

A Writing Ratios

A **ratio** compares quantities of the same kind.

The ratio of two quantities **q** and **r** is written as **q : r**.

Ratios can be simplified the same way as fractions, we can divide both quantities in the ratio by the same number. The numbers in a **simplified ratio** must be whole numbers, not decimals.

Examples :

- a) Our cat has kittens, 4 are black, 2 are ginger.

The ratio black : ginger = 4 : 2 = 2 : 1

- b) On a cellphone the cost of a call at peak-time costs \$1.25 per minute and at off-peak time 75 cents per minute.
peak : off-peak = \$1.25 : 75¢

= 125 : 75 (common unit)

= 25 : 15 (divide by 5)

= 5 : 3 (divide by 5)

- 1 Write ratios for the following situations. Simplify the ratios.

- a) There are 12 girls and 15 boys in class 11GLD.

girls : boys =

- b) On our vacation we had 4 rainy days and 16 sunny days.

sunny : rainy =

- c) At a party with 35 people there were 5 smokers.

smokers : non-smokers =

- 2 The following ratios compare quantities with different units. Choose one common unit and then simplify the ratio.

- a) 20¢ : \$1.40

- b) 15 min : 1 hour

- c) 1 m : 85 cm

- d) 600 mL : 1.5 L

- e) 3 km : 450 m

- 3 Write these ratios with whole numbers in simplest form.

- a) $2\frac{1}{2} : 5\frac{1}{3}$

- b) $2 \times 10^{-1} : 4 \times 10^{-2}$

B Equivalent Ratios

Example :

The ratio boys to girls in class 11BCH is 4 : 5.

If there are 12 boys, how many girls are there?

Working : Find the missing numbers in $4 : 5 = 12 : \dots$

Answer : 15 girls.

x3 x3

- 1 Below are sets of equivalent ratios. Fill in the missing number.

- a) $5 : 6 = \dots : 90$

- b) $7\frac{1}{2} : 8\frac{1}{2} = 30 : \dots$

- c) $0.6 : 0.04 = \dots : 2$

- d) $3\frac{3}{4} : 5 = 45 : \dots$

- 2 Five minutes of fast walking expends as much energy as 2 minutes of running cross country. How long should Chloe walk to burn as much energy as Max who runs cross country for 30 minutes?

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- 3 The ratio female teachers to male teachers at a high school is 4 : 3. There are 36 male teachers, how many teachers are there in total?

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- 4 Jarrod emptied a jar of jelly beans and sorted them. He found white : black : coloured = 2 : 3 : 9. Jarrod counted 12 black jelly beans. Calculate the total amount of jelly beans.

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A Ratios and Fractions

Examples :
Of 40 plants in the garden, 24 are natives, the rest are exotic.
a) Write the ratio native to exotic in simplest form.
b) What fraction of the plants are natives? Simplify the fraction.

Answers : a) native : exotic = $24 : 16 = 3 : 2$
b) 24 out of 40 are natives, $\frac{24}{40} = \frac{3}{5}$

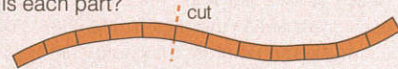
- 1 Many people work 5 of the 7 week days.
 - a) The ratio working days : free days =
 - b) What fraction of the week are free days?
- 2 There are 32 houses in Aaron's street. Twelve of the householders use rubbish bags, the others use wheelie bins.
 - a) Write the ratio bags : bins in simplest form.
 - b) Write the fraction of the householders using wheelie bins in simplest form.
- 3 A rugby team won $\frac{5}{12}$ of its games and lost $\frac{4}{12}$.
 - a) What fraction ended in a draw?
 - b) Complete. win : draw : loss =
- 4 Sarah planted twice as many red tulips as white tulips.
 - a) Write the ratio red : white.
 - b) What fraction of the tulips is red?
- 5 In the pet shop there are 3 times as many yellow budgies as blue budgies and there are twice as many green budgies as yellow budgies.
 - a) Write the ratio green : yellow : blue
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 - b) What fraction of the budgies is yellow?
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B Sharing in a Given Ratio

Many ratio problems are of a type called 'sharing in a given ratio'.

Examples : A ribbon, 60 cm long is cut in the ratio 5 : 7.
How long is each part?



Working : Divide the ribbon into 12 equal sections.
The smaller part will be $\frac{5}{12}$ of 60 cm,
the larger part $\frac{7}{12}$ of 60 cm.

Answer : Short section 25 cm, long section 35 cm

- 1 A bag with 35 jelly beans is shared between Talia and Simon in the ratio 2 : 3.
 - a) What fraction of the jelly beans are for Talia?
 - b) How many jelly beans does she get?
- 2a) Share \$105 in the ratio of 3 : 4.
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- b) Divide 4.5 litres in the ratio 7 : 2.
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- c) Divide \$184.80 in the ratio 1 : 2 : 3.
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- 3 Hannah and her sister bought a lotto ticket for \$10. Hannah paid \$6.25.
 - a) Write the ratio Hannah's share to sister's share in simplest form.
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.....
 - b) If their ticket wins the 3.5 million dollar prize, what would be Hannah's share?

A Read and Solve

Example : An energy mixture is made with dried fruit, chocolate and nuts in the following weight ratio
dried fruit : chocolate : nuts = 7 : 2 : 4.

- a) If the mixture contains 80 g of chocolate, what is the weight of the dried fruit and what is the weight of the entire mixture?
- b) If you want to make 1 kg of energy mixture, how much chocolate should it contain?

Working :

a) fruit : choc : nuts = 7 : 2 : 4 = x : 80 : y
Since $80 = 2 \times 40$, then $x = 7 \times 40 = 280$, $y = 4 \times 40 = 160$.

Answer : The mixture contains 280 g of dried fruit and the total mixture weighs 520 g.

b) $7 + 2 + 4 = 13$, so $\frac{2}{13}$ of the mixture is chocolate; that is $\frac{2}{13}$ of 1000 grams = 154 grams (to nearest gram).

- 1 A student of Bay College did a survey on students' hair colours. In a large random sample of students the ratio of hair colours was as follows :
black : brown : blonde : red = 8 : 11 : 5 : 1.
The sample had 45 blonde-haired students in it.
 - a) How many students in the sample had brown hair?
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.....
 - b) How large was the random sample?
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- 2 The mixing ratio of normal strength cordial drink is :
cordial : water = 2 : 7.
 - a) How much water should be added to 120 mL of cordial?
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 - b) Adam wants to make 2 L of cordial drink. How much cordial should he use?
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.....
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B Try These

- 1 When Anna is up in her room, the ratio of time chatting to friends to time spent doing homework is 2 : 3.
 - a) If the homework will take Anna 75 minutes, how long can we expect Anna to spend up in her room?
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.....
 - b) If Anna spends 75 minutes in her room, how long is she talking to her friends?
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- 2 In a bunch of 15 pink and white tulips the ratio pink : white = 2 : 3. One white tulip is removed. What is the new ratio of colours?
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- 3 Flask A holds 4 times as much water as flask B. Flask C holds $\frac{2}{3}$ of the amount of water in flask A. Write the amount of water in the flasks as a ratio, A : B : C.
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- 4 Two identical jerry-cans contain mixtures of petrol and oil. Both are full, but in one the ratio petrol to oil is 19 : 1, in the other it is 3 : 1. These jerry-cans are poured into an empty tank. Find the ratio petrol to oil in the tank.
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A Being Directly Proportional

The amounts in a baking recipe change in the same proportion if we want to make more (or less) than the standard amount. A problem in which one set of numbers is multiplied by a constant, giving a second set of numbers, is called a **directly proportional** problem. A **ratio table** consists of rows of numbers and each row is directly proportional to the previous.

Example: Here is a table with 2 rows and 3 columns, we call it a 2x3 table.

- a) Show that the 2nd row is directly proportional to the 1st row.
b) Blake says, "Also each column is directly proportional to the previous." Is that true?

2	4	10
6	12	30

Working:

- a) The second row is 3 times the first row.
b) Yes (see arrows).

2	4	10
6	12	30

Arrows indicate scaling: 2 to 6 is x3, 4 to 12 is x3, 10 to 30 is x3. Column-wise: 2 to 4 is x2, 4 to 10 is x2.5, 6 to 12 is x2, 12 to 30 is x2.5.

- 1 Work out the scale factors in these ratio tables.

a)

4	20
8	40

 x
b)

1	3	1 1/2
5	15	7 1/2

 x x

- 2 Use a scale factor to complete these ratio tables.

a)

6	18
15	

 x
b)

3	5	
	35	70

 x

C Missing Numbers

- 1 Calculate the missing numbers in these ratio tables.

a)

21	15		27
14		6	

- 2 Make up a ratio table for this problem, then calculate w.

5 : w = 12 : 8

w =

B Reading the Table

- 1 Here is a simple 2x2 ratio table. Use the numbers in the table to write six true number sentences. A start has been made.

2	3
8	12

- i) $\frac{2}{3} = \frac{8}{12}$ ii) $8 \div 2 = 12 \div 3$
iii) iv)
v) vi)

- 2 Select numbers from this ratio table to complete the sentences.

1	6	4	7.5
2.5	15	10	18.75

- a) $15 \div 6 = 10 \div \dots$ b) $1 \times 10 = \dots \times \dots$
c) $\frac{10}{15} = \frac{\dots}{\dots}$ d) $6 \times 18.75 = \dots \times \dots$
e) $7.5 \div 4 = \dots \div \dots$ f) $2.5 : 15 = \dots : \dots$

In column **A** we used scale factors to find missing numbers in a table. Working out these factors is not always easy. Now you will see other simple strategies to work out missing numbers.

- 3 This is a 2x3 ratio table. We will calculate the values of a and b. Complete the working.

5	4	7
a	2.5	b

- a) In this table $4 \times a = 5 \times 2.5$ or $4a = 12.5$
Calculate a
b) In this table $\frac{b}{7} = \frac{2.5}{4}$ or $\frac{b}{7} = 0.625$
Calculate b

b)

35		63
	16	18
	20	

A Conversions

Conversion problems like adjusting proportions in recipes, exchanging money, etc, can be made simple with the use of ratio tables.

Example: To make 20 marshmallow treats, mix 220 g of butter, 36 marshmallows and 5 cups of krispie crumbs.

Write the ingredients for 35 treats.

Working:

treats (number)	butter (g)	marshm. (number)	crumbs (cups)
20	220	36	5
35	x	y	z

Since the scale factor is hard to work out, we use the diagonal multiplication rule.

$20 \times x = 35 \times 220 \Rightarrow x = 35 \times 220 \div 20 = 385$ g butter
 $20 \times y = 35 \times 36 \Rightarrow y = 35 \times 36 \div 20 = 63$ marshmallows
 $20 \times z = 35 \times 5 \Rightarrow z = 35 \times 5 \div 20 = 8 \frac{3}{4}$ cups of crumbs

- 1 Amy wants to make a cake for which she needs to mix 175 g of sugar, 125 g of flour and 50 g butter. Amy has only 140 g of sugar. How should she adjust the flour and butter?

sugar (g)	flour (g)	butter (g)



- 2 Suppose one NZ dollar buys 0.62 Euros or 0.79 US dollars.

- a) How many NZ dollars do you pay for one thousand Euros?

NZ\$	Euro
1	0.62
	1000

- b) At the airport a souvenir costs US\$15.75. How much would that be ...

NZ\$	Euro	US\$

- i) in NZ dollars?
ii) in Euros?

B Rates

Example: A car is travelling at a speed of 70 km per hour.

- a) How long does it take to go a distance of 40 km?
b) How far does it travel in 25 minutes?

Working:

Set up the ratio table as shown. Do the calculations.

e.g. $\frac{a}{60} = \frac{40}{70} \Rightarrow a = \frac{40}{70} \times 60$
 $\frac{b}{70} = \frac{25}{60} \Rightarrow b = \frac{25}{60} \times 70$

Answer: a) time is 34 minutes b) distance is 29 km.

km	min
70	60
40	a
b	25

- 1 6 metres of fabric are needed to make covers for 15 cushions.

- a) How many metres of fabric are needed to cover 25 cushions?

m fabric	# cushions

- b) How many cushions can be made with 14 metres of fabric?

- 2 On average Harry's car uses 8.6 litres of petrol per 100 km.

- a) How many litres does Harry's car use on a 78 km trip?

- b) What is the car's fuel consumption rate measured in km per litre?

- 3 A cup is placed under a leaking tap. The 220 mL cup is filled up with water in 1 hr and 20 min. How many litres of water would leak out in 10 hours?

- 4 A car travels at a constant speed on the motorway; in 8 minutes it covered 12 km.

- a) How long would the car take to travel 22 km?

- b) Calculate the car's speed in km/h.

39 Using a Ratio Table 3

A Sharing in a Given Ratio

- 1 We extend a square ratio table with another column which contains the sum of the previous two columns.

		sum
3	2	5
10.5	7	17.5

Check with the diagonal multiplication rule that both tables are ratio tables.

$$3 \times 7 = 21 \quad \text{also} \quad \dots \times \dots = 21$$

$$3 \times 17.5 = \dots \quad \text{also} \quad \dots \times \dots = \dots$$

- 2 Zoe made fresh orange drink. She used orange juice and water in the ratio 4 : 1. How much water is in 180 mL of this drink?

	juice	water	sum
mL		180
ratio	4	1	5

Example : A ream of photocopying paper (500 sheets) is divided in the ratio 13 : 7. How many sheets in each pile?

Working : Set up an extended ratio table.

	pile 1	pile 2	sum
pages	a	b	500
ratio	13	7	20

$$20 \times a = 13 \times 500$$

$$a = 325$$

$$20 \times b = 7 \times 500$$

$$b = 175$$

Answer : 325 pages ; 175 pages.

- 3 A ribbon, 520 mm long, is cut in the ratio 8 : 5. Work out the length of the shorter piece.

	long	short	sum
m			
ratio			

- 4 A high school has 912 students with boys and girls in the ratio 9 : 10. How many boys attend the school?

	boys	girls	sum
students			
ratio			

B Set Up a Ratio Table

We can extend any table with the sum and also with the difference of the numbers.

Example : The ratio boys to girls in a school is 9 : 7. There are 56 more boys than girls. How many boys are at the school?

Working : Set up a ratio table.

	boys	girls	diff
real	b		56
ratio	9	7	2

$$2 \times b = 9 \times 56$$

$$b = 252$$

Answer : 252 boys.

- 1 A bricklayer and his apprentice share their profits in the ratio of 5 : 2. The boss gets \$738 more than the apprentice. How much does the apprentice get?

- 2 At a party there are red and white balloons in the ratio 7 : 4. There are 12 more red balloons than white ones. How many balloons are there in total? (Hint : your table should show diff and sum.)

- 3 Peter and Paul together delivered pamphlets. Peter delivered 3500 and Paul 2500. Peter's share of the money was \$140 more than Paul's share. How much did they get paid altogether for delivering 6000 pamphlets?

A One Size Fits All

All types of percentage problems can be solved using a ratio table with the layout shown on the right.

Examples :

- a) A 1.5 L bottle of orange drink contains 1.2 L of pure orange juice. What percentage is pure juice?
- b) A bag with jelly beans contains 36 black ones, that is 48% of the beans. How many beans are there in the bag?

Working :

	whole	part
%	100	a
L	1.5	1.2

$$1.5 \times a = 120$$

$$a = 80$$

Answer : 80%

	whole	part
%	100	48
beans	b	36

$$48 \times b = 3600$$

$$b = 75$$

Answer : 75 beans

- 1 At Mountain View High School 755 students voted to select a representative for the board of trustees. Todd got 83 votes. What percentage of the votes did he get?

	whole	part
%	100	
votes		

- 2 Mrs Brown has health insurance and 80% of each doctor's bill is refunded by the insurance company. The company refunded \$72.80 of her last doctor's bill. How much was the total bill?

	whole	part

- 3 In 2010, twenty-one percent of the NZ population was aged under 15, which amounted to about 920 000 people.

- a) What was the population of NZ in 2010?

	whole	part

- b) 520 000 were aged 65 or over. What percentage of the population was that?

	whole	part

Using a Ratio Table 4 40

B Increase / Decrease

The extended table can be used to solve any questions with discounts, GST, profit, etc.

Example : Hayley got a wage increase of 4%. Her new weekly wage is \$858. How much extra money does Hayley now get?

Working : Carefully set up the ratio table.

	old	increase	new
%	100	4	104
\$		a	858

$$104 \times a = 4 \times 858$$

$$a = 33$$

Answer : \$33 extra

- 1 Blake bought a camera at 30% discount. He paid \$329.70. How much was his discount in dollars?

	old	discount	new
%	100	30	70
\$			

- 2 The GST content of our power bill is \$32.45. How much do we need to pay in total?

	GST excl	GST	GST incl
%	100	15	115
\$			

- 3 In the shoe sale Ruby found a great pair of sneakers with 22% discount. She paid \$79.95. How much were these sneakers before the sale?

	whole	part

- 4 A house is sold at 8% above its current valuation of \$460 000. What is the selling price?

	whole	part



A Inversely Proportional

Inversely proportional problems are those where if one variable is multiplied by a factor, the other is divided by that factor.

Examples :

- a) If two painters take 5 hours to paint our garage, how long would 4 painters take to do the job?
- b) If two painters take 6 hours to paint a 200 m² wall, how long would 3 painters take to paint a 500 m² wall?

Working :

- a) Decide - is this a **directly** or an **inversely** proportional problem? Does double the manpower mean double the painting time or does it mean half the painting time? It's an **inversely** proportional problem. The 4 painters take half of 5 hours.
Answer : $2\frac{1}{2}$ hours.

- b) The 200 m² wall takes 12 'painter hours'.
The size of the wall and painter hours are **directly** proportional, so 500 m² takes 30 painter hours (see table).
The variables **number of painters** and **time taken** are **inversely** proportional :
if 1 painter takes 30 hours, then
3 painters take 10 hours.

size wall (m ²)	painter hours
200	12
500	30

Answer : 10 hours

- 1 It takes 3 apprentices 4 hours to nail down a roof.

- a) How many man-hours does it take to nail down the roof?

- b) How long would the job have taken with ...

- i) 2 apprentices?
- ii) 5 apprentices?

- 2 We have enough water in our emergency kit to last 4 people for 5 days. How long would the supply last with 10 people?

- 3 A 500g bag of scroggan (healthy energy food) lasts two trampers for 3 days.

- a) How long will 6 trampers take to finish 500g of scroggan?
- b) How long will a 750g bag of scroggan last with 4 trampers?

B More Problems

- 1 The cat shelter needs 8 tins of food to feed 4 cats for 3 weeks.

- a) How long would these 8 tins feed 1 cat?

- b) How many tins are needed to feed 6 cats for 5 weeks.

- c) How long can they feed 3 cats with 5 tins?



- 2 It takes 12 asparagus pickers 5 hours to fill 240 boxes.

- a) How many boxes would 8 pickers fill in 2 hours?
- b) How many asparagus pickers are needed if 180 boxes must be filled in 3 hours?

A Working Together

Example :

It would take Jake 4 hours to paint a fence on his own, while Peter would take 5 hours. How long would it take them if they worked together?

Here are two possible strategies :

- 1 In 20 hours, Jake would paint 5 fences and Peter would paint 4. That is 9 fences in 20 hours, or 1 fence every $2\frac{2}{9}$ hours.
- 2 In 1 hour Jake paints $\frac{1}{4}$ fence and Peter paints $\frac{1}{5}$ fence. Together in 1 hour $\frac{9}{20}$ of the fence.
The whole fence is done in $1 \div \frac{9}{20} = 2\frac{2}{9}$ hours.

Answer : 2 hours, 14 minutes.

- 1 By herself it would take Gemma 2 hours to wash all the windows in the house. The same job would take Chloe 3 hours. How long would it take them working together?

- 2 Jordan takes 30 minutes to wash the car, while Sean takes 40 minutes to do the same job. How long would it take them if they worked together?



- 3 An experienced wallpaper hanger can finish wallpapering a 3 m by 4 m bedroom in $3\frac{1}{2}$ hours while his apprentice can do the bedroom in 4 hours.
The wallpaper hanger and his apprentice have to wallpaper 3 bedrooms of that size.
Working together, how long would that take them?

B Building Houses Together

- 1 A participant in a DIY home show has 1 hr and 40 min left on the clock to paint the porch of the house he is doing up. He asked 3 painters how long they would take to paint his porch. The first would take 6 hours, the second 5 hours, the third 4 hours.

Assuming there is enough room in the porch for these painters to work together, would they finish in time?

- 2 A builder and his apprentice are putting a roof on a house. The builder would take 20 hours to finish the job on his own, the apprentice on his own would take 25 hours. For the first 8 hours they work together on the job, but then the apprentice calls in sick and the builder has to finish the job on his own.
How long does it take to put the roof on this house?

A The Weather


- 1 Tauranga can expect on average 122 mm of rain in July.
Last July it only had 52 mm of rainfall.
What percentage of the expected rainfall did Tauranga
get last July?



Yesterday the West Coast had 48 mm of rain.
This rain gauge holds 48 mm of rain.
It fills $\frac{3}{10}$ of the gauge.

How many mm of rain would fill the rain gauge to the top.

- 3 Hamilton has on average 1212 mm of rain per year. The ratio of rainfall in winter to rainfall in the rest of the year is 3 : 1. How much rain can Hamiltonians expect in winter?

- 4 On Saturday 7 am the temperature at Mt Cook Village was -2°C . The temperature rose 1.5° per hour reaching its maximum at 1 pm. Then the temperature dropped again. On Sunday at 2 am it reached its minimum of -5°C .
- 



How much did the temperature drop between 1 pm Saturday and 2 am Sunday?

- 5 Blenheim, at the top of the South Island, gets 2470 hours of sunshine per year. Invercargill, at the bottom, gets only 64% of the amount Blenheim gets. How many hours of sunshine does Invercargill get per year?

B Money Matters

- 1 A hardware shop gives 5% discount for cash. Mr McDuff buys 10 L of fence paint for \$98.70 and two brushes at \$7.95 each. Mr McDuff pays cash for this. How much does he pay?



- 2 Judy and Sonya bought a Lucky Dip Lotto ticket for \$5. Judy paid \$2, Sonya \$3. They won a prize which they shared in the same ratio. Judy's share was \$115. How big was the prize?

- 3 Ben and Tony share a flat. Each pays half of the weekly rent of \$315. To pay for food and power the young men keep a kitty. Ben lives full time in the flat but Tony goes home on the weekends so, Tony's share : Ben's share = 5 : 7. Last week Ben's total cost for rent, food and power was \$402. Calculate Tony's cost for rent, food and power that week.

A Good Health

- 1 This is an extract from a newspaper article.
"There are about 500 cases of meningitis each year. One
in twenty sufferers die from the disease."

- Write 1 in 20 as a percentage.
- How many meningitis sufferers are expected to die from the disease each year?

- 2 These are annual statistics provided by a hospital.

Age Group 15 - 18 yr olds		
diagnosis	females	males
accidents	147	348
cancer/tumours	34	26
abdominal	177	90
skin disease	45	62
gynaecology	355	-
muscle/bone	58	69
dental	82	45
respiratory	108	82
infections	37	18
other	92	98
Total	1135	838

- How many 15-18 year olds were admitted to this hospital in that year?
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.....
- What percentage were male?
.....
.....
- What percentage of the males were admitted because of an accident?
.....
.....
- What percentage of the 15-18 year olds were admitted because of respiratory problems?
.....
.....
- What percentage of the 15-18 year old patients with dental problems were female?

B All About Money

- 1 Peter bought some gear for his first car in a sale.
He saved 35% on a seat cover originally priced at \$124.50.
He saved 30% on a tool kit which was reduced to \$52.95
and only paid \$9.95 for a wax-pack, down from \$16.50.

- a) How much did Peter pay for his seat cover?
- b) What was the original price of the tool kit?
- c) What was the percentage discount on the wax-pack?



- 2 Some years ago Aiden and Kieran bought a computer together for \$1250. Kieran paid \$220 more than Aiden. They're now selling their computer for \$300. How much more than Aiden should Kieran's get?