Sustainability - The Outlook for Someday

How might we promote a message of sustainability so that we contribute as active and caring members of our global community?

Walt calculate different averages such as the mean, the mode, the median and the range

Success Criteria: I understand the differences between different types of averages. I know how to organise numbers from lowest to highest or ascending order. I can find the middle number and the difference between the highest and the lowest number. I can perform addition and subtraction.

View mean median mode song

Thinking and working

Find the **(i)** mean, **(ii)** median, **(iii)** mode and **(iv)** range of each of the following sets of test results.

(a) 9, 4, 5, 7, 8, 7, 2 **(b)** 10, 7, 3, 4, 8, 2, 6, 5

Thinking			Working
(a) **(i)**	**1**	Write the rule for finding the mean.	(a) (i) Mean $= \frac{\text{sum of data values}}{\text{number of data values}}$
	2	Substitute data into the rule.	$= \frac{9+4+5+7+8+7+2}{7}$
	3	Simplify and evaluate.	$= \frac{42}{7}$
	4	Write down the mean.	Mean $= 6$
(ii)	**5**	Order the values from smallest to largest. Count the number of values. If the number is odd, select the middle value.	(ii) 2, 4, 5, (7), 7, 8, 9 ↑ 7
	6	Write down the median.	Median $= 7$
(iii)	**7**	Does any value appear more often than any other? This is the mode. (Yes)	(iii)
	8	Write down the mode.	Mode $= 7$
(iv)	**9**	Range is highest data value – lowest data value.	(iv) Range $= 9 - 2$ $= 7$

Further Thinking and working

(b) (i) 1	Write the rule for finding the mean.	(b) (i)	$\text{Mean} = \frac{\text{sum of data values}}{\text{number of data values}}$
2	Substitute data into the rule.		$= \frac{10+7+3+4+8+2+6+5}{8}$
			$= \frac{45}{8}$
3	Simplify and evaluate.		$= 5.625$
4	Write down the mean.		Mean $= 5.625$
(ii) 5	Order the values from smallest to largest. Count the number of values. If the number is even, select the two middle values.	(ii)	2, 3, 4, 5, 6, 7, 8, 10 2, 3, 4, (5, 6), 7, 8, 10
6	Find the mean of these two numbers to find the median.		$\text{Median} = \frac{5+6}{2}$
7	Write down the median.		Median $= 5.5$
(iii) 8	Does any value appear more often than any other? This is the mode. (No)	(iii)	
9	Write that there is no mode.		There is no mode.
(iv) 10	Range is highest data value – lowest data value.	(iv)	Range $= 10 - 2$ $= 8$

Everyone in class does this exercise!

Fluency

1 Find the **(i)** mean, **(ii)** median, **(iii)** mode and **(iv)** range of each of the following sets of test results.

(a) 7, 1, 7

(b) 5, 8, 5

(c) 6, 6, 6, 6, 6

(d) 4, 4, 4, 4, 4, 4, 4

(e) 7, 6, 7, 5, 7, 9, 7, 8

(f) 6, 5, 5, 5, 9, 9, 10

(g) 8, 7, 9, 5, 8, 2, 3

(h) 2, 8, 2, 6, 8, 6, 1, 2, 1, 4

(i) 4, 3, 6, 4, 8, 2, 5, 7, 6, 4

(j) 5, 3, 8, 7, 2, 5, 9, 5

(k) 1, 6, 2, 7, 9, 8, 9

(l) 6, 3, 2, 8, 9, 2, 5

2 Find the **(i)** mean, **(ii)** median, **(iii)** mode and **(iv)** range of each of the following sets of data. Round your answers to two decimal places if necessary.

(a) 29, 36, 29, 23, 32, 25

(b) 67, 73, 72, 82, 50, 45, 76

(c) 56, 72, 39, 47, 89, 81, 63, 81

(d) 12.8, 6.5, 19.3, 14.2, 4.7, 6.8, 6.5, 11.3

(e) 16.3, 15.8, 11.4, 15.2, 12.3, 11.4, 13.5

(f) 11.01, 11.001, 10.1, 10.11, 10.01, 1.11, 11.11, 1.01, 11

Understanding

3 *Without doing any calculations*, in each case, choose which one of the alternatives could be the mean.

(a) 9, 7, 8, 7, 7, 8, 6, 6, 9

A 3 **B** 5 **C** 7 **D** 10

(b) 6, 6, 6, 7, 6, 7, 4, 7, 2, 7

A 2 **B** 4 **C** 6 **D** 8

(c) 12, 19, 14, 18, 11, 17

A 10 **B** 12 **C** 16 **D** 19

(d) 86, 74, 85, 68, 91, 76, 80, 90

A 74 **B** 81 **C** 89 **D** 91

Apply your understanding - All students work on this

4 The Korea–Japan 2002 World Cup in soccer was played in more stadiums than ever before. This record was not beaten in Germany in 2006 (12 stadiums) or in South Africa in 2010 (10 stadiums). The following table shows the city and capacity of the different stadiums.

Korea	Capacity
Seoul	63 961
Incheon	52 179
Suwon	43 188
Daejeon	40 407
Daegu	68 014
Jeonju	42 391
Gwangju	42 880
Ulsan	43 550
Busan	55 982
Seogwipo	42 256

Japan	Capacity
Sapporo	42 000
Miyagi	49 000
Niigata	42 300
Ibaraki	42 000
Saitama	63 000
Shizuoka	50 600
Kobe	42 000
Osaka	50 000
Oita	43 000
Yokohama	70 000

(a) Find the **(i)** mean, **(ii)** median and **(iii)** range for the stadium capacities in Korea.

(b) Which stadium has closest to the mean capacity in Korea?

(c) Find the **(i)** mean, **(ii)** median and **(iii)** range for the stadium capacities in Japan.

(d) Which stadium has closest to the mean capacity in Japan?

(e) Use the mean and median to compare the sizes of the stadiums in the two countries.

(f) Find the **(i)** mean, **(ii)** median and **(iii)** range for all 20 stadium capacities.

(g) Which stadium is closest to the mean capacity for all 20 stadiums?

Are you ready to take this "Challenge"

Reasoning

5 Consider the following test results: 7, 5, 6, 8, 3

(a) Do you expect the mean to be an actual data value? Explain your answer.

(b) Find the mean of these results. Is the mean an actual data value?

(c) Do you expect the median to be an actual data value? Explain your answer.

(d) Find the median for the test results above.

6 **(a)** Explain what is meant by the statement 'The average family has 2.3 children'.

(b) Explain why the median of a group of whole-number values can only ever either end in .5 or be a whole number.

(c) If the data values are all whole numbers, can the mode ever be a decimal? Explain why or why not.

7 A small company has a manager, an assistant manager, two office workers and 10 factory workers. The manager is paid $125 000 per year, the assistant manager $85 000, the office workers $55 000 each, and the factory workers $50 000 each.

(a) Find the **(i)** mean, **(ii)** median and **(iii)** mode for the annual income of all the people in the company, rounding your answer to the nearest cent.

(b) Which measure of centre gives you the best idea about the salaries paid by this company?

(c) Omit the manager's salary and recalculate the **(i)** mean, **(ii)** median and **(iii)** mode.

(d) Comment on what you found.

Open-ended

8 As well as giving a set of results, make sure you include a short explanation of how you found your answers for each of the following.

(a) Find five test results for which the mean, median and mode are all equal to 6 if the test results are not all the same.

(b) Find five test results for which the mean and the median are 7 and the mode is 9.

(c) Find five test results for which the mean is 7 and the median and mode are 6.

9 Three students are finding the median of the following data values.
20, 17, 28, 23, 19, 21, 22, 20

Sam says 'The median is the fourth value, so is 23.'

Veran says 'No, you need to put the data values in order first, so the median is 20.'

Lakme says 'No, you are both wrong.'

Help Lakme explain to Sam and Veran why they are incorrect.

10 Nine friends went tree planting at their local creek. The median number of trees planted was 5, the mean was 6 and the mode was 4. Find a data set for the number of trees each person planted to match the given statistics.

Extension

Outside the Square — Problem solving

Running the race

1 In a sprint, the first three runners had times of 12.1, 12.1 and 10.9 seconds. Find the mean, median and mode for the three times.

2 In a different race, the first three runners had a mean time of 11.7 seconds, a median time of 10.8 seconds and a modal time of 10.8 seconds. Find the times for the three runners.

3 In a third race, the first three runners had a mean time of 11.8 seconds and a median time of 11.9 seconds. If all three times are different what can you say about the first and third placegetters?

Give at least three pairs of possible times for these two runners, using one decimal place accuracy in your solutions. Explain how you found your answers.

Strategy options

- Work backwards.
- Test all possible combinations.

Check Your Answers

1	(a)	(i) 5	(ii) 7	(iii) 7	(iv) 6
	(b)	(i) 6	(ii) 5	(iii) 5	(iv) 3
	(c)	(i) 6	(ii) 6	(iii) 6	(iv) 0
	(d)	(i) 4	(ii) 4	(iii) 4	(iv) 0
	(e)	(i) 7	(ii) 7	(iii) 7	(iv) 4
	(f)	(i) 7	(ii) 6	(iii) 5	(iv) 5
	(g)	(i) 6	(ii) 7	(iii) 8	(iv) 7
	(h)	(i) 4	(ii) 3	(iii) 2	(iv) 7
	(i)	(i) 4.9	(ii) 4.5	(iii) 4	(iv) 6
	(j)	(i) 5.5	(ii) 5	(iii) 5	(iv) 7
	(k)	(i) 6	(ii) 7	(iii) 9	(iv) 8
	(l)	(i) 5	(ii) 5	(iii) 2	(iv) 7
2	(a)	(i) 29	(ii) 29	(iii) 29	(iv) 13
	(b)	(i) 66.43	(ii) 72	(iii) No mode exists.	(iv) 37

(c) (i) 66 (ii) 67.5 (iii) 81 (iv) 50

(d) (i) 10.26 (ii) 9.05 (iii) 6.5 (iv) 14.6

(e) (i) 13.7 (ii) 13.5 (iii) 11.4 (iv) 4.9

(f) (i) 8.50 (ii) 10.11 (iii) No mode exists. (iv) 10.1

3 (a) C; only one within the range of scores

(b) C; 8 too high and 4 too low

(c) C; only one of each score, so mean will be close to median

(d) B; close to median as single results relatively evenly spread

4 (a) (i) 49 480.8 (ii) 43 369 (iii) 27 607

(b) Incheon

(c) (i) 49 390 (ii) 46 000 (iii) 48 000

(d) Miyagi

(e) The mean capacity is slightly higher in Korea (49 480.8 compared to 49 390), but the median capacity is higher in Japan (46 000 compared to 43 369). Half of the stadiums in each country have a capacity less than 45 000 (i.e. smaller than the mean value).

(f) (i) 49 435.4 (ii) 43 369 (iii) 29 593

(g) Miyagi (again) (It is interesting to note there are two stadiums in Japan quite close to the mean—Miyagi and Osaka—but none of the Korean stadiums are very close at all.)

5 (a) No, because the mean is the sum of all the data values divided by the number of values and so it will not necessarily be a data value.

(b) 5.8, no it is not a data value.

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6 (a) This statement means that the average number of children of all families is 2.3. Most families have 2 children, but because many families have more than two children, the average of all these numbers of children is 2.3.

(b) The middle number will always be a whole number. If there are two middle numbers, then the only decimal you can get by dividing them by 2 is one that ends in .5.

(c) No, as the mode is the value that occurs most often in a data set, and the data set only contains whole numbers.

7 (a) (i) $58 571.43 (ii) $50 000 (iii) $50 000

(b) In this case, the mode does a good job as there are so many of this category of worker—well over half. However, the mean does take account of the other workers as well.

(c) (i) $53 461.54 (ii) $50 000 (iii) $50 000

(d) The mean is the only value that changes, and it changes by a large amount (over $5000).

Open-ended – Sample answers

8 (a) 5, 6, 6, 6, 7. As the median is 6, this is the middle number and we need at least one more 6 to make it the mode.

The total of the five results is 30, so we have 18 left, which we can break up any way we like as long as we don't move the median number.

(b) 4, 6, 7, 9, 9. As the median is 7, this has to be the middle number and we need two 9s after that. This adds to 25 and the total required is 35, so we only have 10 left to play with.

(c) 4, 5, 6, 6, 14. The middle number is 6 and one more is 6. The total is 35, so we still have 23 to play with that can be allocated in many ways.

9 While Veran is correct to say the values must be ordered first when there is an even number of values, the median is found exactly midway between two of the values. The ordered list is: 17, 19, 20, 20, 21, 22, 23, 28, so the median is midway between 20 and 21 and so is 20.5.

10 As the median is 5, we know this must be in the middle; as the mean is 6, we know that the values must add up to 54, and 4 must be the most frequent. Sample answer: 1, 4, 4, 4, 5, 6, 6, 7, 17.