Volume

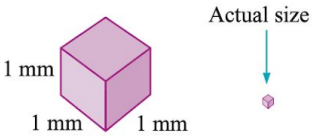
View the video given

[The volume of prisms and cylinders](#) Video

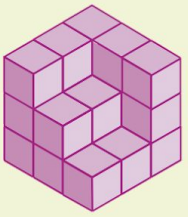
Volume refers to the amount of space that an object occupies. The volume of a solid is the number of **cubic units** that it occupies. The most common metric units for measuring volume are:

- cubic millimetres (mm^3)
- cubic centimetres (cm^3)
- cubic metres (m^3).

A cubic millimetre (1 mm^3) is the amount of space occupied by a cube of side length 1 mm.



Find the volume of this solid by counting cubes.



Method 1: Count the cubes in each layer separately.

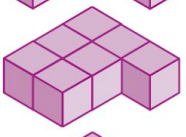
3rd layer = 5 cubes

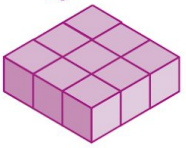
2nd layer = 7 cubes

1st layer = 9 cubes

Total = 21 cubes


3rd layer


2nd layer


1st layer

Method 2: Alternatively, make a plan of the solid and count the number of cubes in each stack.

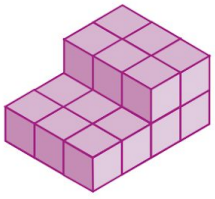
Total = $(3 \times 5) + (2 \times 2) + (1 \times 2) = 21$ cubes

By both methods, the volume of the solid is 21 cubic units.

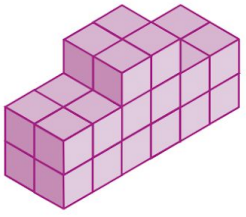
3	3	3
2	2	3
1	1	3

1 If the solids below are made from 1 cm^3 cubes, calculate the volume of each solid.

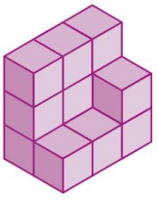
a



b



c



2 **a** Explain the meaning of a cubic centimetre. Illustrate with a diagram.

b Explain the meaning of a cubic metre.

Volumes in life

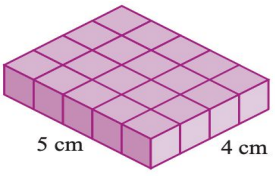
1 **a** Make a cube of side length 1 m. Use the cube to estimate the volume of your classroom, in cubic metres.
b Write a short report on how you made the cube and the method your group used to find the volume of your classroom.

2 Discuss which unit (mm^3, cm^3 or m^3) would be the most suitable to measure the volume of a:

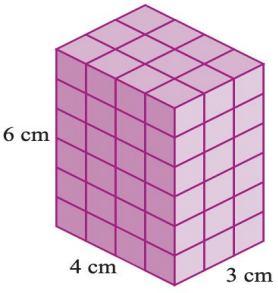
a fruit juice carton **b** key **c** caravan
d refrigerator **e** garage **f** house brick

Volume of a rectangular prism

1 **a** Calculate the volume of this layer of cubes.
b What would be the volume of the rectangular prism formed by stacking up:
i 3 layers? **ii** 5 layers? **iii** 10 layers?

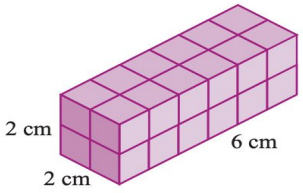


2 This rectangular prism was formed by stacking layers of cubes on top of each other.
a How many cubic centimetres are there in the bottom layer?
b How many layers are stacked on top of each other?
c What is the volume of this prism?

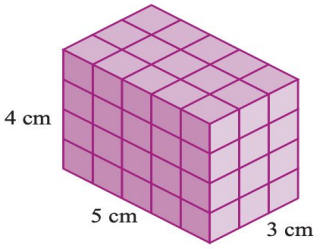


3 Repeat question **2** for each of these rectangular prisms.

a

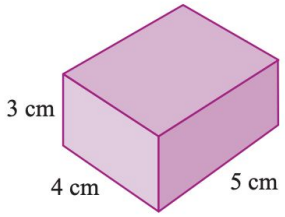


b



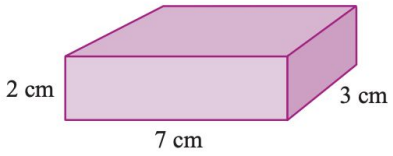
1 Complete the following to calculate the volume of each rectangular prism.

a



$V = lbh$
$= 5 \times \_\_\_\_ \times \_\_\_\_$
$= \_\_\_\_ \text{ cm}^3$

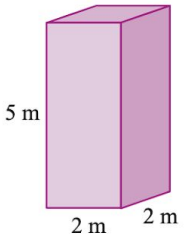
b



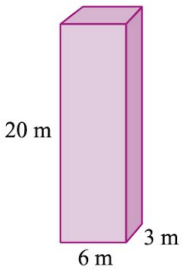
$V = lbh$
$= \_\_\_\_ \times 3 \times \_\_\_\_$
$= \_\_\_\_ \text{ cm}^3$

2 Calculate the volumes of these rectangular prisms.

a



b



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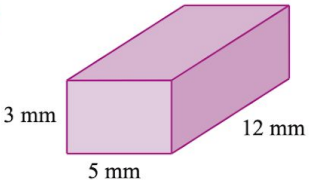
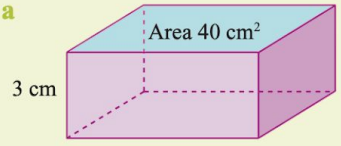


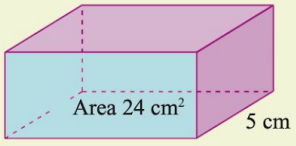
EXAMPLE 2

Calculate the volume of this rectangular prism. A different face has been selected as the base each time.

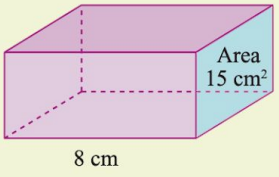
a



b



c



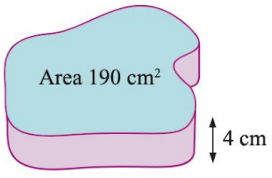
a V = area of base × height
$= 40 \times 3$
$= 120 \text{ cm}^3$

b V = area of base × height
$= 24 \times 5$
$= 120 \text{ cm}^3$

c V = area of base × height
$= 15 \times 8$
$= 120 \text{ cm}^3$

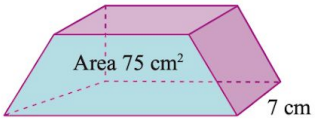
3 Calculate the volume of each solid, given the area of its base.

a



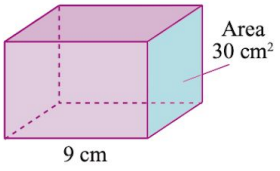
V = area of base × height
$= 190 \times \_\_\_\_$
$= \_\_\_\_ \text{ cm}^3$

b



V = area of base × height
$= \_\_\_\_ \times 7$
$= \_\_\_\_ \text{ cm}^3$

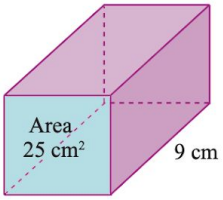
c



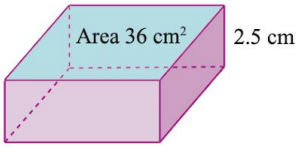
V = area of base × height
$= \_\_\_\_ \times \_\_\_\_$
$= \_\_\_\_ \text{ cm}^3$

4 Calculate the volume of each solid, given the area of its base.

a



b



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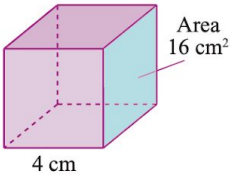

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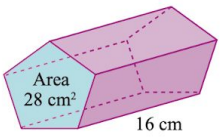

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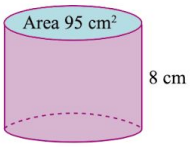

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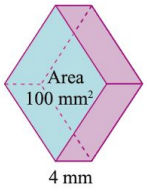
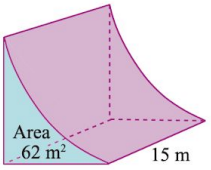
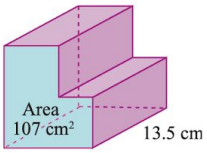


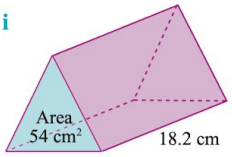
g

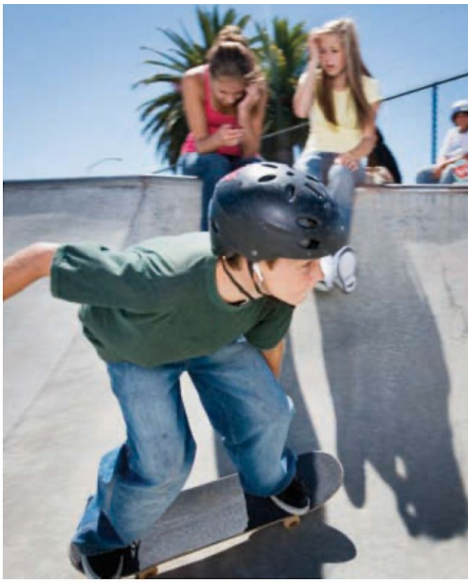


h



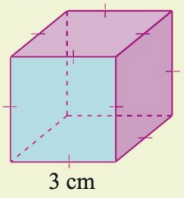
i





Calculate the volume of each rectangular prism.

a



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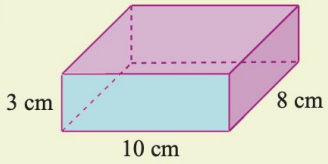

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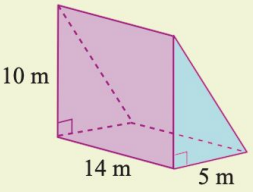


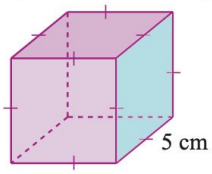
a V = area of base × height
$= (3 \times 3) \times 3$
$= 9 \times 3 = 27$ cm³

b V = area of base × height
$= (10 \times 8) \times 3$
$= 80 \times 3 = 240$ cm³

c V = area of base × height
$= \frac{1}{2}(5 \times 10) \times 14$
$= 25 \times 14 = 350$ cm³

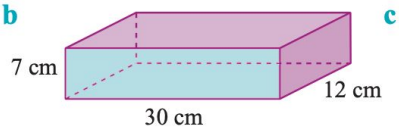
5 Complete the following to calculate the volume of each prism.

a



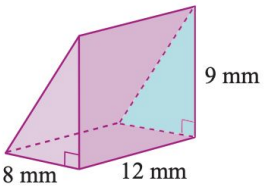
V = area of base × height
$= (5 \times 5) \times$ ____
= ____ cm³

b



V = area of base × height
$= (30 \times$ ____$) \times$ ____
= ____ cm³

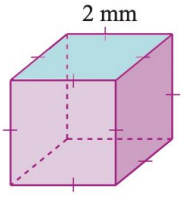
c



V = area of base × height
$= \frac{1}{2}(8 \times$ ____$) \times$ ____
= ____ mm³

6 Calculate the volumes of the following prisms.

a



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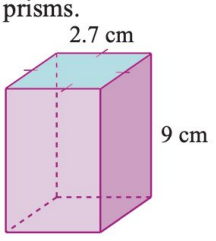

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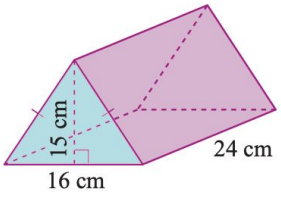

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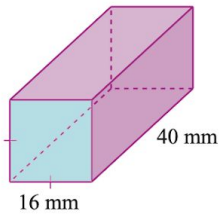
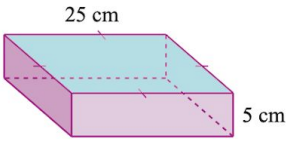


e



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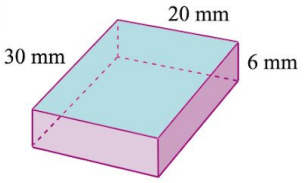

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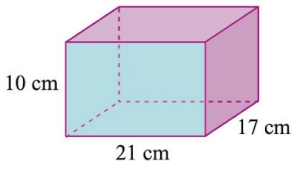

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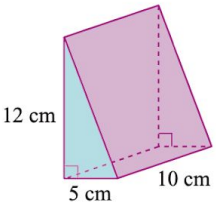
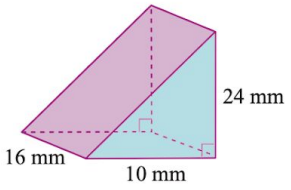


i



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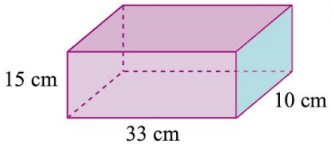

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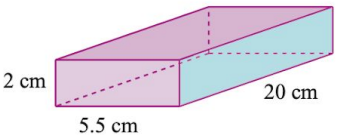

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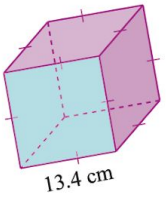
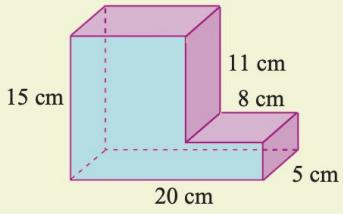
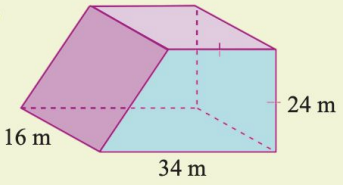


Calculate the volume of each **composite solid**.

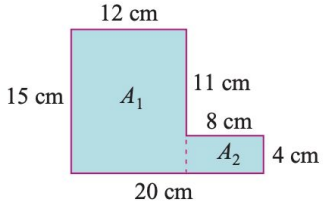
a

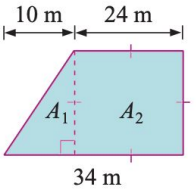


b



First draw the base of the solid. Use addition or subtraction to find any missing dimensions.





a Area of base $= A_1 + A_2$

$= (15 \times 12) + (8 \times 4)$

$= 180 + 32$

$= 212 \text{ cm}^2$

Height $= 5$ cm

Volume $=$ area of base $\times$ height

$= 212 \times 5$

$= 1060 \text{ cm}^3$

b Area of base $= A_1 + A_2$

$= \frac{1}{2}(10 \times 24) + (24 \times 24)$

$= 120 + 576$

$= 696 \text{ m}^2$

Height $= 16$ m

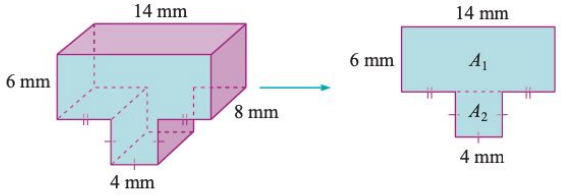
Volume $=$ area of base $\times$ height

$= 696 \times 16$

$= 11\,136 \text{ m}^3$

7 Complete the calculate the volumes of these composite shapes.

a

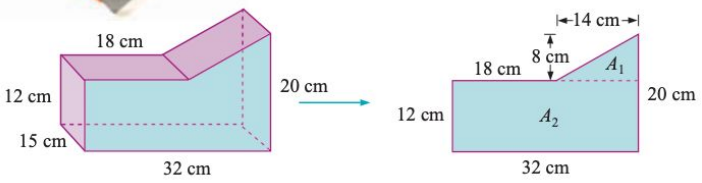


Area of base $= A_1 + A_2$
$= (14 \times \_\_\_) + (4 \times \_\_\_)$
$= \_\_\_ + \_\_\_ = \_\_\_ \text{ mm}^2$

Volume = area of base × height
$= \_\_\_ \times \_\_\_ = \_\_\_ \text{ mm}^3$

Height = ___ mm

b



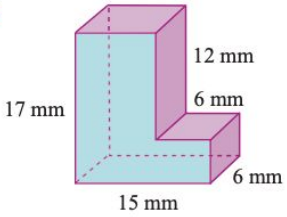
Area of base $= A_1 + A_2$
$= \frac{1}{2}(14 \times \_\_\_) + (32 \times \_\_\_)$
$= \_\_\_ + \_\_\_ = \_\_\_ \text{ cm}^2$

Volume = area of base × height
$= \_\_\_ \times \_\_\_ = \_\_\_ \text{ cm}^3$

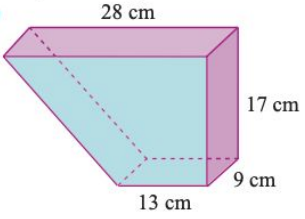
Height = ___ cm

8 Calculate the volumes of the following composite shapes.

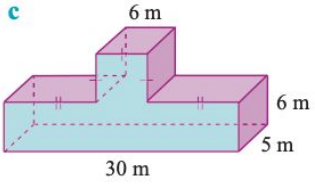
a



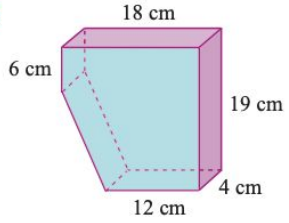
b



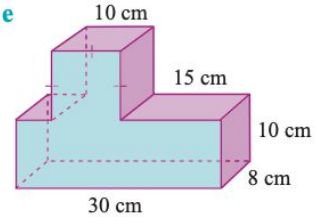
c



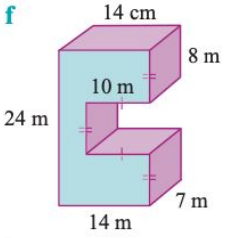
d



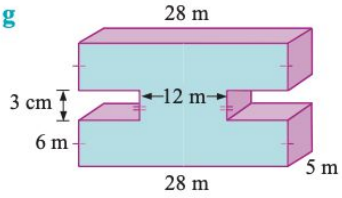
e



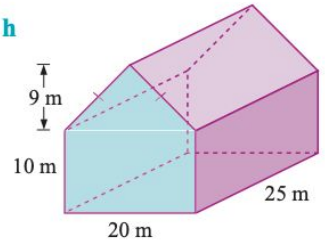
f



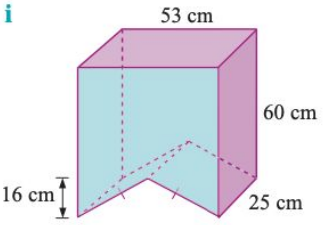
g



h



i



Investigation

Volume and capacity

1 *Step 1*: Take an empty milk carton and open the top so that it is in the shape of a rectangular prism.
Step 2: Measure the length and breadth of the base (bottom).
Step 3: Pour one litre (1 L) of water into the carton and measure the height of the water in the carton.
Step 4: Calculate the volume of water in cubic centimetres. How much space does 1 L of water occupy?

2 *Step 1:* Take a small box of fruit-drink that is in the shape of a rectangular prism.
Step 2: Measure the dimensions of the box, and calculate its volume in cubic centimetres.
Step 3: Compare this volume with the number of millilitres (mL) marked on the box (the **capacity** of the box). What do you notice?

3 Repeat question **2** for a 1 L box of fruit-drink or milk.

4 *Step 1*: Pour some water into a measuring cylinder and accurately note the level of the water in the cylinder in millilitres (mL).
Step 2: Drop 10 plastic centicubes into the cylinder. Record the new level of the water.
Step 3: Calculate how much water has been displaced by the 10 centicubes. This is the amount of water that occupies 10 cm^3. How much water would occupy 1 cm^3?

5 Repeat question **4** for 20 and 30 centicubes. How many centicubes would it take to displace:
a 100 mL of water? **b** 500 mL of water? **c** 1 L of water?

6 From the investigation, what conclusions can be drawn about the quantities 1 mL and 1 L and the volume that each occupies?

There are 1000 mL in 1 L. !

7 Complete these definitions of volume and capacity:
a _____ is the amount of space occupied by a solid or quantity of liquid.
b _____ is the volume of liquid that a container can hold.

Check your answers

1 **a** $V = 5 \times 4 \times 3 = 60\ \text{cm}^3$

b $V = 7 \times 3 \times 2 = 42\ \text{cm}^3$

2 **a** $20\ \text{m}^3$ **b** $360\ \text{m}^3$ **c** $180\ \text{mm}^3$

3 **a** $V = 190 \times 4 = 760\ \text{cm}^3$

b $V = 75 \times 7 = 525\ \text{cm}^3$

c $V = 30 \times 9 = 270\ \text{cm}^3$

4 **a** $225\ \text{cm}^3$ **b** $90\ \text{cm}^3$ **c** $64\ \text{cm}^3$

d $448\ \text{cm}^3$ **e** $760\ \text{cm}^3$ **f** $400\ \text{mm}^3$

g $930\ \text{m}^3$ **h** $1444.5\ \text{cm}^3$ **i** $982.8\ \text{cm}^3$

5 **a** $V = (5 \times 5) \times 5 = 125\ \text{cm}^3$

b $V = (30 \times 12) \times 7 = 2520\ \text{cm}^3$

c $V = \frac{1}{2}(8 \times 9) \times 12 = 432\ \text{mm}^3$

6 **a** $8\ \text{mm}^3$ **b** $65.61\ \text{cm}^3$ **c** $2880\ \text{cm}^3$

d $10\,240\ \text{mm}^3$ **e** $3125\ \text{cm}^3$ **f** $3600\ \text{mm}^3$

g $3570\ \text{cm}^3$ **h** $300\ \text{cm}^3$ **i** $1920\ \text{mm}^3$

j $4950\ \text{cm}^3$ **k** $220\ \text{cm}^3$

l $2406\ \text{cm}^3$ (rounded)

7 **a** Area of base $= (14 \times 6) + (4 \times 4)$

$= 84 + 16 = 100\ \text{mm}^2$

Height $= 8$ mm

Volume $= 100 \times 8 = 800\ \text{mm}^3$

b Area of base $= \frac{1}{2}(14 \times 8) + (32 \times 12)$

$= 56 + 384 = 440\ \text{cm}^2$

Height $= 15$ cm

Volume $= 440 \times 15 = 6600\ \text{cm}^3$

8 **a** $1098\ \text{mm}^3$ **b** $3136.5\ \text{cm}^3$ **c** $1080\ \text{m}^3$

d $1212\ \text{cm}^3$ **e** $3200\ \text{cm}^3$ **f** $1792\ \text{m}^3$

g $1860\ \text{m}^3$ **h** $7250\ \text{m}^3$ **i** $68\,900\ \text{cm}^3$