For greater accuracy, we can measure angles not only in **degrees** (°) but also in parts of degrees, as decimals or using the units **minutes** (') and **seconds** ('').

1 degree = 60 minutes 1 minute = 60 seconds

We will be measuring angles accurate to the nearest minute. Angle 53 degrees 18 minutes is written 53°18'.

# EXAMPLE 1

Find the following correct to 4 decimal places.

a cos 84.3°

**b** sin 68.7°

c tan 15.5°

d cos 78°15′

e sin 11°12′

f tan 17°58'

	Solve	Think	Apply
a	cos 84.3° ≈ 0.0993	cos 84.3 =	Make sure your calculator is in degree mode.  Some calculators have a key instead of a DMS key.
b	$\sin 68.7^{\circ} \approx 0.9317$	sin 68.7 =	
c	$\tan 15.5^{\circ} \approx 0.2773$	tan 15.5 =	

#### EXAMPLE 1 CONTINUED

	Solve	Think	Apply
d	$\cos 78^{\circ}15' \approx 0.2036$	cos 78 DMS 15 DMS =	On some calculators the second DMS is not required.
e	$\sin 11^{\circ}12' \approx 0.1942$	sin 11 DMS 12 DMS =	
f	$\tan 17^{\circ}58' \approx 0.3243$	tan 17 DMS 58 DMS =	

- 1 Find the following correct to 4 decimal places.
  - a sin 36.8°

- **b** cos 14.23°
- c tan 8.11°

- d cos 65.25°
- e cos 89°21′
- f tan 18°23'

- g tan 68°23'
- h sin 45°21′
- i cos 57°51′

- cos 33°21′
- k tan 21°33′
- I sin 11°11′

### EXAMPLE 2

Evaluate the following correct to 4 decimal places.

- **a** 12 cos 15.6°
- **b** 5 sin 11°15′
- $\frac{3 \tan 11^{\circ}51'}{\cos 23^{\circ}15'}$

	Solve	Think	Apply
a	$12 \cos 15.6^{\circ} \approx 11.5580$	12 × cos 15.6 =	Remember that the fraction line is a grouping symbol. Insert brackets if you are not sure of the order of operations.
b	$5 \sin 11^{\circ}15' \approx 0.9755$	5 × sin 11 DMS 15 DMS =	
c	$\frac{3 \tan 11^{\circ}51'}{\cos 23^{\circ}15'} \approx 0.6851$	3 × tan ( 11 DMS 51 DMS	
		÷ cos ( 23 DMS 15	
		DMS ) =	

2 Evaluate the following correct to 4 decimal places.

a 8 cos 23.1°
d 23 sin 75°12′
g sin 11°51′

e 8.3 tan 58°51' h  $\frac{8 \tan 16°16'}{\sin 15°}$ 

**b** 5 tan 16.4°

f 12.3 cos 27°48′ 12.3 cos 48°

sin 16°15′

c 15 sin 48.18°

 $\frac{8.7 \tan 75^{\circ}14'}{13.2}$ 

 $\frac{4.2 \cos 18.3^{\circ}}{6.8}$ 

 $\frac{3 \sin 83^{\circ}12'}{16.5}$ 

 $\mathbf{m} \,\, \frac{4 \sin 18^{\circ} \cos 18^{\circ}}{3}$ 

 $\frac{11 \tan 16^{\circ} \cos 14^{\circ}}{\sin 12^{\circ}}$ 

 $\frac{8.3 \cos 11^{\circ}15'}{\sin 11^{\circ}15'}$ 

## Using trigonometric ratios to find angles

You can work backwards on a calculator to find an angle from one of the trigonometric ratios, by using one of the key combinations  $\begin{array}{c} \text{SHIFT} \end{array}$   $\begin{array}{c} \text{cos} \end{array}$ . These may appear on your calculator display as  $\begin{array}{c} \text{tan}^{-1} \text{ or } \text{sin}^{-1} \text{ or } \text{cos}^{-1}. \end{array}$ 

For example, if  $\sin \theta = 0.4369$ then  $\theta = \sin^{-1} 0.4369$ 

where  $\sin^{-1} 0.4369$  means 'the angle whose sine is 0.4369'.

Similarly,  $\cos^{-1}$  means 'the angle whose cosine is' and  $\tan^{-1}$  means 'the angle whose tangent is'.

#### **EXAMPLE 3**

Find  $\theta$  to the nearest: i degree ii minute. **a**  $\sin \theta = 0.6314$ **b**  $\tan \theta = 3.6$  $\cos \theta = 0.8$ Solve Think **Apply** Make sure your a i  $\sin \theta = 0.6314$  $\sin 0.6314 =$  $\theta = 39.153...$ calculator is in degree mode. ≈ 39° Ensure that SHIFT  $\theta = 39^{\circ}9'12.55...''$ ii DMS is pressed before the  $\approx 39^{\circ}9'$  to the nearest minute As the seconds are less than 30, trigonometric ratios so as 12.55... < 30 round the minutes down. that the answer is an b i  $\tan \theta = 3.6$ SHIFT tan 3.6 angle. The half-way  $\theta = 74.475...$ point for rounding is  $\approx 74^{\circ}$ 30 seconds. Below 30 seconds round  $\theta = 74^{\circ}28'33.20...''$ ii DMS down; 30 seconds or  $\approx 74^{\circ}29'$  to the nearest minute As the seconds are greater than or more, round up. as 33.2 > 30equal to 30, round the minutes up. Note that some c i  $\cos \theta = 0.8$ cos 0.8 calculators require  $\theta = 36.869...$ SHIFT before DMS ≈ 37° to convert to minutes  $\theta = 36^{\circ}52'11.63...''$ ii DMS and seconds.  $\approx 36^{\circ}52'$  to the nearest minute as 11.63 < 30

3 Write these calculator displays as angles to the nearest minute.

- **a** 43°27′14.2″
- **b** 62°15′58.13″
- c 14°3′0″

- d 81°53′30′′
- e 21°59′48.72″
- f 10°1′28.42′′

- g 35°28′18.3″
- h 72°51′38.5″
- i 27°53′58.1″

- j 39°35′11.3″
- k 68°54′41.2″

**b**  $\cos \theta = 0.1445$ 

**e**  $\tan \theta = 4.1371$ 

 $\mathbf{h} \sin \theta = 0.5512$ 

 $k \tan \theta = 23.7215$ 

 $\mathbf{n} \sin \theta = 0.6262$ 

0°3′34.2′′

4 Find the value of  $\theta$  to the nearest:

- i degree
- $\mathbf{a} \quad \sin \theta = 0.3625$
- **d**  $\cos \theta = 0.6731$
- $g \tan \theta = 0.0371$
- $\mathbf{j} \sin \theta = 0.0027$
- $m \cos \theta = 0.6614$

- ii minute.
  - -----
    - c  $\tan \theta = 2.1351$
    - $\mathbf{f} \quad \sin \theta = 0.1113$
    - $\cos \theta = 0.0314$
    - $\cos \theta = 0.9811$
    - o  $\tan \theta = 0.2222$

## **EXAMPLE 4**

Find  $\theta$  to the nearest:

- i degree
- $a \sin \theta = \frac{5}{9}$

- ii minute.
- **b**  $\cos \theta = \frac{6}{13}$
- c  $\tan \theta = \frac{18}{7}$

	Solve	Think	Apply
a i	$\sin \theta = \frac{5}{9}$ $\theta = 33.74$ $\approx 34^{\circ}$	SHIFT Sin (5 ÷ 9 ) =	Make sure that the calculator is in degree mode.
ii	θ = 33°44′56.35" ≈ 33°45′	As the seconds are greater than 30, round the minutes up.	Press SHIFT first to obtain an angle.  Put the fraction in brackets before pressing Round accordingly.  Note that some calculators require  SHIFT DMS to convert to minutes and seconds.
b i	$\cos \theta = \frac{6}{13}$ $\theta = 62.51$ $\approx 63^{\circ}$	SHIFT COS ( 6 ÷ 13 ) =	
ii	$\theta = 62^{\circ}30'48.86''$ $\approx 62^{\circ}31'$	As the seconds are greater than 30, round the minutes up.	
c i	$\tan \theta = \frac{18}{7}$ $\theta = 68.74$ $\approx 69^{\circ}$	SHIFT (tan ( 18 ÷ 7 ) =	
ii	$\theta = 68^{\circ}44'58.18''$ $\approx 68^{\circ}45'$	As the seconds are greater than 30, round the minutes up.	

5 Find the value of  $\theta$  to the nearest:

i degree

ii minute.

 $i \quad \tan \theta = \frac{11.27}{15}$ 

**a**  $\tan \theta = \frac{14}{3}$  **b**  $\cos \theta = \frac{3}{11}$  **c**  $\sin \theta = \frac{11}{18}$  **d**  $\sin \theta = \frac{4}{29}$  **e**  $\tan \theta = \frac{6}{7}$  **f**  $\cos \theta = \frac{14}{17}$  **g**  $\sin \theta = \frac{0.013}{0.214}$  **h**  $\cos \theta = \frac{6.2}{15}$  $\mathbf{j} \cos \theta = \frac{1}{3}$ 

 $\mathbf{k} \sin \theta = \frac{3}{4}$ 

 $1 \quad \tan \theta = \frac{4}{3}$ 

**6** Find angle A to the nearest minute given that:

$$a \cos A = 0.7$$

**b** 
$$\sin A = 0.642$$

c 
$$\tan A = 3.265$$

#### EXELCISE OF

1 a 0.5990

**b** 0.9693

c 0.1425

**d** 0.4187 g 2.5236 e 0.0113

f 0.3323

j 0.8353

**h** 0.7114 k 0.3949

i 0.5321 0.1939

2 a 7.3586

c 11.1786

d 22.2369

b 1.4716e 13.7320

f 10.8803

g 3.5884

h 9.0191

i 29.4119

i 2.5004

k 0.5864 n 14.7202

1 0.1805

m 0.3919

o 41.7269

**3** a 43°27′ **b** 62°16′

e 22°0′

c 14°3′ f 10°1′

d 81°54′ g 35°28′

h 72°52′

i 27°54′

c 72°58′

6 a 45°34'

i 39°35′ k 68°55′

1 0°4′

4 a i 21° ii 21°15′ b i 82° ii 81°42′ ii 64°54′ **d** i 48° ii 47°42′ i 65° f i 6° i 76° ii 76°25′ ii 6°23′ ii 2°7′ i 2° h i 33° ii 33°27′ i 88° i 0° ii 88°12′ ii 0°9′ i 88° ii 11°9′ ii 87°35′ 1 i 11° i 49° n i 39° ii 38°46′ ii 48°36′ m i 13° ii 12°32′ 0 i 78° ii 77°54′ **b** i 74° ii 74°10′ 5 a ii 7°56′ i 38° ii 37°40′ d i 8° f i 35° ii 34°34′ i 41° ii 40°36′ e i 3° ii 3°29′ h i 66° ii 65°35′ i 37° ii 70°32′ ii 36°55′ i i 71° 1 i 53° k i 49° ii 48°35′ ii 53°8′

**b** 39°56′