Area and Perimeter

AREA AND PERIMETER

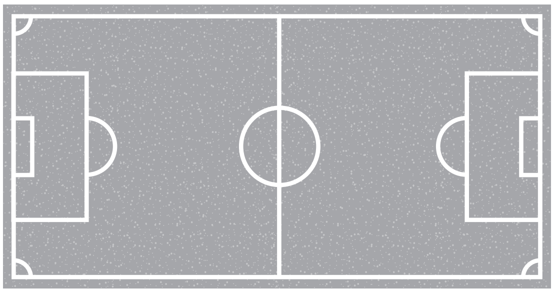
Series H

MATHLETICS

www.mathletics.co.nz

This booklet shows how to calculate the area and perimeter of common plane shapes.

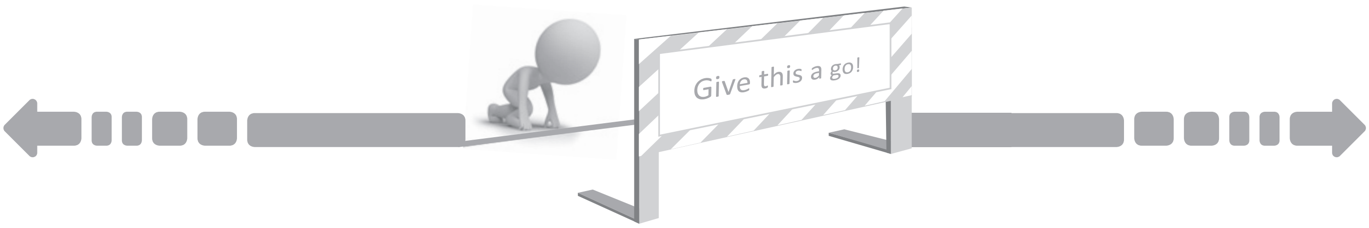
Football fields use rectangles, circles, quadrants and minor segments with specific areas and perimeters to mark out the playing field.



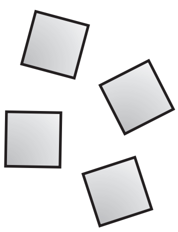
Write down the name of another sport that uses a playing field or court and list all the plane shapes used to create them below (include a small sketch to help you out):

Sport:

Shapes list:



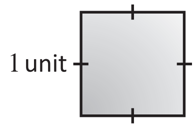
Q Use **all** four squares below to make **two** shapes in which the number of sides is also equal to four.
Compare the distance around the outside of your two shapes.
Write down what you discovered and whether or not it was different from what you expected.



Area using unit squares

Area is just the amount of flat space a shape has inside its edges or boundaries.

A unit square is a square with each side exactly one unit of measurement long.

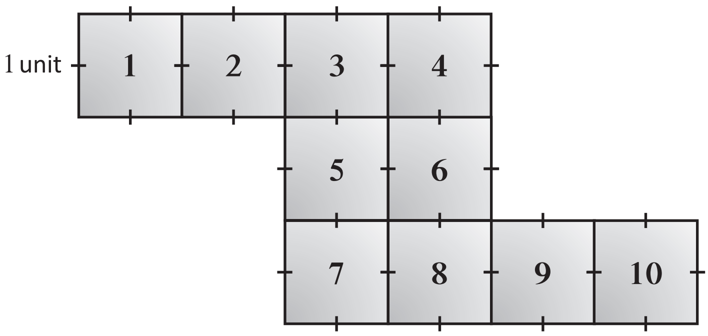


Little dashes on each side mean they are all the same length.

Area (A) $= 1$ square unit

$= 1$ unit^2 (in shorter, units form)

So the area of the shaded shape below is found by simply counting the number of unit squares that make it.



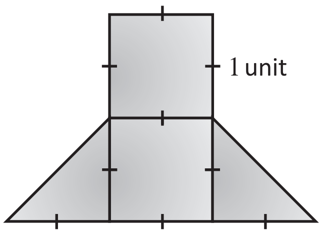
Area (A) $= 10$ square units

$= 10$ unit^2

Here are some examples including halves and quarters of unit squares:

Calculate the area of these shapes

(i)



Area (A) $=$ 2 whole square units + 2 half square units

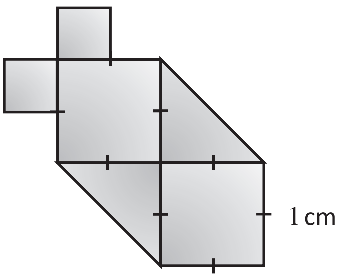
$=$ 2 square units + $2 \times \frac{1}{2}$ square units

$= (2 + 1)$ square units

$= 3$ units^2

When single units of measurement are given, they are used instead of the word 'units'.

(ii)



Area (A) $=$ 2 whole squares + 2 half squares + 2 quarter squares

$=$ 2 square cm + $2 \times \frac{1}{2}$ square cm + $2 \times \frac{1}{4}$ square cm

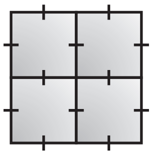
$= (2 + 1 + 0.5)$ square centimetres

$= 3.5$ cm^2

Area using unit squares

1 Calculate the area of all these shaded shapes:

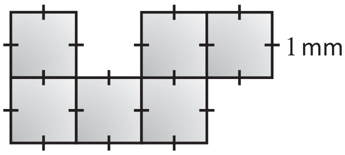
a



Area = ____ whole squares

= ____ units^2

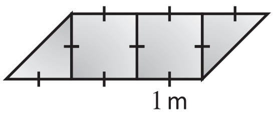
b



Area = ____ whole squares

= ____ mm^2

c

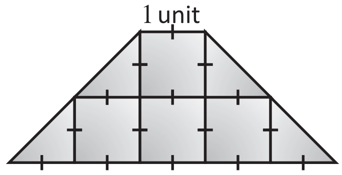


Area = ____ whole + ____ half squares

= ____ m^2 + ____ $\times \frac{1}{2}$ m^2

= ____ m^2

d

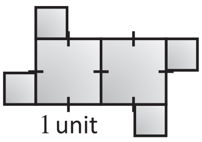


Area = ____ whole + ____ half squares

= ____ units^2 + ____ $\times \frac{1}{2}$ units^2

= ____ units^2

e

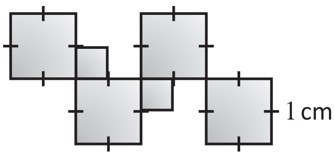


Area = ____ whole + ____ quarter squares

= ____ units^2 + ____ $\times \frac{1}{4}$ units^2

= ____ units^2

f

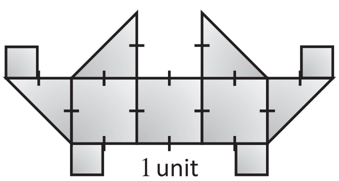


Area = ____ whole + ____ quarter squares

= ____ cm^2 + ____ $\times \frac{1}{4}$ cm^2

= ____ cm^2

g



Area = ____ whole + ____ half + ____ quarter squares

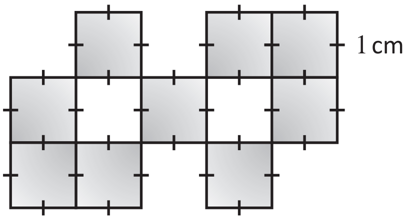
= ____ units^2 + ____ $\times \frac{1}{2}$ units^2 + ____ $\times \frac{1}{4}$ units^2

= ____ units^2

Area using unit squares

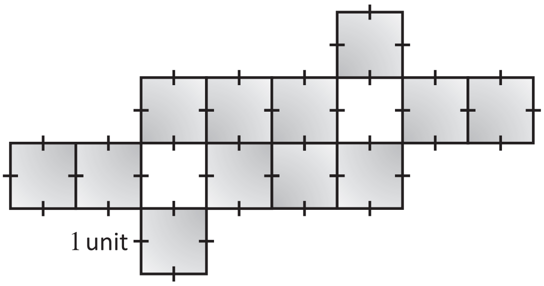
2 Calculate the area of these shaded shapes, using the correct short version for the units:

a



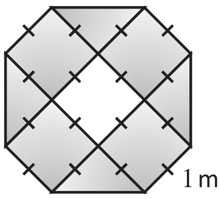
Area =

b



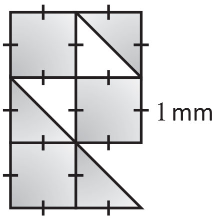
Area =

c



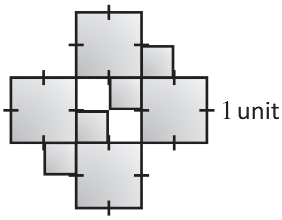
Area =

d



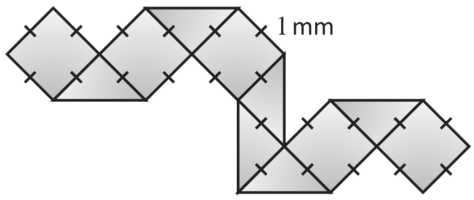
Area =

e



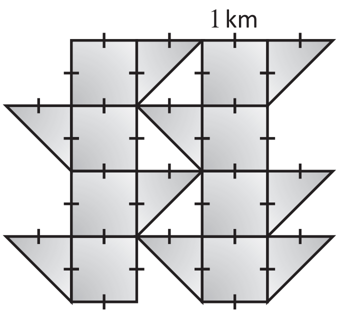
Area =

f



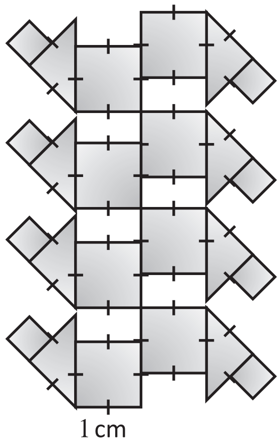
Area =

g



Area =

h

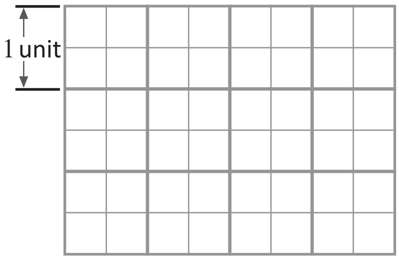


Area =

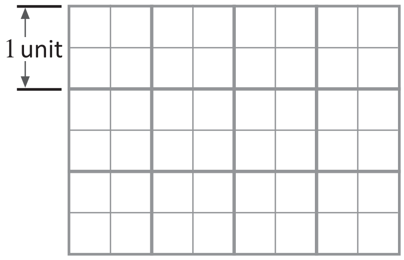
Area using unit squares

3. Shade shapes on these square grids to match the area written in square brackets.

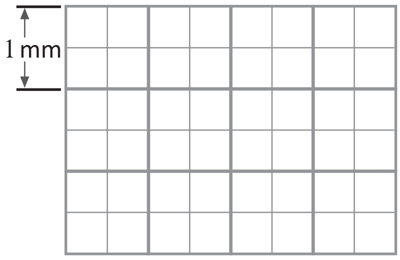
a. [$8\ \text{units}^2$], using whole squares only.



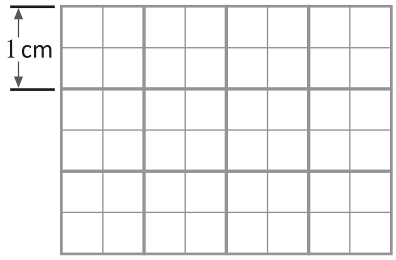
b. [$5\ \text{units}^2$], include half squares in your shape.



c. [$3\ \text{mm}^2$], include quarter squares in your shape.

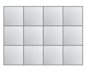
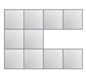
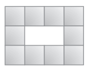


d. [$4.5\ \text{cm}^2$], include halves and quarters.

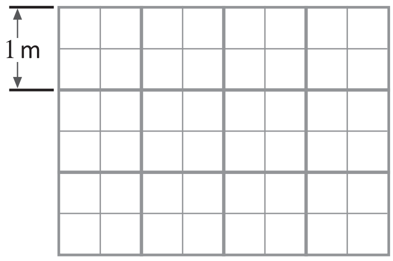


4. An artist has eight, $1\ \text{m}^2$, square-shaped panels which he can use to make a pattern.
The rules for the design are:

- the shape formed cannot have any gaps/holes.

i.e.  or  ✓  ✗

- it must fit entirely inside the display panel shown,
- all the eight panels must be used in each design.



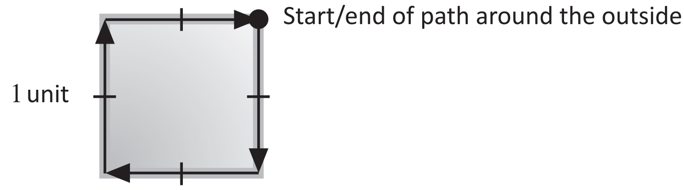
How many different designs can you come up with?
Sketch the main shapes to help you remember your count.

Number of different designs you found =

Perimeter using unit squares

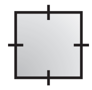
The word perimeter is a combination of two Greek words peri (around) and meter (measure).

So finding the perimeter (P) means measuring the distance around the **outside**!



$$\text{Perimeter}(P) = 1\,\text{unit} + 1\,\text{unit} + 1\,\text{unit} + 1\,\text{unit}$$
$$= 4 \times 1\,\text{unit}$$
$$= 4\,\text{units}$$

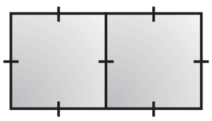
Remember, little dashes on each side mean they are all the same length.

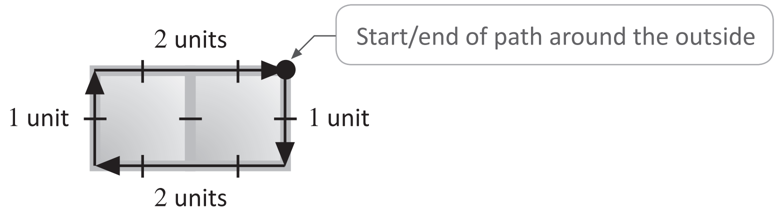


These examples shows that we only count all the outside edges.

Calculate the perimeter of these shapes formed using unit shapes

(i)



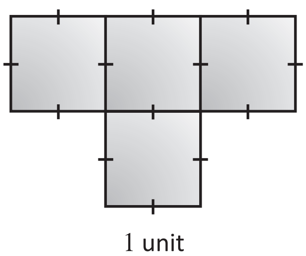


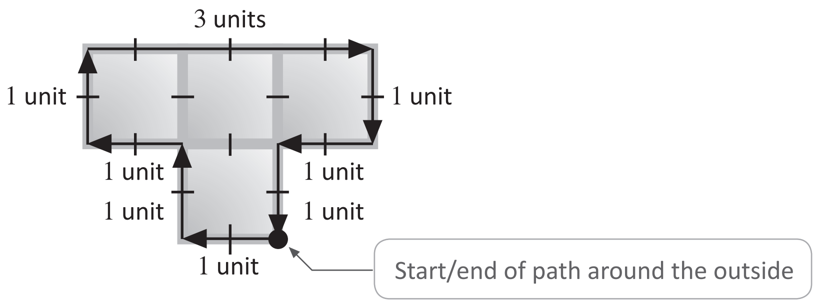
$$\text{Perimeter}(P) = 1 + 2 + 1 + 2\ \text{units}$$
$$= 6\,\text{units}$$

Sides of unit squares inside the shape not included

It does not matter where you start/finish, but it is usually easiest to start from one corner.

(ii)

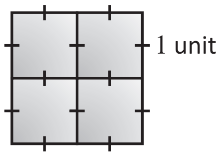




$$\text{Perimeter}(P) = 1 + 1 + 1 + 1 + 3 + 1 + 1 + 1\ \text{units}$$
$$= 10\,\text{units}$$

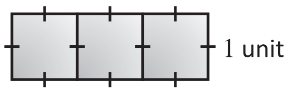
Perimeter using unit squares

1 Calculate the perimeter of these shaded shapes:

a 

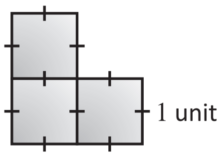
Perimeter = ☐ + ☐ + ☐ + ☐ units

= ☐ units

b 

Perimeter = ☐ + ☐ + ☐ + ☐ units

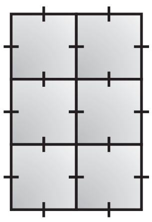
= ☐ units

c 

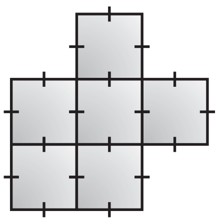
Perimeter = ☐ + ☐ + ☐ + ☐ + ☐ + ☐ units

= ☐ units

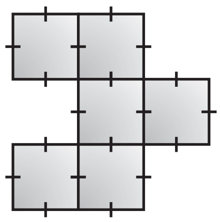
2 Write the length of the perimeter (*P*) for each of these shaded shapes:

a 

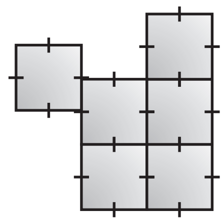
$P =$ ☐ units

b 

$P =$ ☐ units

c 

$P =$ ☐ units

d 

$P =$ ☐ units

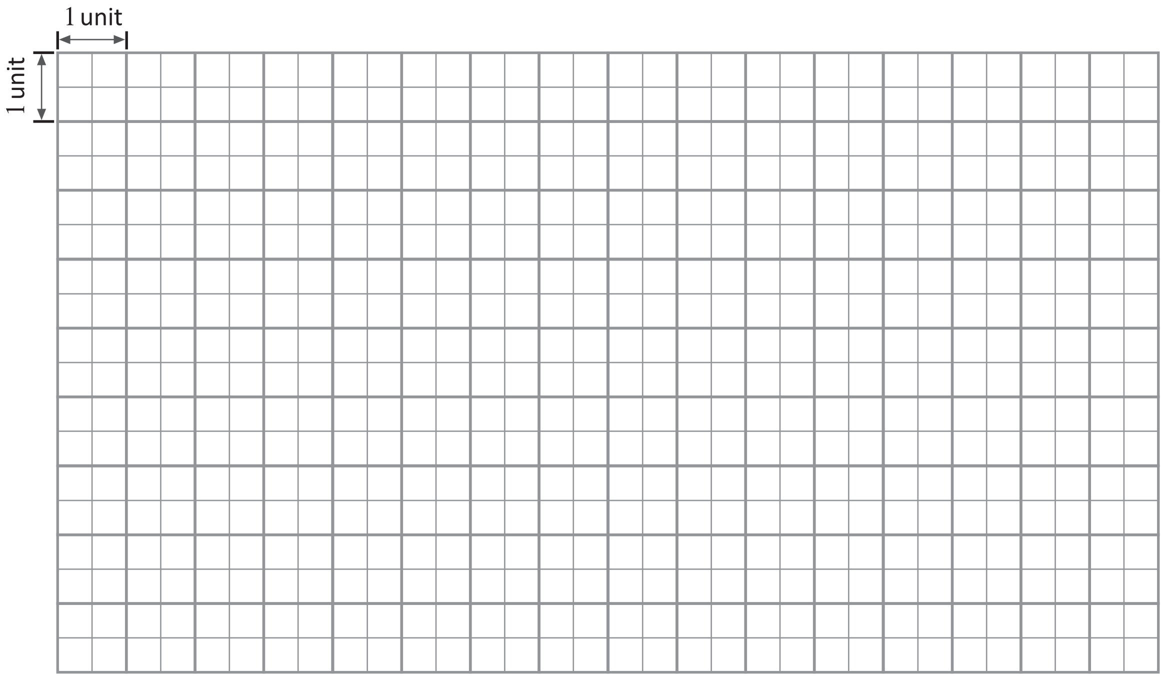
3 The shaded shapes in **2** all have the same area of 6 units^2.
Use your results in question 2 to help you explain briefly whether or not all shapes with the same area have the same perimeter.

Perimeter using unit squares

4 **a** Draw six patterns on the grid below which:

- all have an area of 5 units^2 and,
- have a different perimeter from each other.

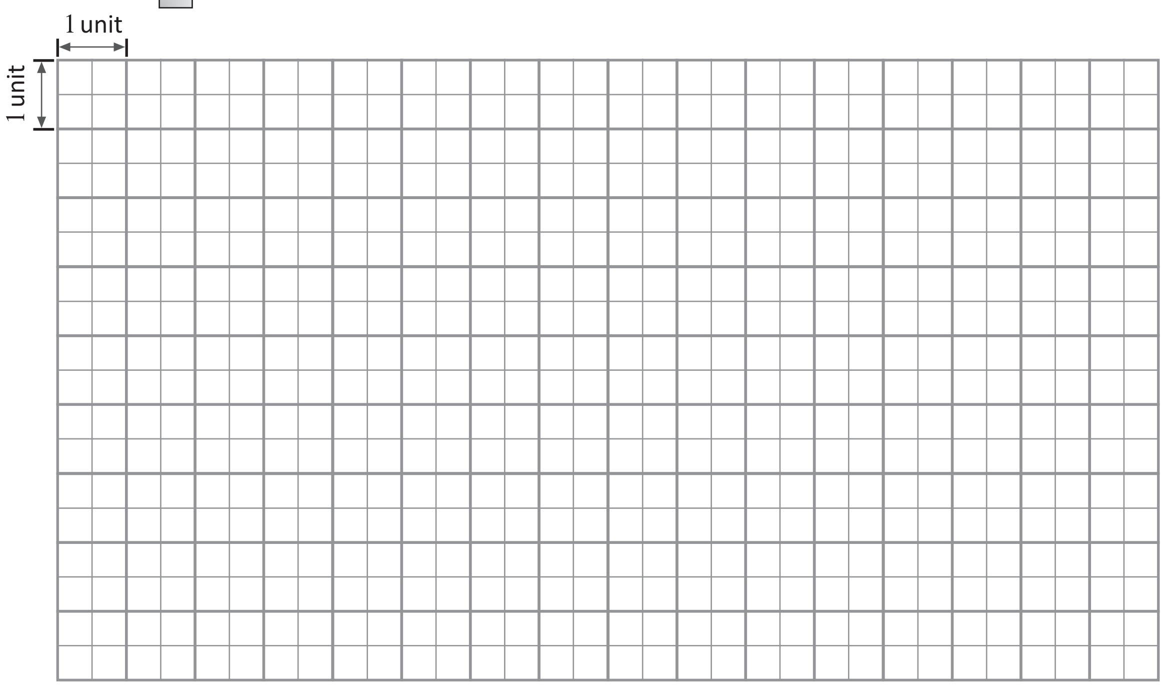
All squares used for each pattern must share at least one common side or corner point.



b Draw another five patterns on the grid below which:

- all have an area of 5 units^2 and,
- have a different perimeter than the shapes formed in part **a**.

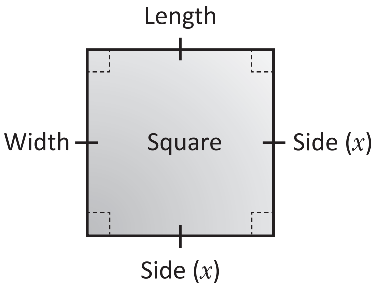
All squares used for each pattern must share at least **half** of a common side or a corner point.

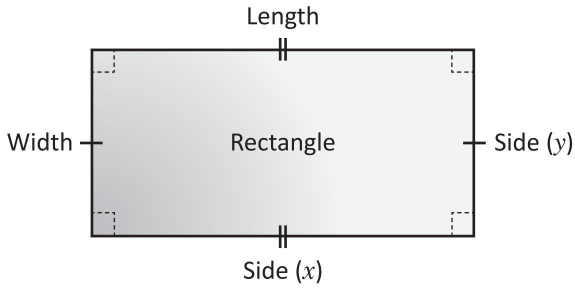


Area: Squares and rectangles

A simple multiplication will let you calculate the area of squares and rectangles.

For squares and rectangles, just multiply the length of the perpendicular sides (Length and width).





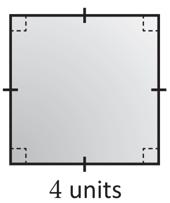
$$\begin{aligned}\text{Area} &= \text{length} \times \text{width}\\ &= \text{Side }(x)\text{ units} \times \text{Side}(x)\text{ units}\\ &= x \times x \text{ units}^2\\ &= x^2 \text{ units}^2\end{aligned}$$

$$\begin{aligned}\text{Area} &= \text{length} \times \text{width}\\ &= \text{Side }(x)\text{ units} \times \text{Side}(y)\text{ units}\\ &= x \times y \text{ units}^2\\ &= xy \text{ units}^2\end{aligned}$$

Here are some examples involving numerical lengths:

Calculate the area of these shaded shapes

(i)

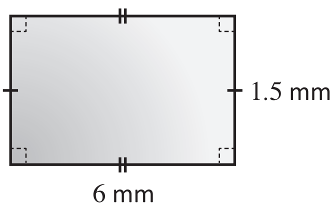


$$\begin{aligned}\text{Area} &= \text{length} \times \text{width}\\ &= 4 \text{ units} \times 4 \text{ units}\\ &= 4^2 \text{ units}^2\\ &= 16 \text{ units}^2\end{aligned}$$

So why units squared for area?

$$\begin{aligned}4 \text{ units} \times 4 \text{ units} &= 4 \times 4 \text{ units} \times \text{units}\\ &= 4^2 \times \text{units}^2\\ &= 16 \text{ units}^2\end{aligned}$$

(ii)

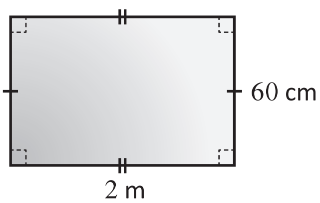


$$\begin{aligned}\text{Area} &= \text{length} \times \text{width}\\ &= 6 \text{ mm} \times 1.5 \text{ mm}\\ &= 9 \text{ mm}^2\end{aligned}$$

Units of area match units of side length

All measurements (or **dimensions**) must be written in the same units before calculating the area.

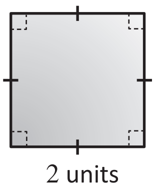
(iii)



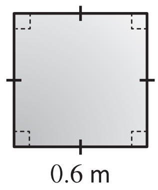
$$\begin{aligned}\text{Area} &= \text{length} \times \text{width}\\ &= 2 \text{ m} \times 60 \text{ cm}\\ &= 200 \text{ cm} \times 60 \text{ cm} && \text{Write both lengths using the same unit}\\ &= 12\,000 \text{ cm}^2 && \text{Units of area match units of side length}\end{aligned}$$

Area: Squares and rectangles

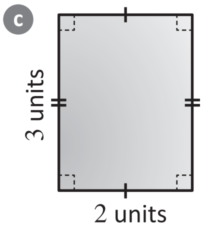
1. Calculate the area of these squares and rectangles, answering using the appropriate units.

a


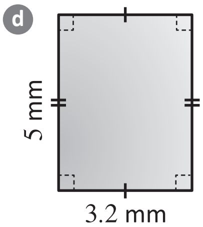
Area = ☐ × ☐ units^2
length width
= ☐ units^2

b


Area = ☐ × ☐ m^2
length width
= ☐ m^2

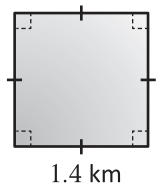
c


Area = ☐ × ☐ units^2
length width
= ☐ units^2

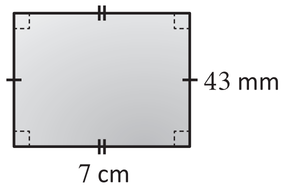
d


Area = ☐ × ☐ mm^2
length width
= ☐ mm^2

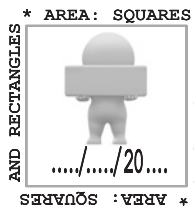
2. Calculate the area of these squares and rectangles. Round your answers to nearest whole square unit.

a


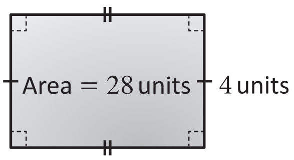
Area = ☐ × ☐ km^2
length width
= ☐ km^2
≈ ☐ km^2 (to nearest whole km^2)

b


Area = ☐ × ☐ cm^2
length width
= ☐ cm^2
≈ ☐ cm^2 (to nearest whole cm^2)



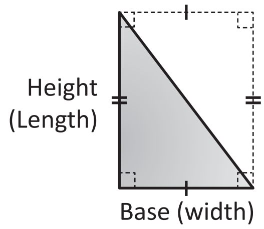
3. What is the length of this rectangle?



4. What are the dimensions of a square with an area of $121\,m^2$?
Psst: remember the opposite of squaring numbers is calculating the square root.

Area: Triangles

Look at this triangle drawn inside a rectangle.



The triangle is exactly half the size of the rectangle

$\therefore$ Area of the triangle = half the area of the rectangle units^2

$= \frac{1}{2}$ of width (base (b) for a triangle) $\times$ Length (height (h) for a triangle) units^2

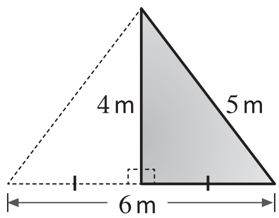
$= \frac{1}{2} \times b \times h$ units^2

This rule works to find the area for all triangles!

Here are some examples involving numerical dimensions:

Calculate the area of the shaded triangles below

(i)



Area $= \frac{1}{2} \times$ base $\times$ height

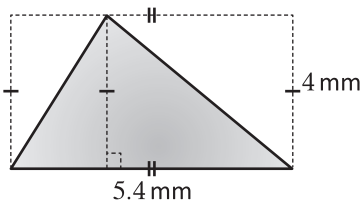
$= \frac{1}{2} \times 3\text{ m} \times 4\text{ m}$

$= 6\text{ m}^2$

Height = use the perpendicular height

The rule also works for this next triangle which is just the halves of two rectangles combined.

(ii)



Area $= \frac{1}{2} \times$ base $\times$ height

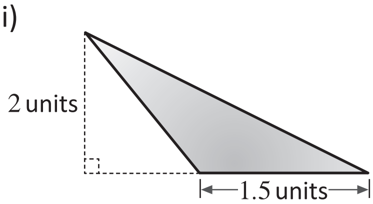
$= \frac{1}{2} \times 5.4\text{ mm} \times 4\text{ mm}$

$= 10.8\text{ mm}^2$

Here, we say the height is the perpendicular distance of the third vertex from the base.

For unusual triangles like this shaded one, we still multiply the base and the perpendicular height and halve it.

(iii)



Area $= \frac{1}{2} \times$ base $\times$ height

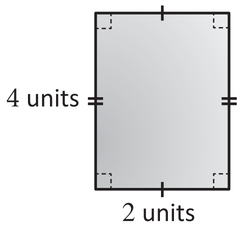
$= \frac{1}{2} \times 1.5\text{ units} \times 2\text{ units}$

$= 1.5\text{ units}^2$

Area: Triangles

1. Calculate the area of the triangle that cuts these two shapes in half.

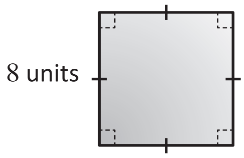
a



Area $= \frac{1}{2} \times \square \times \square$ units^2
(base) (height)

$= \square$ units^2

b

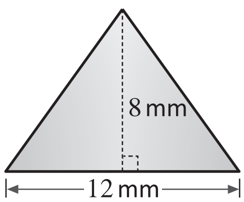


Area $= \frac{1}{2} \times \square \times \square$ units^2
(base) (height)

$= \square$ units^2

2. Calculate the area of these shaded triangles:

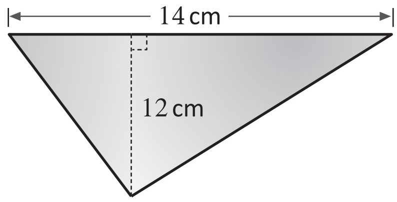
a



Area $= \square \times \square \times \square$ mm^2

$= \square$ mm^2

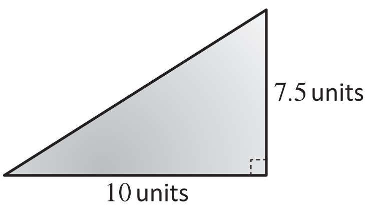
b



Area $= \square \times \square \times \square$ cm^2

$= \square$ cm^2

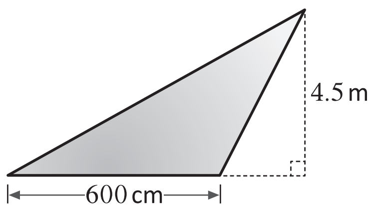
c



Area $= \square \times \square \times \square$ units^2

$= \square$ units^2

d

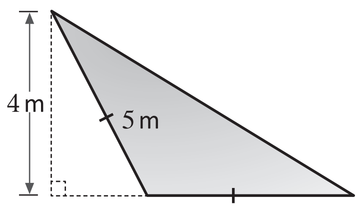


Area $= \square \times \square \times \square$ m^2

$= \square$ m^2

Remember: same units needed.

e

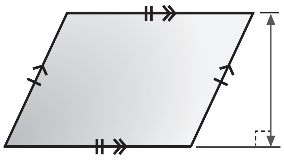


Area $= \square \times \square \times \square$ m^2

$= \square$ m^2

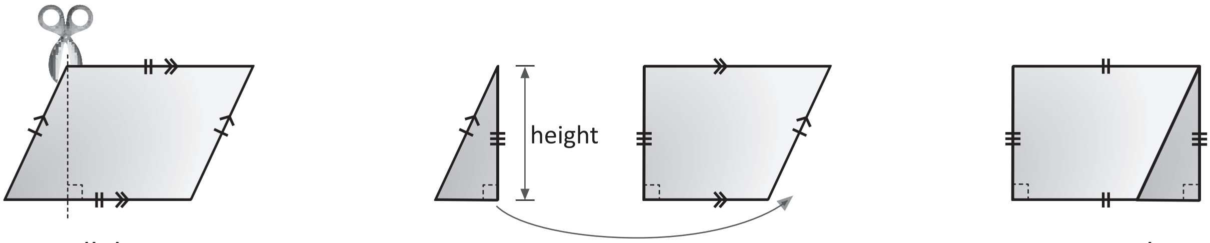
Area: Parallelograms

Parallelograms have opposite sides equal in length and parallel (always the same distance apart).



The shortest distance between a pair of parallel sides is called the perpendicular height

We can make them look like a rectangle by cutting the triangle off one end and moving it to the other.



Parallelogram | Rectangle

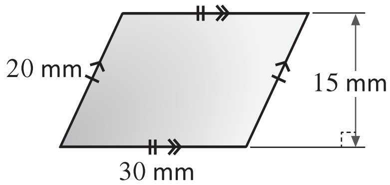
$\therefore$ Area of a parallelogram $=$ Area of the rectangle formed after moving triangle

$= \text{length} \times \text{perpendicular height units}^2$

$= l \times h \text{ units}^2$

Calculate the area of these parallelograms

(i)

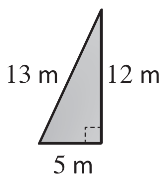


$\text{Area} = \text{length} \times \text{height}$

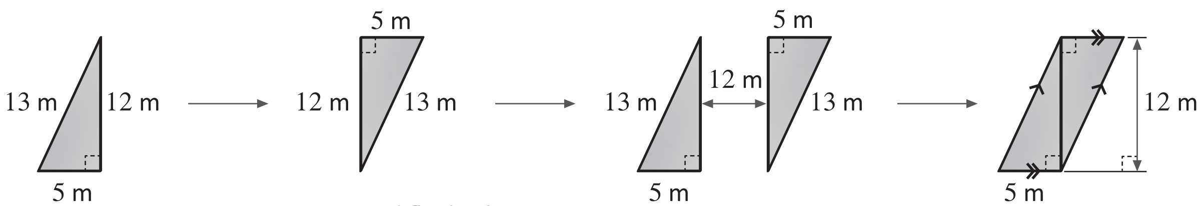
$= 30\text{ mm} \times 15\text{ mm}$

$= 450\text{ mm}^2$

A parallelogram can also be formed joining together two identical triangles.



(ii) Find the area of the parallelogram formed using two of these right angled triangles:



Copy and flip both vertically and horizontally | Bring them together | Parallelogram

$\text{Area} = 2 \times \text{area of the triangle}$

$= 2 \times \frac{1}{2} \times 5\text{ m} \times 12\text{ m}$

$= 60\text{ m}^2$

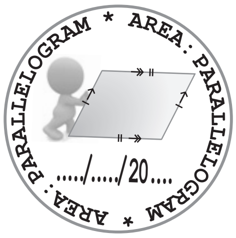
OR

$\text{Area} = \text{length} \times \text{perpendicular height}$

$= 5\text{ m} \times 12\text{ m}$

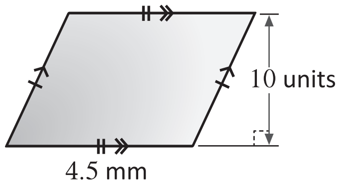
$= 60\text{ m}^2$

Area: Parallelogram



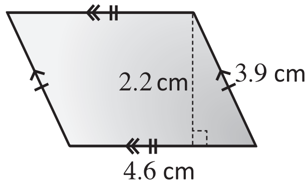
1 Complete the area calculations for these parallelograms:

a



Area = ☐ × ☐ units^2
length height
= ☐ units^2

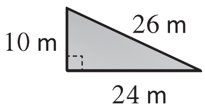
b



Area = ☐ × ☐ cm^2
length height
= ☐ cm^2

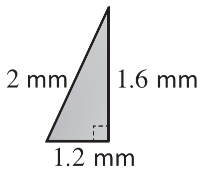
2 Calculate the area of the parallelograms formed using these triangles.

a



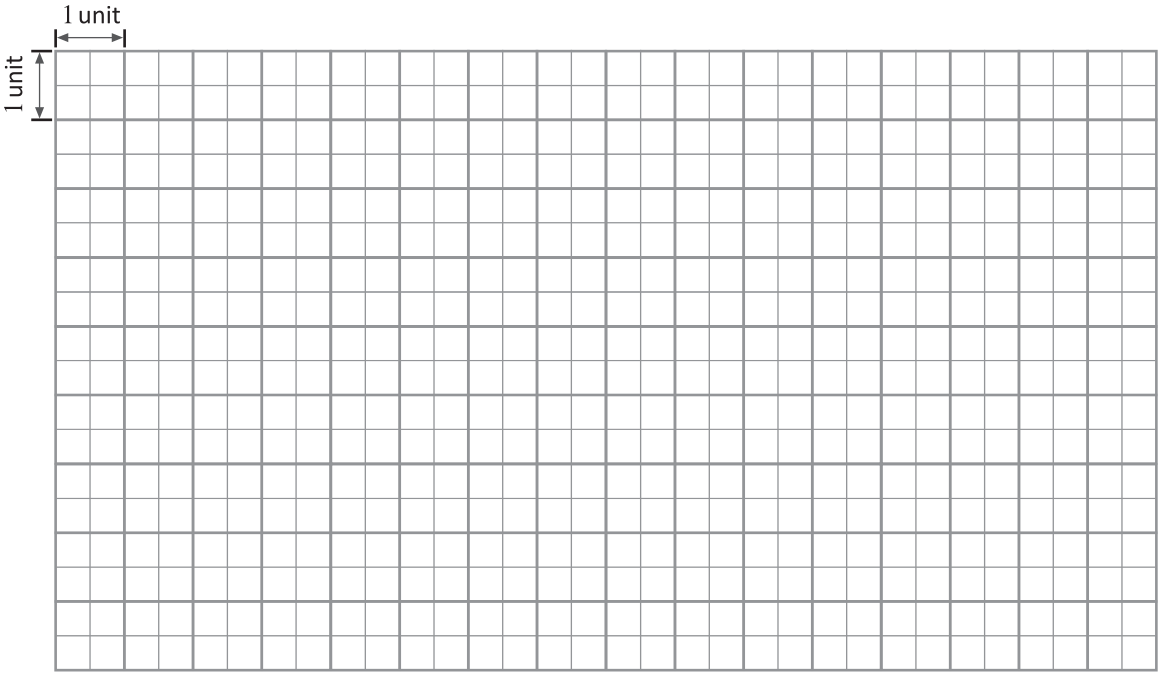
Area = ☐ m^2

b



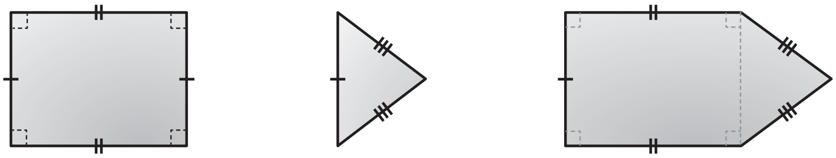
Area = ☐ mm^2

3 Fill the grid below with as many **different** parallelograms as you can which have an area of 4 units^2.



Area of composite shapes

When common shapes are put together, the new shape made is called a composite shape.



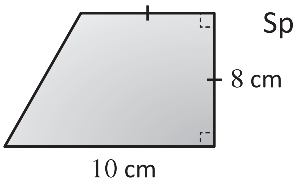
Common shape (Rectangle) + Common shape (Isosceles triangle) = Composite shape (Rectangle + Isosceles triangle)

Composite just means it is made by putting together separate parts

Just calculate the area of each shape separately then add (or subtract) to find the total composite area.

Calculate the area of these composite shapes

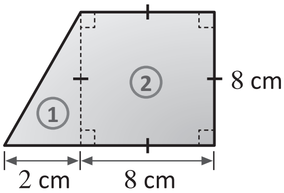
(i)



Split into a triangle ① and a square ②.

Area ① $= \frac{1}{2} \times 2\,\text{cm} \times 8\,\text{cm} = 8\,\text{cm}^2$

Area ② $= 8\,\text{cm} \times 8\,\text{cm} = 64\,\text{cm}^2$



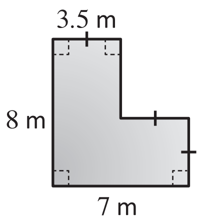
$\therefore$ Total area = Area ① + Area ②

$= 8\,\text{cm}^2 + 64\,\text{cm}^2$ Add area ① and ② for the composite area

$= 72\,\text{cm}^2$

This next one shows how you can use addition or subtraction to calculate the area of composite shapes.

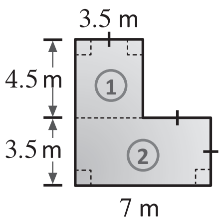
(ii)



- **method 1**: Split into two rectangles ① and ②

Area ① $= 4.5\,\text{m} \times 3.5\,\text{m} = 15.75\,\text{m}^2$

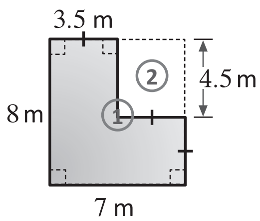
Area ② $= 3.5\,\text{m} \times 7\,\text{cm} = 24.5\,\text{m}^2$



$\therefore$ Total area $= 15.75\,\text{m}^2 + 24.5\,\text{m}^2$ Add area ① and area ② together

$= 40.25\,\text{m}^2$

- **method 2**: Large rectangle ① minus the small 'cut out' rectangle ②



Area ① $= 8\,\text{m} \times 7\,\text{m} = 56\,\text{m}^2$

Area ② $= 3.5\,\text{m} \times 4.5\,\text{m} = 15.75\,\text{m}^2$

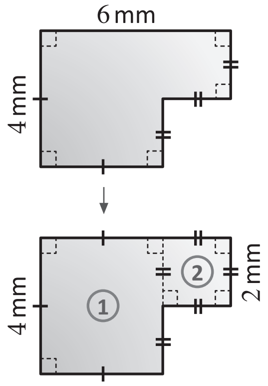
$\therefore$ Total area $= 56\,\text{m}^2 - 15.75\,\text{m}^2$ Subtract area ② from area ①

$= 40.25\,\text{m}^2$

Area of composite shapes

1 Complete the area calculations for these shaded shapes:

a

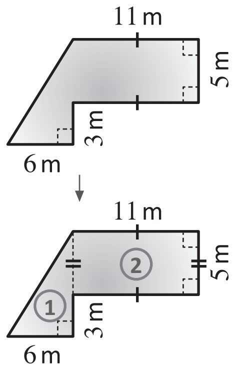


Area ① = ____ mm × ____ mm = ____ mm^2

Area ② = ____ mm × ____ mm = ____ mm^2

∴ Composite area = ① + ② mm^2 = ____ mm^2

b

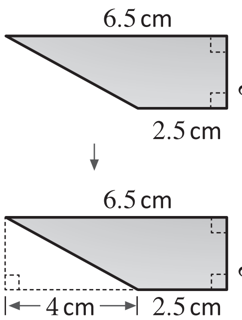


Area ① = ____ × ____ × ____ m^2 = ____ m^2

Area ② = ____ × ____ m^2 = ____ m^2

∴ Composite area = ① + ② m^2 = ____ m^2

c

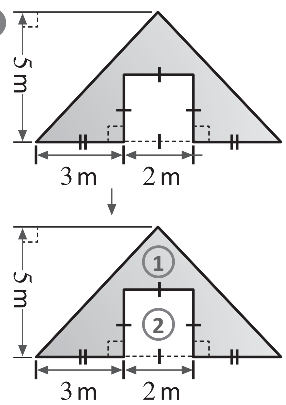


Area ① = ____ × ____ cm^2 = ____ cm^2

Area ② = ____ × ____ × ____ cm^2 = ____ cm^2

∴ Composite area = ① − ② cm^2 = ____ cm^2

d

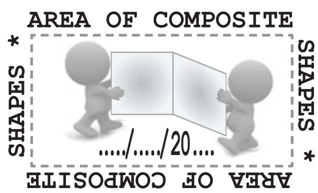


Area ① = ____ × ____ × ____ m^2 = ____ m^2

Area ② = ____ × ____ m^2 = ____ m^2

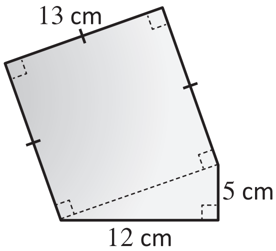
∴ Composite area = ① − ② m^2 = ____ m^2

Area of composite shapes



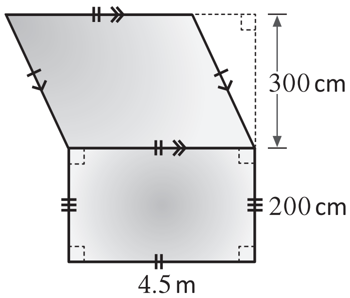
2 Calculate the area of these composite shapes, showing all working:

a



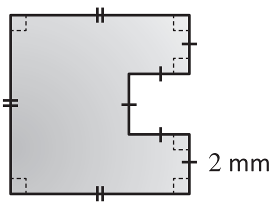
Area = ... cm^2

b psst: change all the units to metres first.



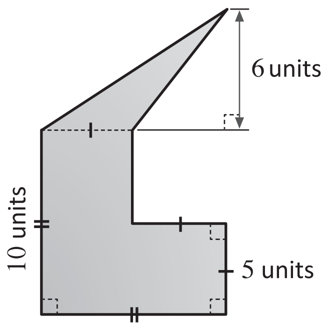
Area = ... m^2

c



Area = ... mm^2

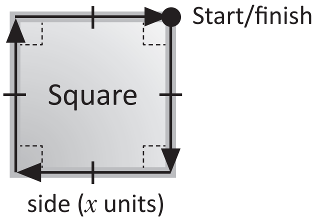
d psst: this one needs three area calculations



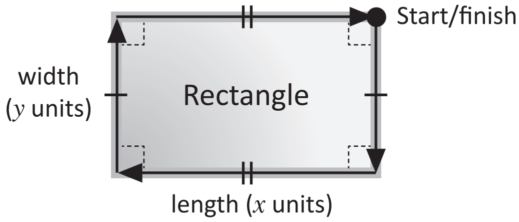
Area = ... $units^2$

Perimeter of simple shapes

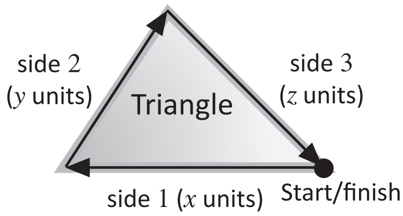
By adding together the lengths of each side, the perimeter of all common shapes can be found.



$P = 4 \times$ side length
$= 4 \times x$ units
$= 4x$ units



$P =$ width + length + width + length
$= (y + x + y + x)$ units
$= (2 \times x) + (2 \times y)$ units
$= 2x + 2y$ units



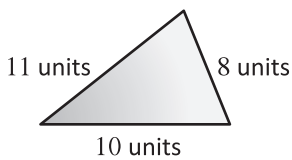
$P =$ side 1 + side 2 + side 3
$= x + y + z$ units

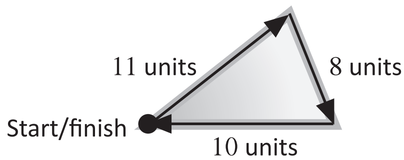
You can start/end at any vertex of the shape

Here are some examples involving numerical dimensions:

Calculate the perimeter of these common shapes

(i)

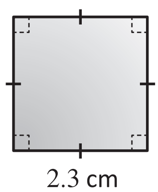


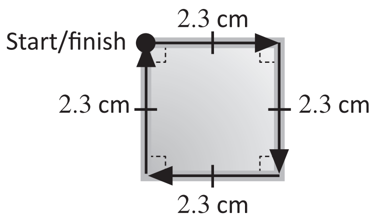


Sum of all the side lengths

Perimeter $= 11$ units $+ 8$ units $+ 10$ units
$= 29$ units

(ii)



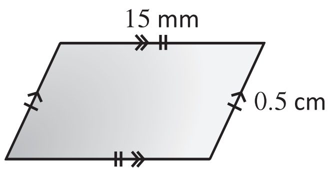


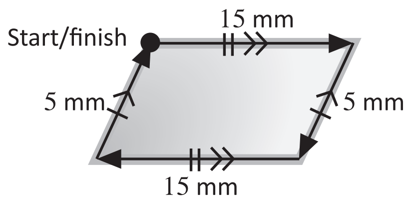
Four lots of the same side length

Perimeter $= 4 \times 2.3$ cm
$= 9.2$ cm

All measurements must be in the same units before calculating perimeter.

(iii) The perimeter for parallelograms is done the same as for rectangles. Calculate this perimeter in mm.



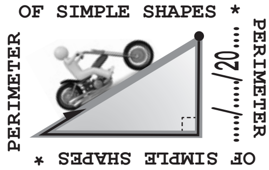


All side lengths in mm

Perimeter $= 2 \times 15$ mm $+ 2 \times 5$ mm
$= 30$ mm $+ 10$ mm
$= 40$ mm

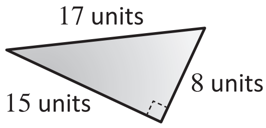
Opposite sides in pairs

Perimeter of simple shapes



1 Complete the perimeter calculations for these shapes:

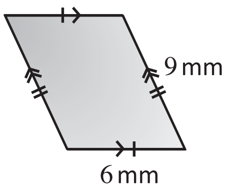
a



Perimeter = ______ units + ______ units + ______ units

= ______ units

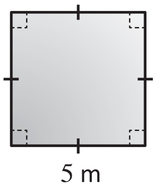
b



Perimeter = 2 × ______ mm + 2 × ______ mm

= ______ mm

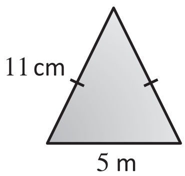
c



Perimeter = ______ × ______ m

= ______ m

d

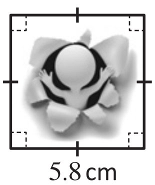


Perimeter = 2 × ______ cm + ______ cm

= ______ cm

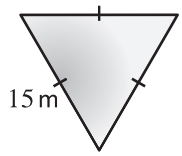
2 Calculate the perimeter of the shapes below, using the space to show all working:

a



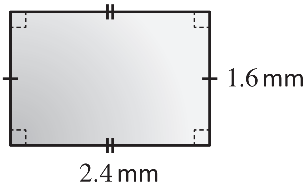
Perimeter = ______ cm

b



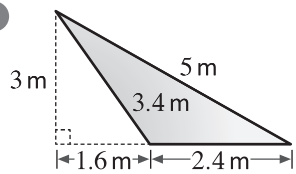
Perimeter = ______ m

c



Perimeter = ______ mm

d

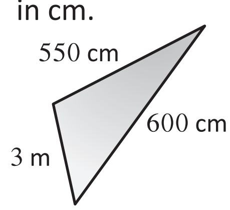


Perimeter = ______ m

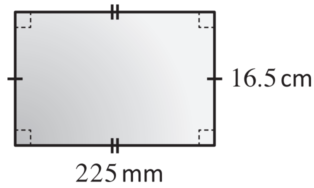
Perimeter of simple shapes

3 Find the perimeter of each shape written using the smaller units of measurement in each diagram.

a in cm.



b in mm.

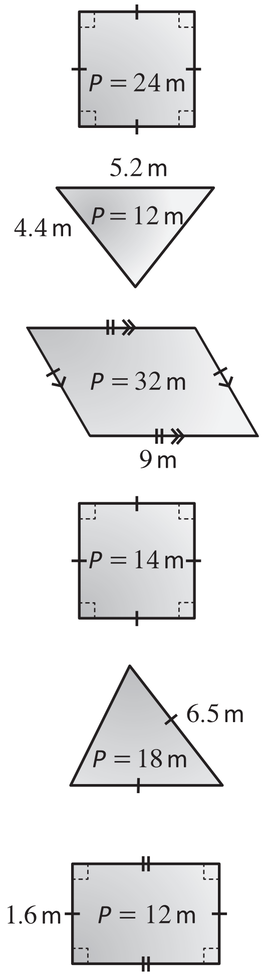


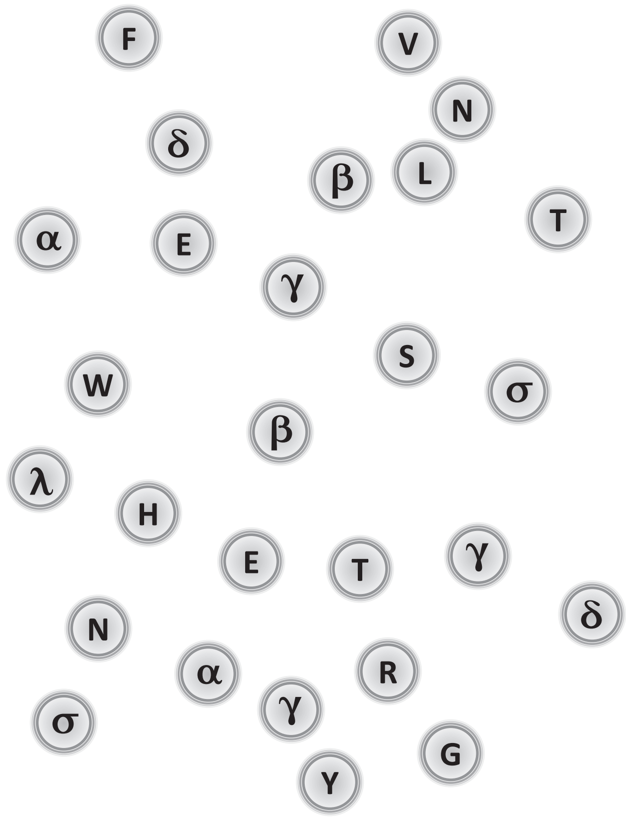
Perimeter = ☐ cm

Perimeter = ☐ mm

4 Each shape below has its perimeter written inside and is missing one of the side length values.
Rule a straight line between each shape and the correct missing side length on the right to answer:

How many straight sides does an icosagon have?



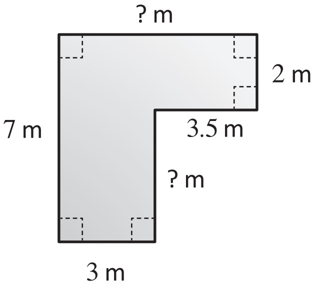


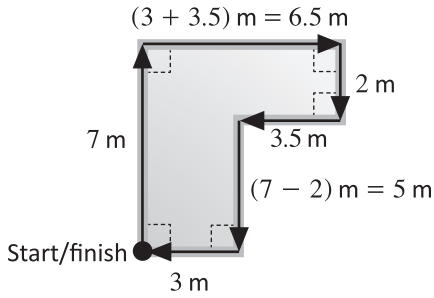
- 8 m
- 2.4 m
- 3.5 m
- 380 cm
- 440 cm
- 2 m
- 7 m
- 650 cm
- 1.1 m
- 6 m
- 5 m

☐ α ☐ β ☐ γ ☐ δ ☐ λ ☐ σ

Perimeter of composite shapes

The lengths of the unlabelled sides must be found in composite shapes before calculating their perimeter.



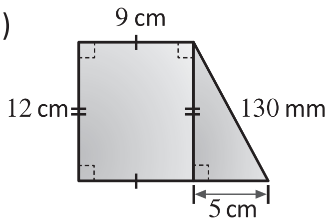


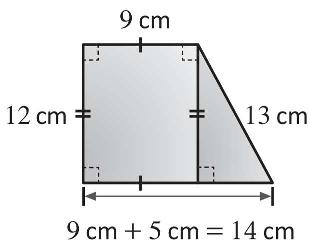
$$\therefore \text{ Perimeter} = 7\text{ m} + 6.5\text{ m} + 2\text{ m} + 3.5\text{ m} + 5\text{ m} + 3\text{ m}$$
$$= 27\text{ m}$$

Here are some more examples.

Calculate the perimeter of these composite shapes

(i)

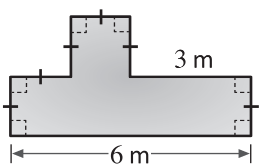




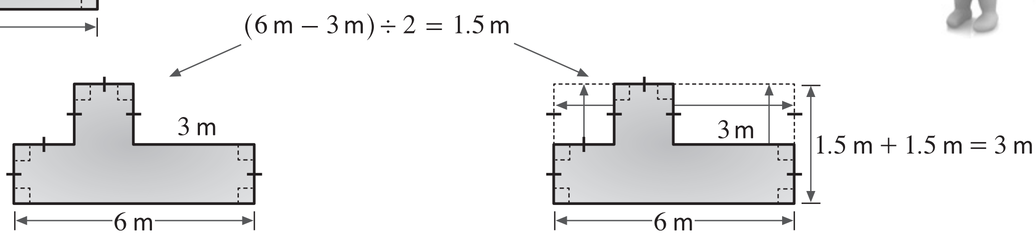
Calculate each side length of the shape in the same units

$$\therefore \text{Perimeter} = 9\text{ cm} + 13\text{ cm} + 14\text{ cm} + 12\text{ cm}$$
$$= 48\text{ cm}$$

(ii)



You can also imagine the sides re-positioned to make the calculation easier



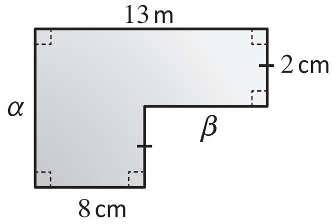
$$\therefore \text{Perimeter} = 6 \times 1.5\text{ m} + 3\text{ m} + 6\text{ m}$$
$$= 18\text{ m}$$

$$\therefore \text{Perimeter} = 2 \times 6\text{ m} + 2 \times 3\text{ m}$$
$$= 18\text{ m}$$

Perimeter of composite shapes

1 Calculate the value of the sides labelled α and β in each of these composite shapes:

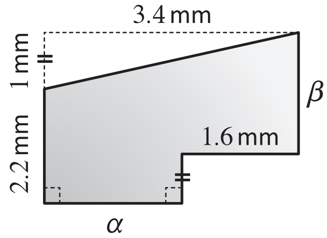
a



$\alpha =$ ______ cm

$\beta =$ ______ cm

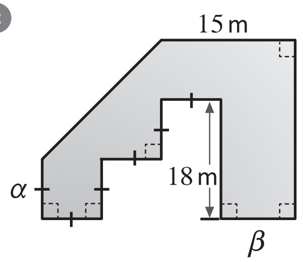
b



$\alpha =$ ______ mm

$\beta =$ ______ mm

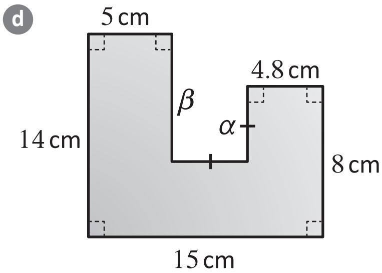
c



$\alpha =$ ______ m

$\beta =$ ______ m

d

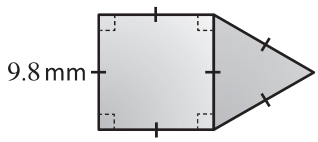


$\alpha =$ ______ cm

$\beta =$ ______ cm

2 Calculate the perimeter of these composite shapes:

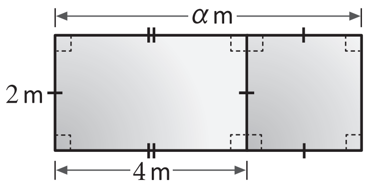
a



Perimeter = ______ $\times$ ______ cm

= ______ cm

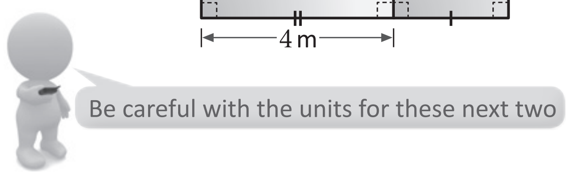
b



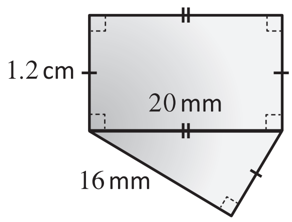
Perimeter = $2 \times \alpha$ m + $2 \times$ ______ m

= ______ m + ______ m

= ______ m



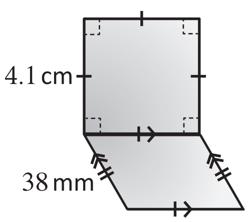
c



Perimeter = $3 \times$ ______ mm + ______ mm + ______ mm

= ______ mm

d



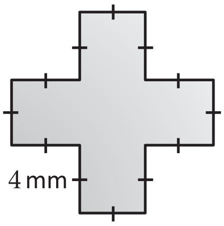
Perimeter = ______ $\times$ 4.1 cm + ______ $\times$ ______ cm

= ______ cm

Perimeter of composite shapes

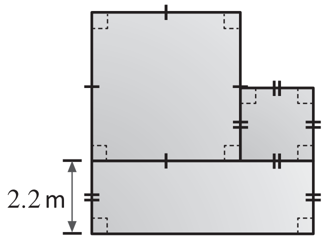
3 Calculate the perimeter of these composite shapes in the units given in square brackets.
Show all working.

a [mm]



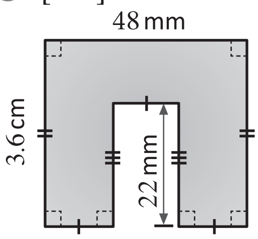
Perimeter = ____ mm

b [m]



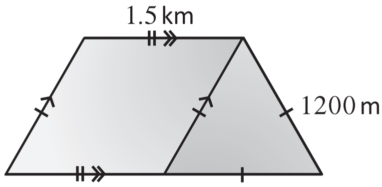
Perimeter = ____ m

c [cm]



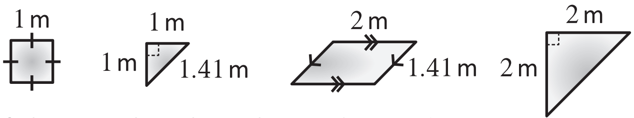
Perimeter = ____ cm

d [km] psst: 1 km = 1000 m

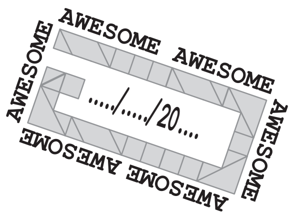


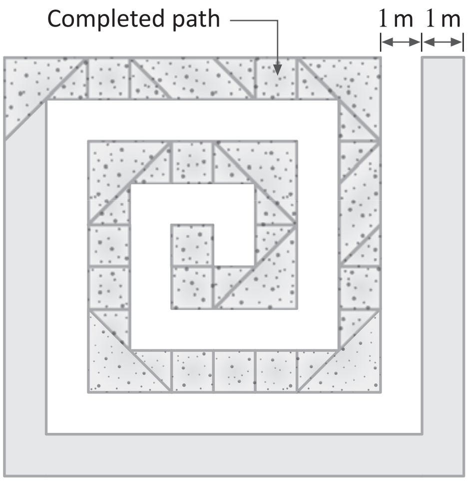
Perimeter = ____ km

4 Earn an awesome passport stamp for this one!
The incomplete geometric path shown below is being constructed using a combination of the following shaped pavers:



The gap in between each part of the spiral path is always 1 m wide.
Calculate what the total perimeter of this path will be when finished.

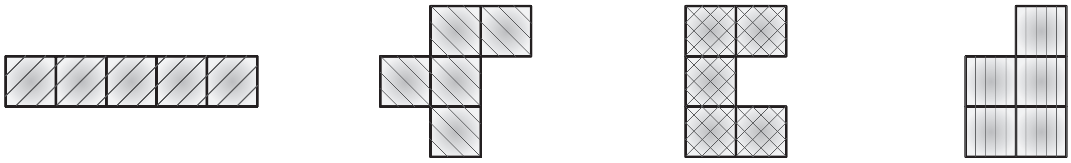




Total perimeter of completed path = ____ m

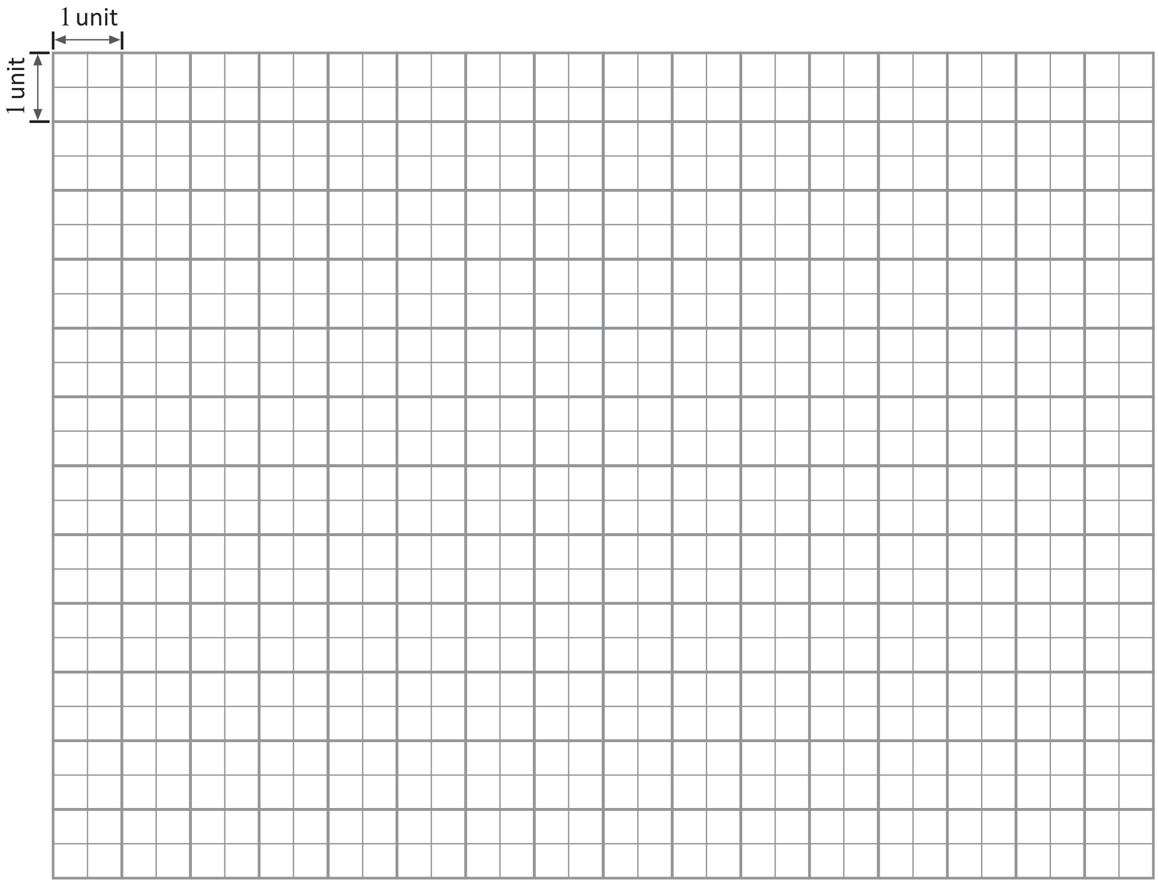
Perimeter of composite shapes

5 The four composite shapes below have been formed using five, unit squares.



a Using your knowledge of perimeter and the grid below, combine all four pieces to create two different shapes so that:
- One shape has the smallest possible perimeter.
- The other has the largest possible perimeter.

All shapes must be connected by **at least** one whole side of a unit square.



b Briefly describe the strategy you used to achieve each outcome below:

- A shape with the smallest possible perimeter.

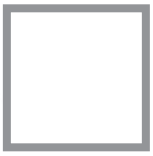
- A shape with the largest possible perimeter.

Simple word problems involving area and perimeter

Sometimes we can only communicate ideas or problems through words.

So it is important to be able to take written/spoken information and turn it into something useful.

For example,
Miguel wants to paint a square. He has just enough paint to create a line 240 cm long.
What is the longest length each side of the square can be if he wishes to use all of the paint?

To use up all the paint, the total perimeter of the square must equal 240 cm.

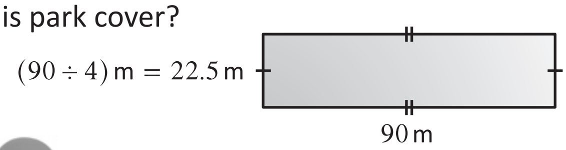
$$\text{So each side length} = 240\text{ cm} \div 4$$
$$= 60\text{ cm}$$

$\therefore$ The longest length each side of the square painted by Miguel can be is 60 cm.

This is useful for Miguel to know because if he painted the first side too long, he would run out of paint!

Here are some more examples

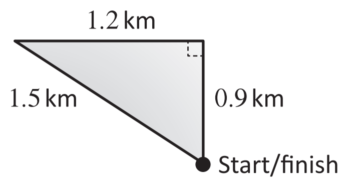
(i) A rectangular park is four times longer than it is wide. If the park is 90 m long, how much area does this park cover?



Draw diagram to illustrate problem

$$\text{Area} = \text{length} \times \text{width}$$
$$= 90\text{ m} \times 22.5\text{ m}$$
$$= 2025\text{ m}^2$$

(ii) At a fun run, competitors run straight for 0.9 km before turning left 90 degrees to run straight for a further 1.2 km. The course has one final corner which leads back to the start along a straight 1.5 km long street. How many laps of this course do competitors complete if they run a total of 18 km?



Draw diagram to illustrate problem

$$\text{Perimeter of course} = 0.9\text{ km} + 1.2\text{ km} + 1.5\text{ km}$$
$$= 3.6\text{ km}$$

Perimeter will be the length of each lap

$\therefore$ Length of each lap of the course is 3.6 km

$$\therefore \text{Number of laps} = 18\text{ km} \div 3.6\text{ km}$$
$$= 5$$

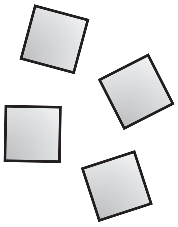
Race distance divided by the length of each lap

$\therefore$ Competitors must complete 5 laps of the course to finish

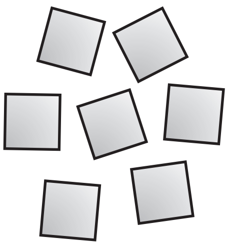
Simple word problems involving area and perimeter

1 Three equilateral triangles, each with sides of length 3 cm have been placed together to make one closed four-sided shape. Each triangle shares at least one whole side with another. Calculate the perimeter of the shape formed.

2 **a** Use **all** four squares below to make **two** shapes in which the number of sides is also equal to four. Compare the distance around the outside of your two shapes and explain what this shows us about the relationship between area and perimeter.



b You have been employed by a fabric design company called Double Geometrics. Your first task as a pattern maker is to design the following using all seven identical squares: "Closed shapes for a new pattern in which the value of their perimeter is twice the value of their area." Draw five possible different patterns that match this design request.



3 The base length of a right-angled triangle is one fifth of its height. If the base of this triangle is 4.2 m, calculate the area of the triangle.

SIMPLE WORD PROBLEMS INVOLVING AREA AND PERIMETER
...../...../20....

Simple word problems involving area and perimeter

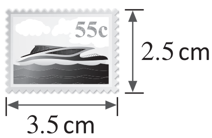
4. An architect is asked to design an art gallery building. One of the design rules is that the floor must be a rectangle shape with an area of $64\,\text{m}^2$.

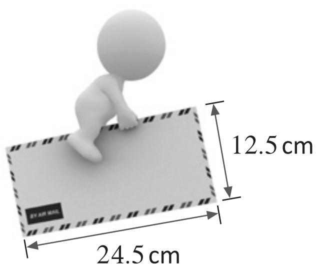
   a. If only whole metre measurements can be used, sketch all the different possible floor dimensions.

   b. Another design rule is to try ensure a large perimeter so there is more space to hang paintings from. Use calculations to show which floor plan will have the largest perimeter.

   c. Would the design with the largest possible perimeter be a good choice? Explain briefly why/why not.

   d. A small art piece at the gallery has one side of an envelope completely covered in stamps like the one pictured below. How many of these stamps were needed to cover one side of an envelope 12.5 cm wide and 24.5 cm long if they all fit perfectly without any edges overlapping?

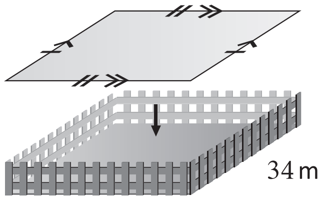




Simple word problems involving area and perimeter

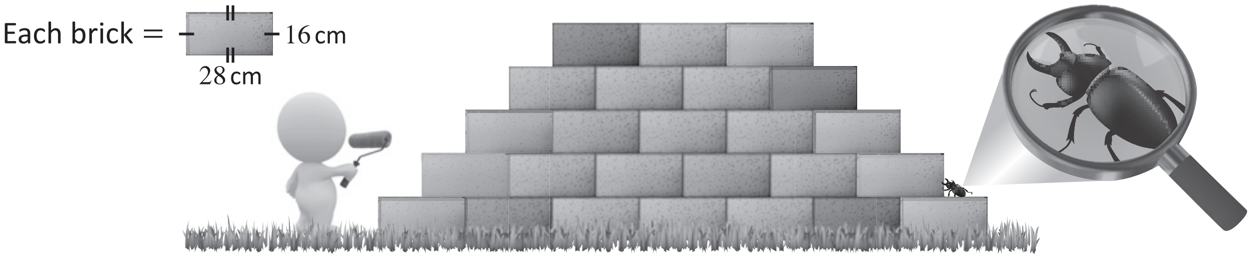
5 A fence used to close off a parallelogram-shaped area is being rearranged to create a square area with the same perimeter. The short side of the area is 34 m long (half the length of the long side).

a How long will each side of the new square area be after using the whole length of this fence?



b If the distance between the longer sides of the original area was 30 m and the length did not change, use calculations to show which fencing arrangement surrounded the largest area.

6 A wall is created by stacking equal-sized rectangular bricks on top of each other as shown. The end of each rectangle sits exactly half-way along the long side of the rectangle underneath it.

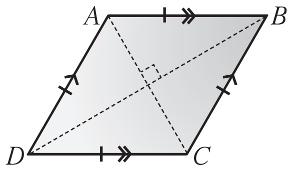


a A 500 mL tin of white paint has been purchased to paint the wall. The instructions on the paint tin say this is enough to cover an area of $11\,500\,\text{cm}^2$.
Use calculations to show that there is enough paint in the tin to cover side of the wall.

b If a beetle walked all around the outside of the wall (including along the ground), how many metres did it walk?

Rhombus and Kite shapes

The area for both of these shapes can be calculated the same way using the length of their diagonals.



- A rhombus is like a square parallelogram.

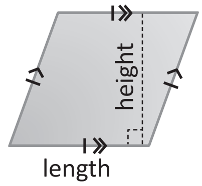
  Area $= (\text{diagonal lengths multiplied together}) \div 2$

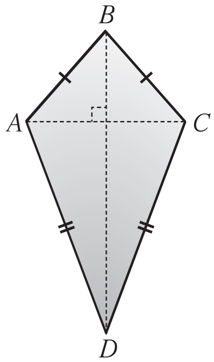
  $= (AC \times BD) \div 2$

  Perimeter $= 4 \times \text{length of one side}$

  $= 4 \times AB$

A Rhombus is a parallelogram, so we can also use the same rule to find the area:





- A kite has two pairs of equal sides which are adjacent (next to) each other.

  Area $= (\text{diagonal lengths multiplied together}) \div 2$

  $= (AC \times BD) \div 2$

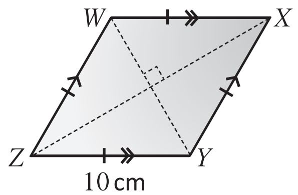
  Perimeter $= 2 \times \text{short side} + 2 \times \text{long side}$

  $= 2 \times AB + 2 \times AD$

Here are some examples:

Calculate the area and perimeter of these shapes

(i) For this rhombus, $WY = 12\,\text{cm}$ and $XZ = 16\,\text{cm}$.



Area $= (\text{diagonal lengths multiplied together}) \div 2$

$= (12\text{ cm} \times 16\text{ cm}) \div 2$

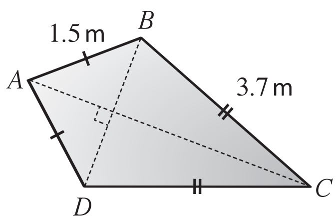
$= 96\text{ cm}^2$

Perimeter $= 4 \times \text{length of sides}$

$= 4 \times 10\text{ cm}$

$= 40\text{ cm}$

(ii) For the kite $ABCD$ shown below, $AC = 4.7\,\text{m}$ and $BD = 2.1\,\text{m}$.



Area $= (\text{diagonal lengths multiplied together}) \div 2$

$= (2.1\text{ m} \times 4.7\text{ m}) \div 2$

$= 4.935\text{ m}^2$

Perimeter $= 2 \times \text{short side} + 2 \times \text{long side}$

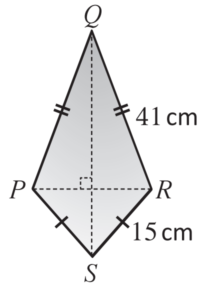
$= 2 \times 1.5\text{ m} + 2 \times 3.7\text{ m}$

$= 10.4\text{ m}$

Rhombus and Kite shapes

1 Calculate the area and perimeter of these shapes:

a $PR = 18$ cm and $QS = 52$ cm



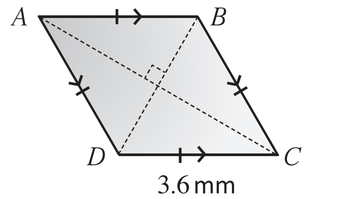
Area = ____ × ____ ÷ ____ cm^2

= ____ cm^2

Perimeter = 2 × ____ + 2 × ____ cm

= ____ cm

b $BD = 1.8$ mm and $AC = 2.4$ mm



Area = ____ × ____ × $\frac{1}{2}$ mm^2 $\div 2 = \times\frac{1}{2}$

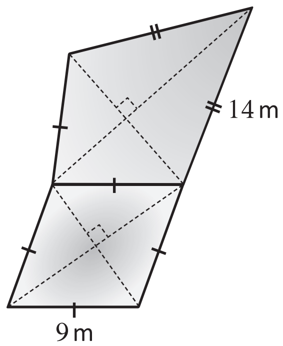
= ____ mm^2

Perimeter = ____ × ____ mm

= ____ mm

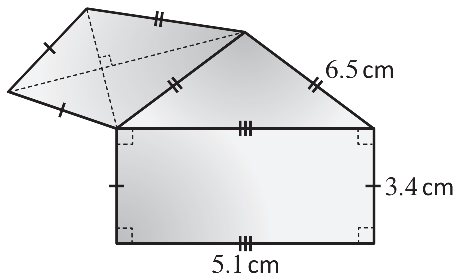
2 Calculate the perimeter of these composite shapes:

a



Perimeter = ____ m

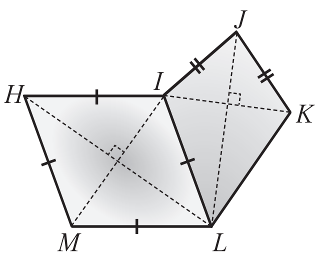
b



Perimeter = ____ cm

3 Calculate the area of this composite shape, showing all working when:

$HL = 30$ m, $IK = IM = 16$ m and $JL = 21$ m



Area = ____ m^2

Trapeziums

A trapezium is a quadrilateral which has at least one pair of parallel sides.

Squares, rectangles, parallelograms and rhombi are all just special types of trapeziums.

So the area formula for a trapezium would also work on all of those shapes.

Two common trapezium shapes

In both shapes, the sides AB (a) and CD (b) are parallel ($AB \parallel CD$).

The height is the perpendicular distance between the parallel sides.

- Area $=$ (sum of the parallel sides) $\times$ height $\div 2$

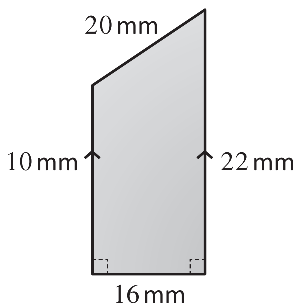
  $= (a + b) \times h \div 2$

- Perimeter $= AB + BD + CD + AC$

Here are some examples:

Calculate the area and perimeter of these shapes

(i)



Area $=$ (sum of the parallel sides) $\times$ height $\div 2$

$= (22\text{ mm} + 10\text{ mm}) \times 16\text{ mm} \div 2$

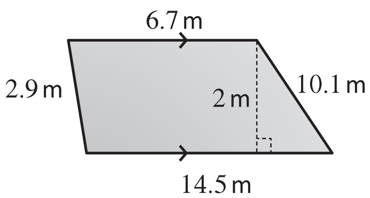
$= 32\text{ mm} \times 16\text{ mm} \div 2$

$= 256\text{ mm}^2$

Perimeter $= 20\text{ mm} + 22\text{ mm} + 16\text{ mm} + 10\text{ mm}$

$= 68\text{ mm}$

(ii)



Area $=$ (sum of the parallel sides) $\times$ height $\div 2$

$= (6.7\text{ m} + 14.5\text{ m}) \times 2\text{ m} \div 2$

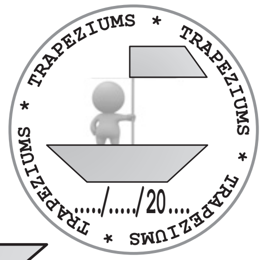
$= 21.2\text{ m} \times 2\text{ m} \div 2$

$= 21.2\text{ m}^2$

Perimeter $= 6.7\text{ m} + 10.1\text{ m} + 14.5\text{ m} + 2.9\text{ m}$

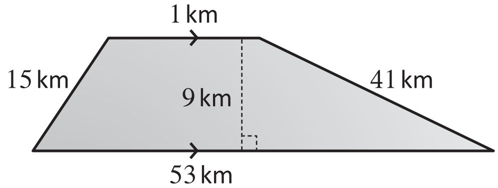
$= 34.2\text{ m}$

Trapeziums



1 Calculate the area and perimeter of these trapeziums:

a

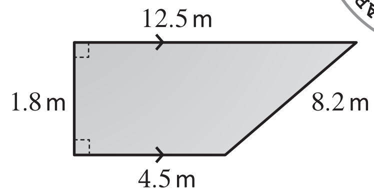


Area = (______ + ______) × ______ ÷ 2 km^2

= ______ km^2

Perimeter = ______ km

b



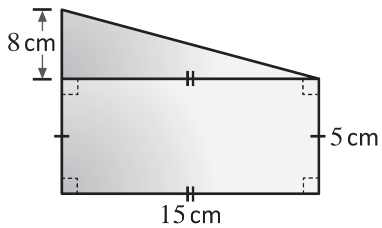
Area = (______ + ______) × ______ ÷ 2 m^2

= ______ m^2

Perimeter = ______ m

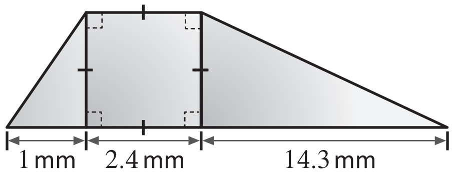
2 Use the trapezium method to calculate the area of these composite plane shapes.

a



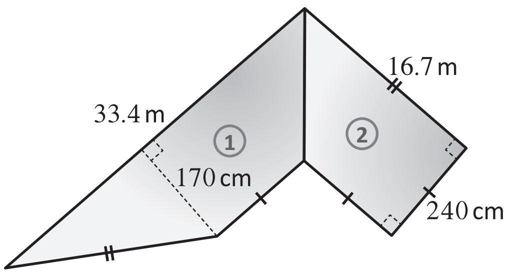
Area = ______ cm^2

b



Area = ______ mm^2

3 Use the trapezium method to calculate the area of this composite plane shapes.

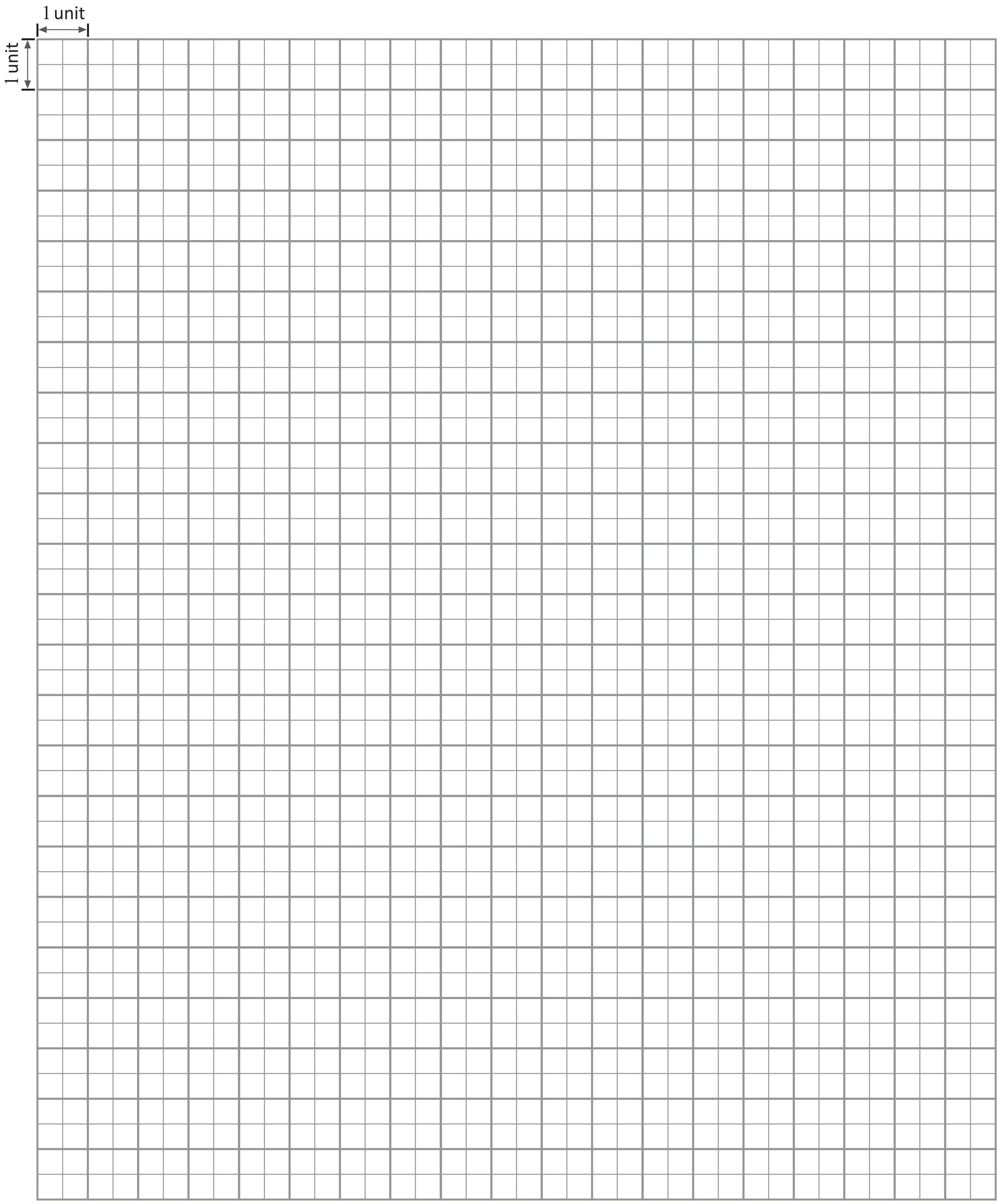


Area = ______ m^2

Perimeter = ______ m

Area challenge

Fill the grid below with as many different squares, triangles, rectangles, parallelograms, rhombi, kites and trapeziums as you can which all have the same area of 8 units^2.



Reflection Time

Reflecting on the work covered within this booklet:

- What useful skills have you gained by learning how to calculate the area and perimeter of plane shapes?

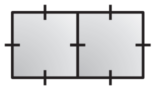
- Write about one or two ways you think you could apply area and perimeter calculations to a real life situation.

- If you discovered or learnt about any shortcuts to help with calculating area and perimeter or some other cool facts/conversions, jot them down here:

Here is what you need to remember from this topic on Area and perimeter

Area using unit squares

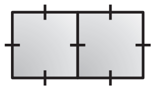
Area is just the amount of flat space a shape has inside its edges or boundaries.
A unit square is a square with each side exactly one unit of measurement long.
Count the total number of whole squares, or fractions of squares to calculate the area.



Area (A) $= 2$ units^2

Perimeter using unit squares

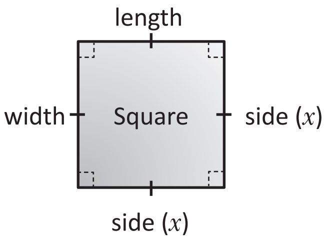
The perimeter with unit squares means count the number of edges around the outside of the shape.

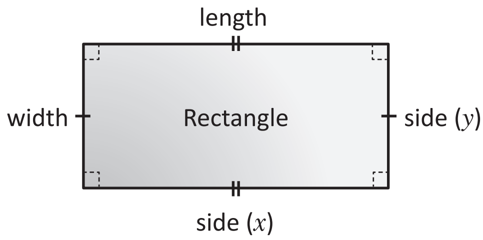


Perimeter (P) $= 6$ units

Area: Squares and rectangles

Just multiply the length of the perpendicular sides (length and width).

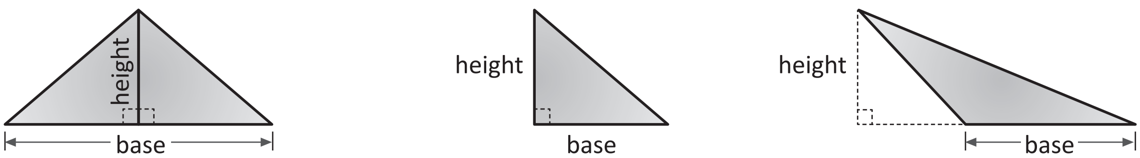




Area $=$ length $\times$ width
$= x^2$ units^2

Area $=$ length $\times$ width
$= xy$ units^2

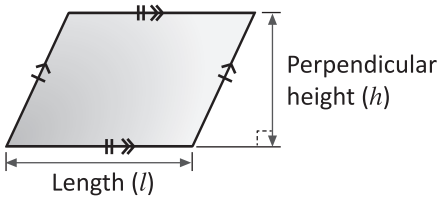
Area: Triangles



$\therefore$ Area of the triangle $=$ (half the base multiplied by the perpendicular height) units^2

$$= \frac{1}{2} \times b \times h \text{ units}^2$$

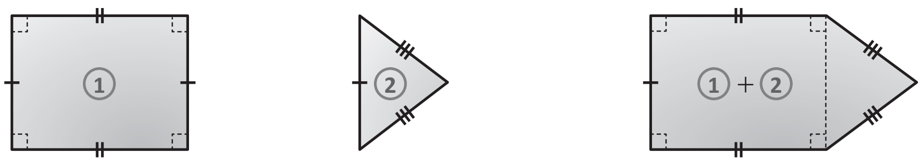
Area: Parallelograms



$\therefore$ Area of a parallelogram $=$ length $\times$ perpendicular height units^2

$$= l \times h \text{ units}^2$$

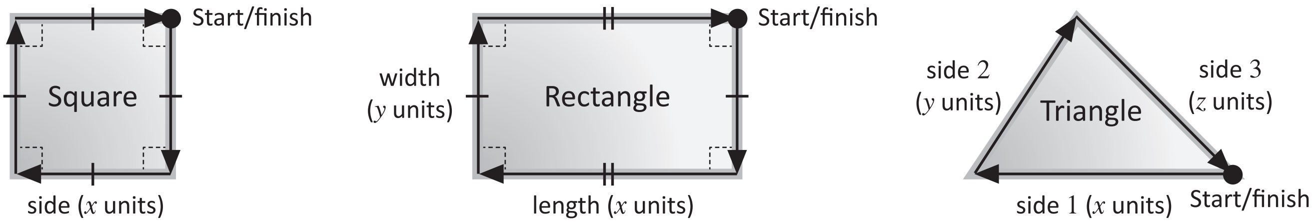
Area of composite shapes



Area ① (Rectangle) + Area ② (Isosceles triangle) = Composite Area = Area ① + Area ② (Rectangle + Isosceles triangle)

Perimeter of simple shapes

Add together the lengths of every side which make the shape.



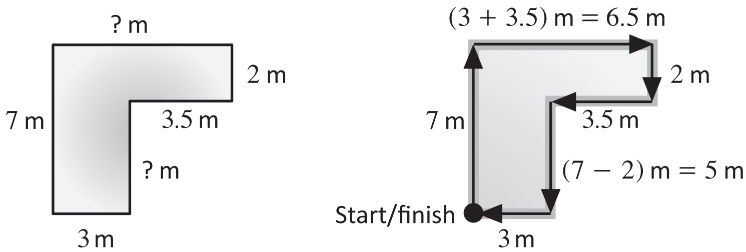
$P = 4 \times$ side length
$= 4x$ units

$P =$ width + length + width + length
$= 2x + 2y$ units

$P =$ side 1 + side 2 + side 3
$= x + y + z$ units

Perimeter of composite shapes

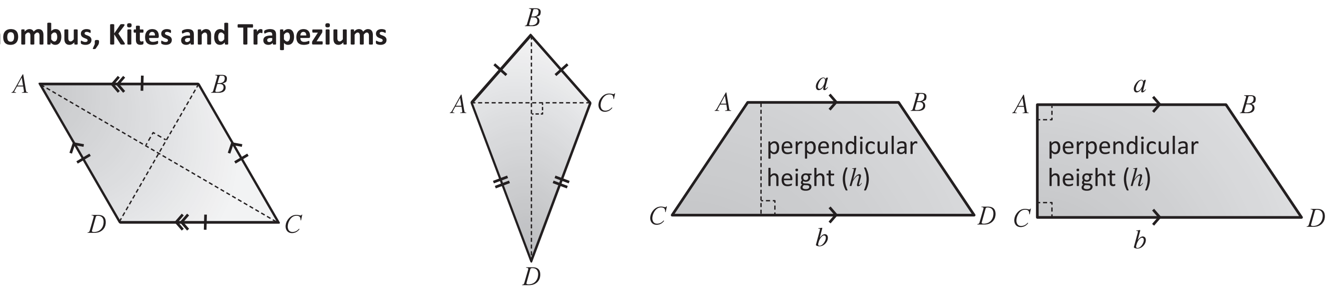
The lengths of all unlabelled sides must be found in composite shapes before calculating their perimeter. It is easier to add them together if the lengths are all in the same units.



$\therefore$ Perimeter $= 7\text{ m} + 6.5\text{ m} + 2\text{ m} + 3.5\text{ m} + 5\text{ m} + 3\text{ m}$

$= 27\text{ m}$

Rhombus, Kites and Trapeziums



Rhombus

Area $= (AC \times BD) \div 2$

Perimeter $= 4 \times AB$

Kite

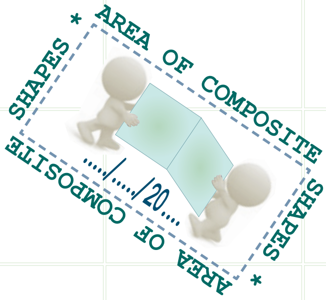
Area $= (AC \times BD) \div 2$

Perimeter $= 2 \times AB + 2 \times AD$

Trapezium

Area $= (a + b) \times h \div 2$

Perimeter $= AB + BD + CD + AC$



SIMPLE WORD PROBLEMS INVOLVING AREA AND PERIMETER

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