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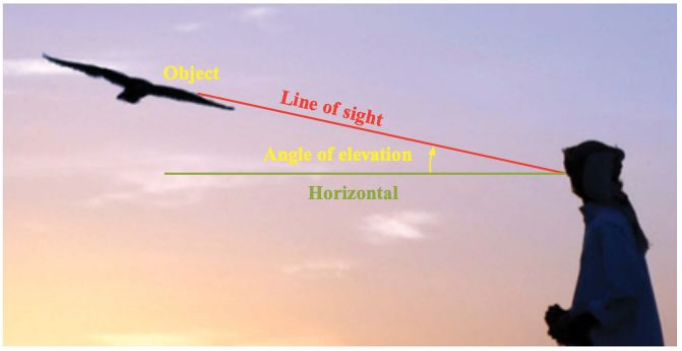
Worded problems using trigonometry

WALT apply angle of elevation and the angle of depression to solve trig problems
Success Criteria I know my trig ratios, I can draw a diagram and determine the sides and the ratio.

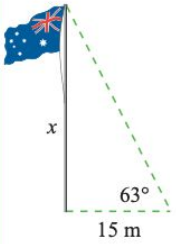
The **angle of elevation** of an object from an observer is the angle between the horizontal and the line of sight *up* to the object.



The **angle of depression** of an object from an observer is the angle between the horizontal and the line of sight *down* to the object.

EXAMPLE 1

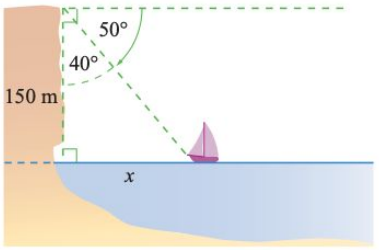
The angle of elevation of the top of a flagpole, as observed from a point 15 m from its base, is 63°. Draw a diagram and find the height of the flagpole.

Solve	Think	Apply
$\tan 63° = \frac{x}{15}$ $x = 15 \tan 63°$ ≈ 29.4 The flagpole is about 29 m high.		Elevation means looking upwards. The angle is at ground level.

EXAMPLE 2

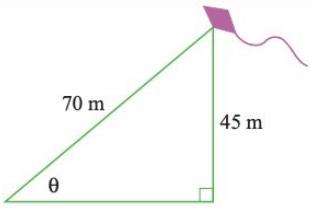
The angle of depression from the top of a vertical cliff, 150 m above sea level, to a boat below is 50°. Draw a diagram and find the distance of the boat from the base of the cliff.

We assume that the angle between the ground (or sea) and a building (or cliff) is always 90°.

Solve	Think	Apply
$\tan 40° = \frac{x}{150}$ $x = 150 \tan 40°$ ≈ 125.86 The boat is about 126 m out from the base of the cliff.		Depression means looking downwards. Either subtract from 90° to find the angle in the triangle or use parallel line properties to label the angle at the bottom as equal.

EXAMPLE 3

A kite is flying at a height of 45 m above the ground at the end of a string of length 70 m. Find, to the nearest minute, the angle of elevation from the ground to the string.

Solve	Think	Apply
$\sin\theta = \frac{45}{70}$ $\therefore \theta = 40.005…°$ $= 40°0'19''$ The angle of elevation is $40°0'$.	 45 ÷ 70 = SHIFT sin SHIFT DMS	Determine the sides required and select the correct ratio.

Draw a diagram for each of the following and find the unknown. For questions **1** to **5** give your answer to the nearest metre where necessary.

1 The angle of elevation of the top of a flagpole from the ground, as observed from a point 50 m from its base, is 38°. Find the height of the flagpole.

2 The angle of depression from the top of a cliff, 100 m above sea level, to a boat is 65°. Find the distance of the boat from the base of the cliff.

3 From a point 35 m from the base of a vertical cliff, the angle of elevation to the top of the cliff is 72°. Find the height of the cliff.

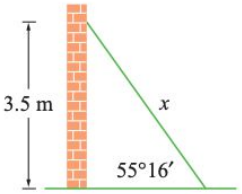
4 When looking down from the top of a building to a person standing in a park 150 m from the base of the building, the angle of depression is 28°. Find the height of the building.

5 The top of a tree, when viewed 40 m from the base of the tree, has an angle of elevation of 37°. Find the height of the tree.

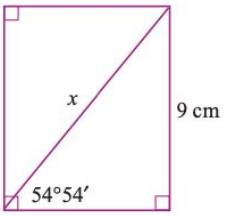
6 A person is standing 200 m from a vertical cliff 265 m high. Find the angle of elevation to the top of the cliff to the nearest minute.

EXAMPLE 4

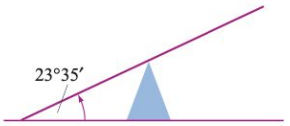
A ladder leaning against a vertical wall reaches 3.5 m up the wall and makes an angle of 55°16′ with the ground. Determine the length of the ladder.

Solve	Think	Apply
$\sin 55°16' = \frac{3.5}{x}$ $\therefore x \sin 55°16' = 3.5$ $x = \frac{3.5}{\sin 55°16'}$ $= 4.25\ldots$ ≈ 4.3 The ladder is 4.3 m long.	Use the opposite side and hypotenuse.  3.5 ÷ sin 55 DMS 16 DMS =	Identify the sides required and select the correct ratio.

7 A rectangle has a longer side of 9 cm. The angle between the diagonal and the shorter side is 54°54′. Find the length of the diagonal.



8 A seesaw is 6.3 m long. When one end is resting on the ground it makes an angle of 23°35′ with the ground. Find the height of the other end above ground level.



9 A rally driver travels 210 km on a bearing of 145°T. How far east of the starting position would the rally driver be now?

10 An isosceles triangle has height 13 cm and base 20 cm. Find the value of the base angles to the nearest minute.

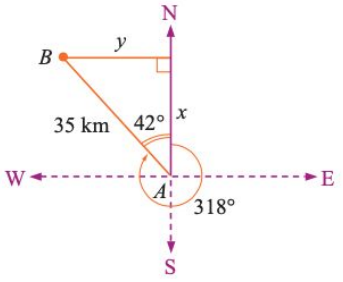
11 A right-angled triangle has non-hypotenuse sides of length 12 cm and 17 cm. Find the value of the other angles in degrees and minutes.

12 An isosceles triangle has a base of length 12 cm and a vertical angle of 70°. Find the lengths of the equal sides.

Extension

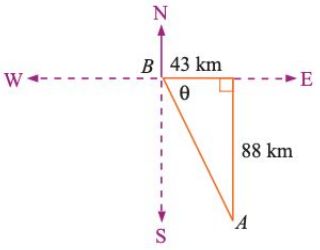
Example 5

A ship sails 35 km from a port A on a bearing of 318°T to a buoy B. Find how far the ship is north and west of A.

Solve	Think	Apply
$\cos 42° = \frac{\text{adjacent}}{\text{hypotenuse}}$ $= \frac{x}{35}$ $\therefore x = 35 \cos 42°$ ≈ 26.01 (2 decimal places) The ship is 26 km north of A. $\sin 42° = \frac{\text{opposite}}{\text{hypotenuse}}$ $= \frac{y}{35}$ $\therefore y = 35 \sin 42°$ ≈ 23.42 (2 decimal places) The ship is 23 km west of A.	$\angle NAB = 360° - 318° = 42°$ Let x be the distance north and y be the distance west. 	Always draw a diagram with north in the vertical direction of the page. Locate all the other bearings or distances.

EXAMPLE 6

Town A is 43 km east and 88 km south of town B. Find the bearing of A from B.

Solve	Think	Apply
$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$ $= \frac{88}{43}$ $\therefore \theta \approx 64°$ The bearing of A from B is $90 + 64 = 154°$T.		Draw a diagram showing all the information, then isolate the right-angled triangle.

13 A ship sails 58 km from a port A on a bearing of 262°T to a buoy B. Find how far the ship is west and south of A.

14 A ship sails 83 km from a port O on a bearing of 131° to another boat X. Find how far the ship is east and south of O.

15 A ship sails from a port P. It travels 55 km west then 30 km south to an atoll A. Find the bearing of A from P.

16 Town X is 185 km west and 260 km north of town Y.
- **a** Find the bearing of Y from X.
- **b** Find the bearing of X from Y.

17 A plane flies 800 km north and 1250 km west. Find the bearing and distance of the plane from its starting point.

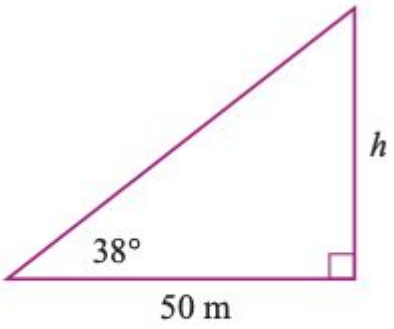
18 A is 40 km due north of B and C is 100 km due east of B. Find the distance and bearing of C from A.

Remember Pythagoras.

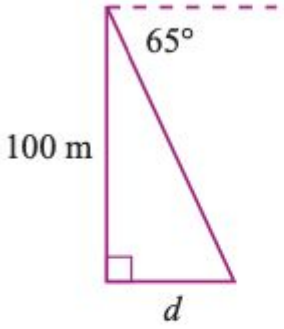
19 A kayaker paddles due west for 1.5 km, then turns due south and covers a further 800 m. How far and in what direction to the nearest degree must she travel to return to her starting point?

Check your answers

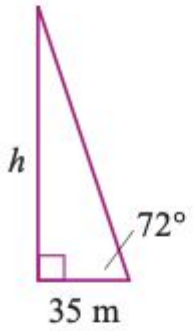
1 $h = 39$ m



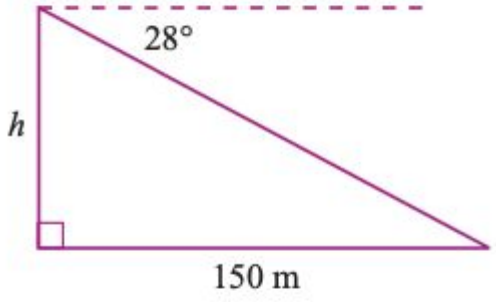
2 $d = 47$ m



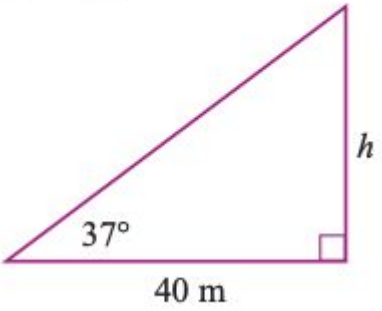
3 $h = 108$ m



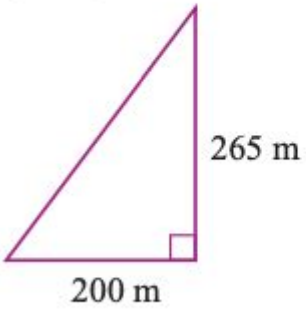
4 $h = 80$ m



5 $h = 30$ m



6 52°27′



7 11 cm
8 2.5 m
9 120 km
10 52°26′
11 35°13′ and 54°47′
12 10.5 cm
13 57 km W, 8 km S
14 63 km E, 54 km S
15 241°T
16 **a** 145°T **b** 325°T
17 302°37′T, 1484 km
18 111°48′T, 108 km
19 1.7 km, 062°T