$$= \frac{32}{0.3746...}$$

$$= 85.422...$$

$$\approx 85.42 \text{ cm} \quad \text{(2 decimal places)}$$

For EC, use $\triangle CDE$.

Since we are missing EC the **adjacent** to $\angle ECB$ and the **opposite** side to $\angle ECB$ has been given as 32 cm, we use $\tan\theta$.

$\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$

$$\frac{DE}{EC} = \tan\angle ECD$$

$$\frac{32}{EC} = \tan 25^\circ$$

$$EC = \frac{32}{\tan 25^\circ}$$ ← Make EC the subject

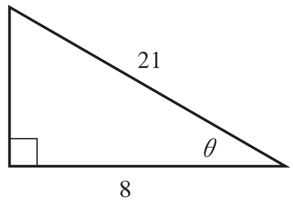
$$= \frac{32}{0.4663}$$

$$= 68.6242$$

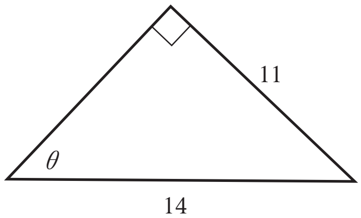
$$\approx 68.62 \text{ cm} \quad \text{(2 decimal places)}$$

1. Find θ in each triangle to the nearest degree:

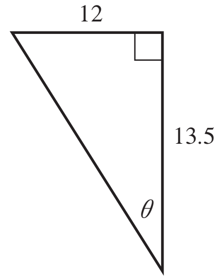
a



b

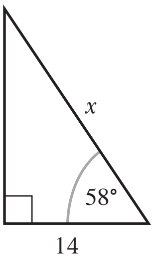


c

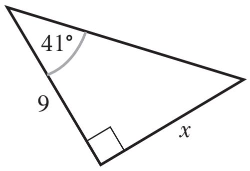


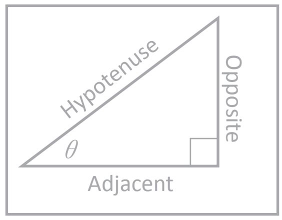
2. Complete the table below if you are solving side labeled x:

a

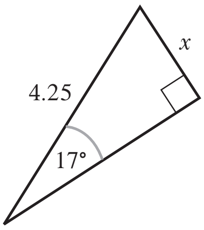


b

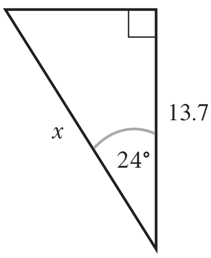




c



d



Triangle	Given side for angle	Missing side for angle (x)	Correct ratio to use (sin, cos, tan)
a	adjacent		cos
b		opposite	
c	hypotenuse		
d			

3. Find the value of x in each of the triangles from the previous question:

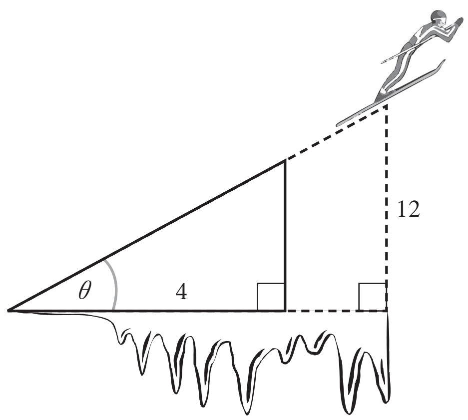
a

b

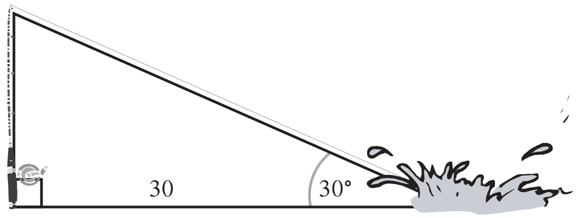
c

d

4. A skier jumps a 4m ramp. 2m after the jump the skier's height is 12m. What is the angle of the ramp?



5. A fisherman casts his line out and keeps his fishing rod pointing straight upwards. If the line touches the water 30 m from the shore at an angle of $30°$, then how long is the fishing line to the nearest metre?

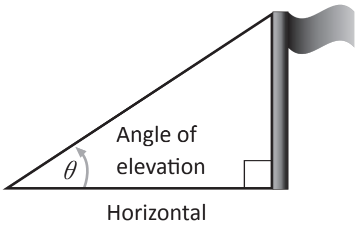


6. If the fishing line is 40 m long and touches the water 33 m from the shore, at what angle will the line touch the water?

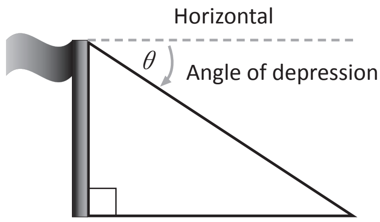
Angles of Depression and Elevation

These angles are always made with a horizontal line.

Angle of Elevation is the 'angle looking up'



Angle of Depression is the 'angle looking down'



These angles will always have the same value, even though they are in different places. (Do you know why?)

A Ferris wheel has a maximum height of 60 m and casts a shadow 100 m long

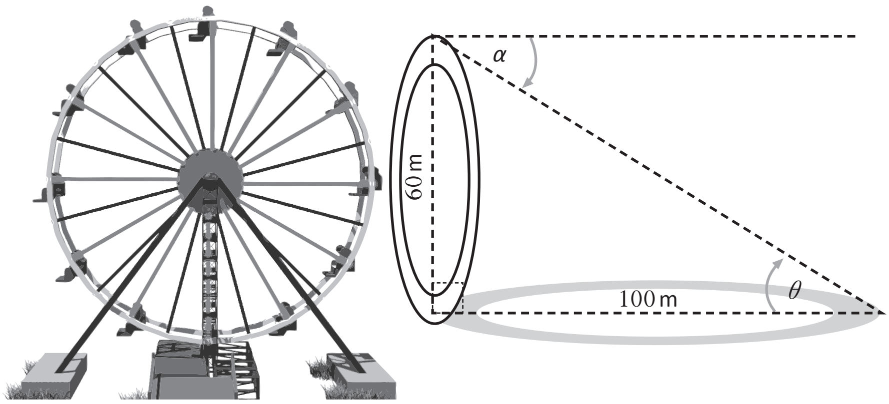
a What is the angle of elevation θ from the tip of the shadow to the top of the ferris wheel? (to the nearest degree)

$$\tan\theta = \frac{60}{100}$$

$$\theta = \tan^{-1}\left(\frac{60}{100}\right)$$

$$\theta = \tan^{-1}0.6$$

$\tan^{-1}$ = **shift** **tan**



$$= 30.9637...^\circ$$

$$\approx 31^\circ \text{ (nearest degree)}$$

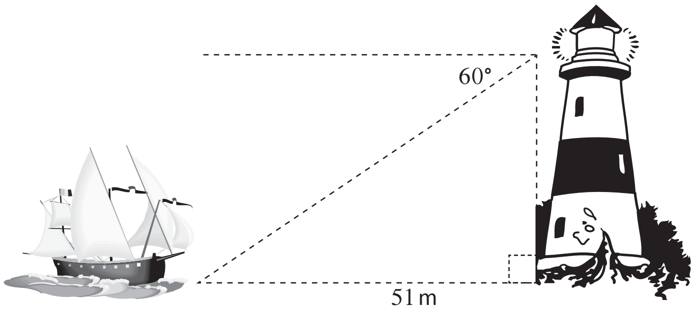
b **What is the angle of depression, α, from the top of the ferris wheel to the tip of the shadow?**

The angle of depression has the same value as the angle of elevation.

$$\alpha = \theta = 31^\circ$$

When working with these questions, the key is to determine the position of the right angled triangle in the diagram.

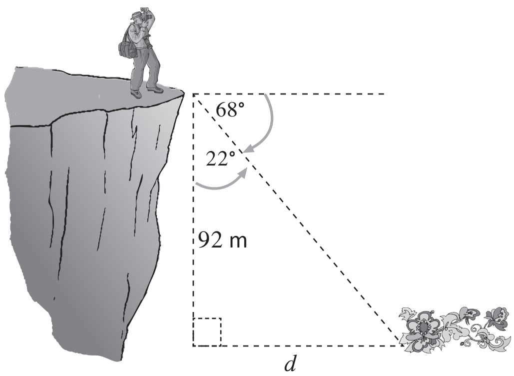
The angle of depression from the top of a lighthouse to a ship is $60°$. The lighthouse is 51 m away from the ship. Draw a diagram to represent this situation:



If the diagram is drawn correctly, then missing sides and angles can be used in the exact same easy way as in the previous section.

A photographer stands on the edge of a 92 m cliff and takes a photo of a flower. If the angle of depression of the camera is $68°$, then what is the distance, d, between the cliff and the flower? (nearest metre)

Method 1

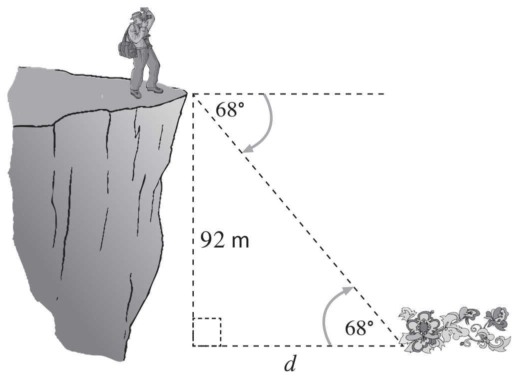


The angle with the vertical is $90^\circ - 68^\circ = 22^\circ$.

So, $\tan 22^\circ = \dfrac{d}{92}$

$$d = 92 \times \tan 22^\circ$$
$$= 92 \times 0.40403...$$
$$= 37.1704...$$
$$\approx 37 \text{ m}$$ (nearest degree)

Method 2

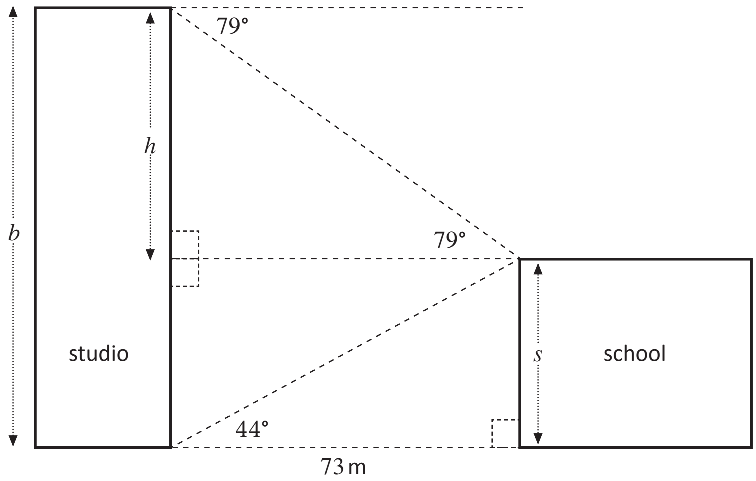


Angle of elevation $=$ Angle of depression $= 68^\circ$.

So, $\tan 68^\circ = \dfrac{92}{d}$

$$d = \frac{92}{\tan 68^\circ}$$
$$= \frac{92}{2.475...}$$
$$= 37.1704...$$
$$\approx 37 \text{ m}$$ (nearest degree)

1. A studio is 73 m to the left of a school. The angle of elevation from the base of the studio to the roof of the school is $44°$. The angle of depression from the roof of the studio to the roof of the school is $79°$.



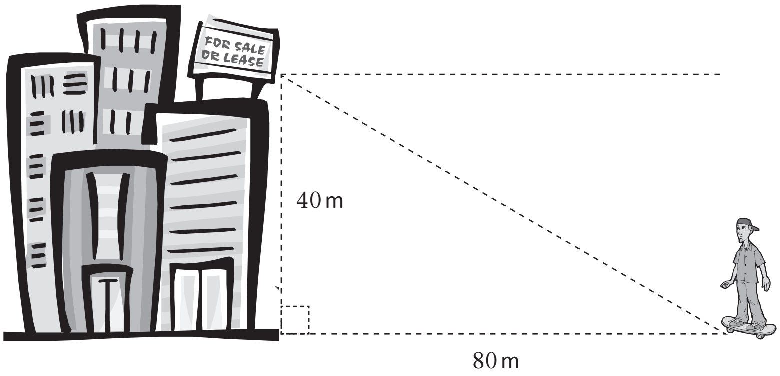
a Find the height of the school to 3 decimal places:

b How much higher is the studio than the school to 3 decimal places?

c What is the total height of the studio to 1 decimal place?

2. A skateboarder reads a sign on top of a 40 m building.

a Identify the angle of elevation and the angle of depression in the following diagram:



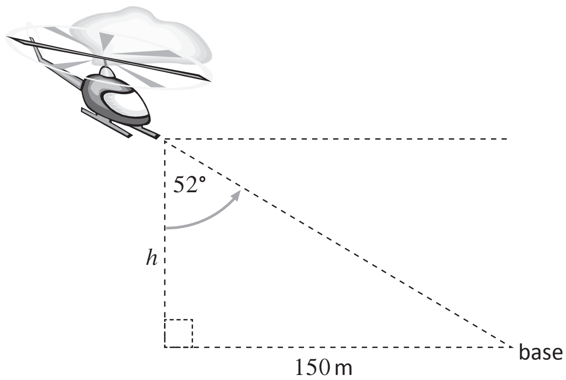
b If he sees the sign when he is 80 m away from the building, what is the angle of elevation from the skater to the sign?

c If the skater continues skating until he is 30 m from the building, will the angle of elevation increase or decrease? By how much?

3. Aiden answered the following question incorrectly. Can you spot his mistake?

The angle of depression from a helicopter to its landing base is $52°$. If the horizontal distance between the helicopter and the landing base is 150 m, then how high is the helicopter (1 decimal place) at this point?

AIDEN'S SOLUTION



$$\tan 52° = \frac{150}{h}$$

$$h = \frac{150}{\tan 52°}$$

$$h = \frac{150}{1.279}$$

$$h = 117.192... \approx 117.2 \text{ m} \quad (1 \text{ decimal place})$$

a What was Aiden's mistake?

b Find the correct height of the helicopter at this point.

4. An aeroplane takes off at an angle of $28°$ to the ground. It flies over a house 900 m from the airport.

a How high is the aeroplane at that point, to 3 decimal places?

b What is the angle of depression at this point?

c After continuing to fly at the same height, the pilot notices that as they are flying over a lake, the airport has a $15°$ angle of depression. How far is the lake away from the airport, to 2 decimal places?

5. A satellite tower is on the right of a post office and they are separated by a distance d. The post office has a height of 12 m. The angle of depression from the roof of the post office to the base of the tower is $23°$. The angle of elevation from the roof of the post office to the roof of the tower is $58°$.

a Draw a diagram to represent this situation:

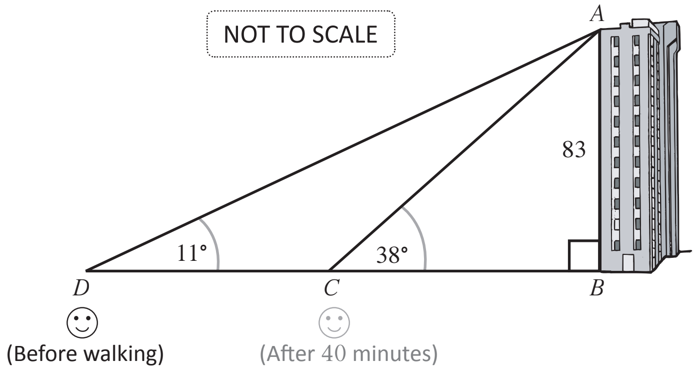
b Find d, the distance between the buildings to 1 decimal place:

c Find the total height of the tower to 1 decimal place:

Problems Involving More Than 1 Triangle

As all mathematicians know, most problems in real life are more complicated. They could involve more than 1 triangle internally or externally.

Let's say you have to be at a building (which is 83 m tall) in 1 hour. The angle of elevation from you to the top of the building is 11°. After 40 minutes of walking closer to the building, the angle of elevation has increased to 38°.



a **Which are the two right angled triangles involved in this problem?**

ΔABD and ΔABC (Highlight these)

b **How far were you from the building before you started walking (nearest metre)?**

$$\tan 11^\circ = \frac{AB}{BD} = \frac{83}{BD}$$

$$BD = \frac{83}{\tan 11^\circ}$$

$$= \frac{83}{0.194}$$

$$= 426.997...$$

$$\approx 427 \text{ m} \text{ (nearest metre)}$$

c **If you keep walking at the same pace, are you going to make it to the building in time?**

First we find BC: $\tan 38^\circ = \frac{AB}{BC}$

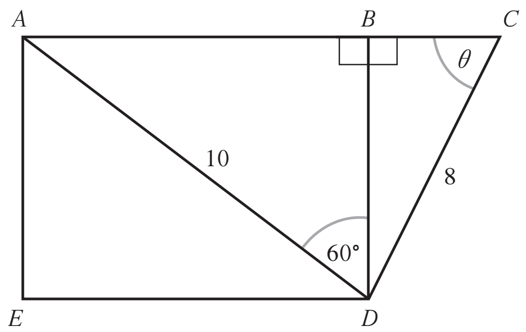
$$BC = \frac{AB}{\tan 38^\circ} = \frac{83}{0.781} = 106.235 \approx 106 \text{ m} \text{ (nearest metre)}$$

This means that the distance walked in 40 minutes $= DC = 427\text{ m} - 106\text{ m} = 321\text{ m}$.
This is 8.025 m per minute. Thus in 20 minutes you would walk $20 \times 8.025\text{ m} = 160.5\text{ m}$.

Since you only have to walk 106 m in 20 minutes, you would make it on time.

Most problems involving more than 1 triangle will ask you to find a common side (or angle) of two triangles and then use that common side (or angle) to find something (side or angle) in a second triangle.

A giant gate needs to be built in the shape below **(all measurements in m)**



a Find the height of the gate:

To find the height of the gate we need AE or BD.

$$\frac{BD}{AD} = \cos 60^\circ$$

$$BD = AD \times \cos 60^\circ$$

$$= 10 \times 0.5$$

$$= 5\text{ m}$$

b Find the value of θ to 1 decimal place:

$$\sin\theta = \frac{BD}{CD} = \frac{5}{8} = 0.625$$

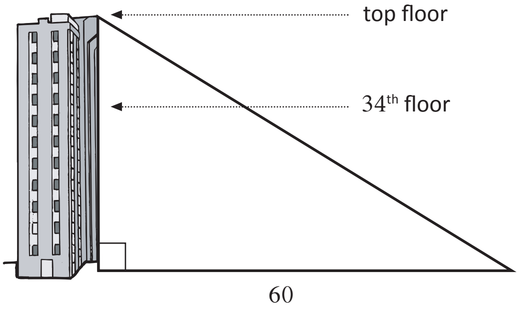
$$= \sin^{-1}(0.625)$$

$$= 38.682...^\circ$$

$$\approx 38.7^\circ \text{ (1 decimal place)}$$

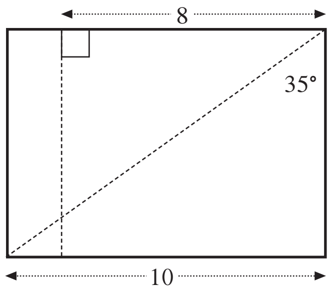
In the example above, we had to find the common side BD using what we knew about $\triangle ABD$ to find the angle in $\triangle BCD$.

1. You and a friend stand in a building with 50 floors, each floor is 2 m high. You are on the 34^{th} floor and your friend is on the top floor. Find the difference in your angles of elevation from 60 m away.



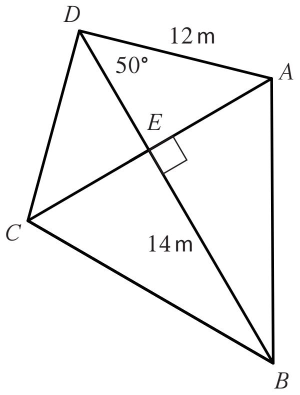
2. As a technician you need to tie rope along the dotted lines in this rectangle:

a How many right triangles are involved in this problem?



b Find the total length of rope needed if all measurements are in m (nearest m):

3. In order for a certain kite to fly it needs to look like this.
Find the length of AB and angle $\angle ABE$ each to 1 decimal place.

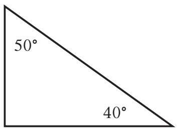


Basics:

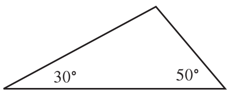
1. **a** Triangles **a**, **c**, **d**, **e** are right angled indicated by the small square in the corner of each triangle.

2. The sum of the interior angles of a triangle is $180°$.

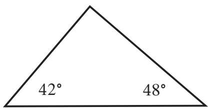
3.



is right angled

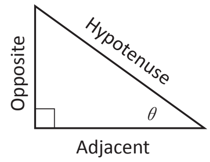


is not right angled

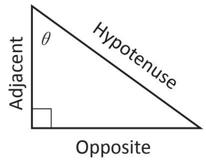


is right angled

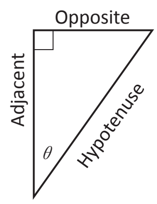
4. **a**



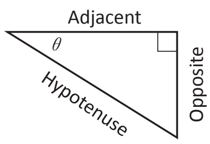
b



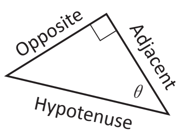
c



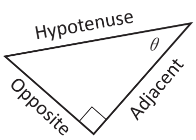
d



e



f



5.

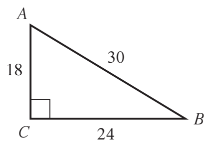
Triangle	Opposite to θ	Adjacent to θ	Opposite to α	Adjacent to α	Hypotenuse
ΔABC	AC	BC	BC	AC	AB
ΔDEF	EF	DE	DE	EF	DF
ΔLMN	MN	LM	ML	MN	LN
ΔPQR	PR	QR	QR	PR	PQ
ΔWXY	WX	WY	WY	WX	XY

Knowing More:

1.

Triangle	Opposite to θ	Adjacent to θ	Hypotenuse	$\sin\theta$	$\cos\theta$	$\tan\theta$
1	18	24	30	$\frac{18}{30} = \frac{3}{5}$	$\frac{24}{30} = \frac{4}{5}$	$\frac{18}{24} = \frac{3}{4}$
2	5	2	$\sqrt{29}$	$\frac{5}{\sqrt{29}}$	$\frac{2}{\sqrt{29}}$	$\frac{5}{2}$
3	$\sqrt{51}$	7	10	$\frac{\sqrt{51}}{10}$	$\frac{7}{10}$	$\frac{\sqrt{51}}{7}$
4	10	24	26	$\frac{10}{26} = \frac{5}{13}$	$\frac{24}{26} = \frac{12}{13}$	$\frac{10}{24} = \frac{5}{12}$
5	$\sqrt{32}$	2	6	$\frac{\sqrt{32}}{6}$	$\frac{2}{6} = \frac{1}{3}$	$\frac{\sqrt{32}}{2}$

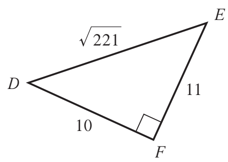
2.



$\sin\angle A = \frac{4}{5}$

$\cos\angle B = \frac{4}{5}$

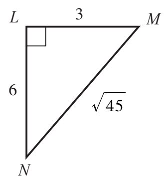
$\tan\angle B = \frac{3}{4}$



$\cos\angle E = \frac{11}{\sqrt{221}}$

$\tan\angle D = \frac{11}{10}$

$\sin\angle D = \frac{11}{\sqrt{221}}$



$\tan\angle N = \frac{1}{2}$

$\sin\angle N = \frac{3}{\sqrt{45}}$

$\cos\angle N = \frac{6}{\sqrt{45}}$

Knowing More:

3. (a) $MN = 10$

$\sin \angle N = \frac{3}{5}$ $\tan \angle M = \frac{4}{3}$

$\cos \angle M = \frac{3}{5}$ $\tan \angle N = \frac{3}{4}$

(b) $b = 5$

$\sin \theta = \frac{12}{13}$ $\tan \alpha = \frac{5}{12}$

$\sin \alpha = \frac{5}{13}$ $\cos \theta = \frac{5}{13}$

(c) $PQ = 15$

$\sin \angle Q = \frac{8}{17}$ $\cos \angle Q = \frac{15}{17}$

$\cos \angle R = \frac{8}{17}$ $\tan \angle R = \frac{15}{8}$

(d) $c = \sqrt{106}$

$\tan \theta = \frac{5}{9}$ $\cos \alpha = \frac{5}{\sqrt{106}}$

$\sin \theta = \frac{5}{\sqrt{106}}$ $\tan \alpha = \frac{9}{5}$

4. (a) $\sin 40° = 0.643$ (b) $\cos 30° = 0.866$

(c) $\cos 60° = 0.5$ (d) $\tan 20° = 0.364$

(e) $\tan 50° = 1.192$ (f) $\sin 85° = 0.996$

(g) $3\cos 45° = 2.121$ (h) $\sqrt{2}\sin 45° = 1$

(i) $\sqrt{3}\tan 30° = 1$ (j) $4\sin 73° = 3.825$

(k) $\frac{\cos 23°}{2} = 0.460$ (l) $\frac{3\tan 80°}{4} = 4.253$

5. (a) 60° (b) 14° (c) 60°

(d) 77° (e) 36° (f) 45°

(g) 30° (h) 144° (i) 45°

Using Our Knowledge:

1. (a) 68° (b) 52° (c) 42°

2.

Triangle	Given side for angle	Missing side for angle (x)	Correct ratio to use (sin, cos, tan)
a	Adjacent	Hypotenuse	cos
b	Adjacent	Opposite	tan
c	Hypotenuse	Opposite	sin
d	Adjacent	Hypotenuse	cos

3. (a) 26.4 (1 d.p.) (b) 7.8 (1 d.p.)

(c) 1.2 (1 d.p.) (d) 15.0 (1 d.p.)

4. $\theta = 63°$ (nearest degree)

5. 35 m (nearest degree)

6. Angle = 34° (nearest degree)

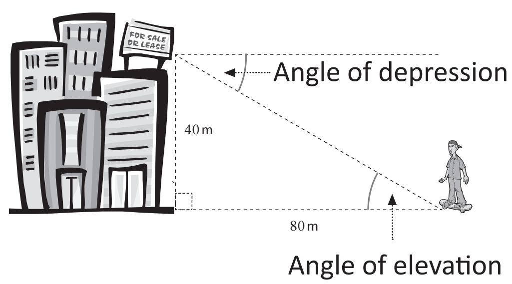
Thinking More:

1. (a) $s = 70.495$m (3 d.p.)

(b) $h = 375.552$m (3 d.p.)

(c) $b = 446.0$m (1 d.p.)

2. (a)



(b) 26.6° (1 d.p.)

(c) The angle of elevation will increase. It will increase by 26.5°. (1 d.p.)

Thinking More:

3. (a) Aiden incorrectly labelled the angle of depression. The angle of depression is formed between the upper horizontal and hypotenuse not h and the hypotenuse.

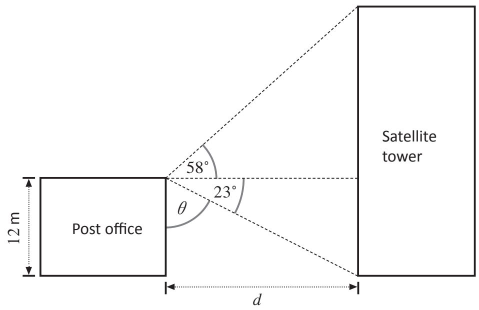
(b) $h = 192.0$ m (1 d.p.)

4. (a) 478.538 m (3 d.p.)

(b) Angle of depression $= 28°$

(c) 1785.93 m (2 d.p.)

5. (a)



(b) $d = 28.3$ m (1 d.p.)

(c) Total height $= 57.3$ m (1 d.p.)

Thinking Even More:

1. Difference of angles $= 10.4°$ (1 d.p.)

2. (a) There are 3 right angled triangles.

(b) Total length of rope $= 32$ m

3. $AB = 16.7$ m

$\angle ABE = 33.3°$