National Certificate of Educational Achievement
TAUMATA MĀTAURANGA Ā-MOTU KUA TAEA

Exemplar for Internal Achievement Standard

Mathematics and Statistics Level 1

This exemplar supports assessment against:

Achievement Standard 91035

Investigate a given multivariate data set using the statistical enquiry cycle

An annotated exemplar is an extract of student evidence, with a commentary, to explain key aspects of the standard. It assists teachers to make assessment judgements at the grade boundaries.

New Zealand Qualifications Authority

To support internal assessment

	Grade Boundary: Low Excellence
1.	For Excellence, the student needs to investigate a given multivariate data set using the statistical enquiry cycle, with statistical insight.

This involves integrating statistical and contextual knowledge throughout the statistical enquiry cycle, and may involve reflecting on the process or considering other explanations for the findings.

This student's evidence comes from the TKI assessment resource 'Sporting success'.

The student has posed an appropriate comparison question (1), selected and used appropriate displays (2), given summary statistics (3), discussed features of distributions comparatively (4), and communicated an informal inference in their conclusion (5).

The student has provided evidence of statistical insight throughout the statistical enquiry cycle, and demonstrated statistical knowledge in the comment about the use of 0.41 for DBM/OVS (6). The student has also started to reflect on the process (7).

For a more secure Excellence, the student could strengthen the reflection on the process and consider other explanations. For example, the student could consider the current world rankings of the teams. They could also strengthen the depth of the contextual discussion. |

I wonder if the All Blacks tended to score more points in test matches against Northern than Southern Hemisphere teams between 1992 and 2011? I am going to look at a random sample of 100 data points taken from all test matches, 1992 to 2011. I think the All Blacks score is likely to be bigger against Northern Hemisphere teams than Southern hemisphere teams because they always seem to score lots of points on overseas tours but the southern hemisphere Four Nation matches seem a lot closer when I watch them on Sky Sports. Because the matches are usually closer, the points scored are not high very often.

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NZ.score by Opposition.from.Northern.or.Southern.Hemisphere
Opposition.from.Northern.or.Southern.Hemisphere
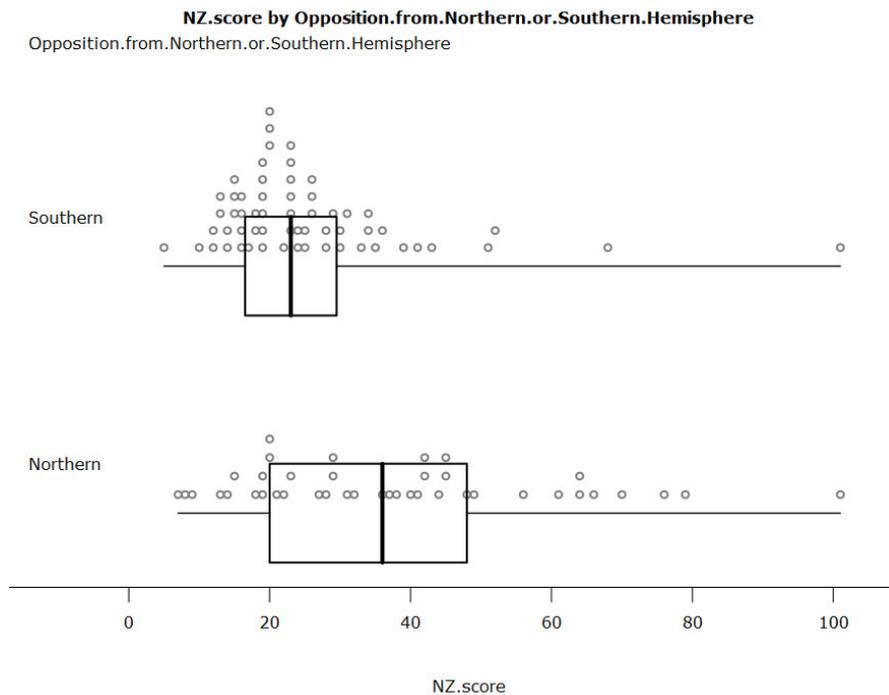
②

This table gives a summary of the numbers I have used when describing the graphs

```
Summary of NZ.score by Opposition.from.Northern.or.Southern.Hemisphere
         Min. 1st Qu. Median  Mean 3rd Qu. Max. Std.dev Sample.Size
Northern    7    20.0     36 37.76    48.0  101  21.766          41
Southern    5    16.5     23 25.56    29.5  101  14.950          59
```

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The graph shows that for this sample the average All Blacks score against Northern Hemisphere teams was bigger than against Southern hemisphere teams. Looking at the graph this is because the median for matches against northern teams is about 35 points whilst the median against southern teams is about 24 points (shown by the thick black lines on the graph). This agrees with my initial thoughts that they always score more against Northern Hemisphere teams

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The middle 50% of All Blacks scores against Northern hemisphere are much more spread out than against Southern hemisphere teams. I can see this because the box on the graph

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(the middle 50%, the inter-quartile range) is much larger for the northern games than the southern games. The width of these boxes are 28 points for northern games and 13 points for southern games. I think this is because most of the matches against the southern hemisphere teams are against Australia and South Africa and the teams are quite equal in strength, whilst the Northern hemisphere teams vary quite widely. For example, Italy are very weak.

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There is quite an overlap in the boxes. The left hand box of the north (from 20 to 36 points) contains almost all the box of the south (16 to 23 points). This shows there are a lot of middle points scores in common for both hemispheres.

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Both graphs have a long right tail with a few games with a much larger score than the others. In both cases the biggest score is 101 points. There are fewer of these very big points on the southern graph (the last 2 points) than the northern graph (the last four points). These very large scores are likely to have come from playing the weaker teams, for example one of the island teams in the southern hemisphere and Italy in the northern hemisphere. I looked this up on the All Blacks website and this game was against Italy during the 1999 world cup.

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The scores against the southern hemisphere are very tightly bunched together at the lower end of the graph. This shown by the clump of points between 5 and 40 points with tall columns around 20 points. The tall columns mean there must have been several matches when the All Blacks scored the same number of points. There is not the same shape in the northern hemisphere graph, where the points are much more evenly spread out between 7 and 55 points.

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The median is given in the table, The interquartile range is the 3rd Qu – the 1st Qu., 28 for the northern hemisphere and 13 for the southern hemisphere, this is the width of the middle 50%. The table shows that I only looked at 41 games against northern hemisphere teams and 59 against southern hemisphere teams. I don't think this matters as it reflects the fact that the All Blacks do play more games against southern hemisphere teams than the northern hemisphere teams. The smallest points (7 and 5) and the largest points (101 and 101) are very similar in both hemispheres.

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Conclusion

I want to make a conclusion about what happens in all the test matches the All Blacks played from 1992 to 2011. Because the sample sizes are close to 50 I have to use the DBM/OVS rule. The DBM is 13 and the OVS is 31.5. The ratio is 13/31.5 = 0.41. The critical ratio for sample sizes around 30 is 1/3 and this will be bigger than the critical ratio for my sample sizes, because my samples are bigger. Because the DBM/OVS ratio for my samples is bigger than the critical ratio for sample sizes of 30 it must be bigger than the critical ratio for sample sizes around 50. I can conclude that for the test matches between 1992 and 2011 the All Blacks are likely to score more points against northern teams than southern teams.

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I am reasonably confident about this call but I might get a different answer if I took another sample. Another sample would have different data points and would produce different graphs that could lead me to a different answer, although I think this is very unlikely. It is unlikely because of the big shift between the medians in my samples and the high value of the DBM/OVS ratio.

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	Grade Boundary: High Merit
2.	For Merit, the student needs to investigate a given multivariate data set using the statistical enquiry cycle, with justification.
	This involves linking aspects of the statistical enquiry cycle to the context and the population and making supporting statements which refer to evidence such as summary statistics, data values, trends or features of visual displays.
	The student has posed an appropriate comparison question (1), selected and used appropriate displays (2), given summary statistics (3), discussed features of distributions comparatively (4), and communicated an informal inference in their conclusion (5).
	The student has provided justification for comments made by referring to supporting evidence. This is shown in the student's discussion of the sample distributions and when making an informal inference (6).
	The student has also justified their call through the correct use of the DBM/OVS ratio (7), and started to integrate statistical and contextual knowledge in the response by linking their findings to a local paper article about trout size (8).
	To reach Excellence, the student would need to integrate the comments made with contextual knowledge throughout the response, and link the critical value of 0.2 to the sample sizes (close to 100).

The local paper has been full of stories about Lake Taupo being polluted and the trout fishing not being so good because the fish are smaller than they used to be. I am going to investigate this by looking at the data I have been given of trout caught in Lake Taupo in 1997 and 2011. I wonder if trout tend to be shorter in 2011 than in 1997?

(1)

Length..mm. by year

year

year2011

year1997

(2)

| | | | |
400   500   600   700

Length..mm.
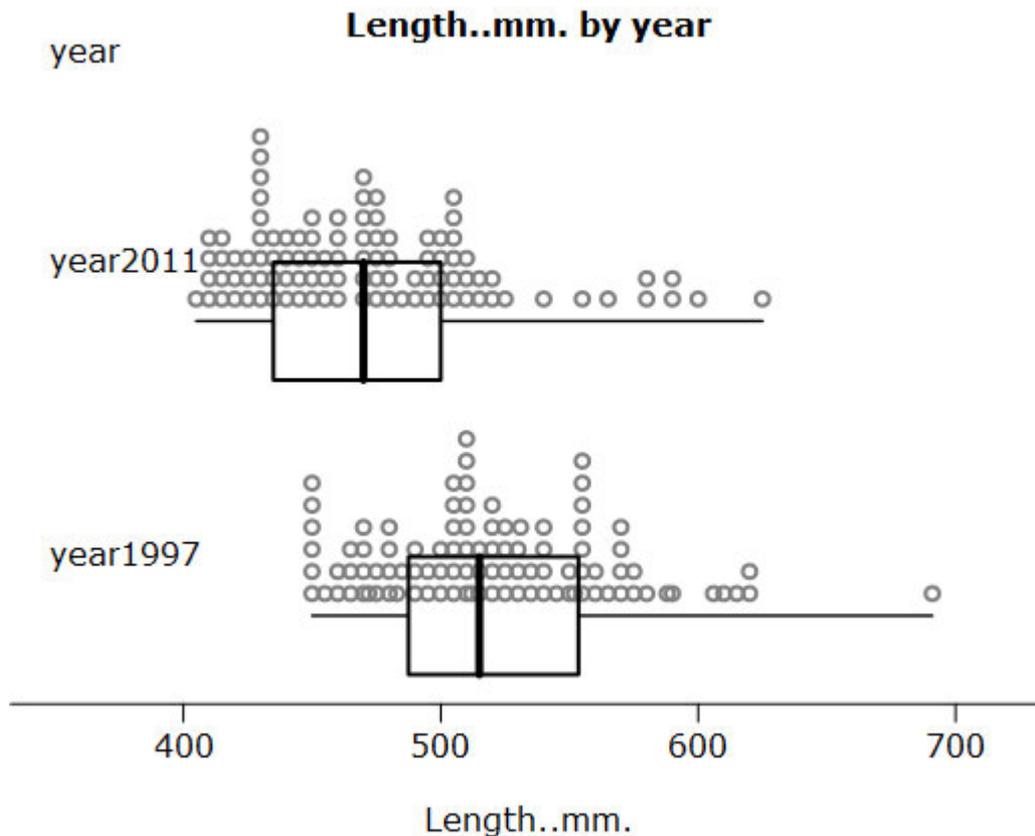
I can see from the graph that trout are not on average as long in 2011 as in 1997. This is shown by the middle line in the boxes, and the middle line for 1997 is further up the graph with a higher value. If you look in the table this is agreed because the median in 1997 was 515mm and it is now only 470mm. This agrees with the report in the local paper which reported that the lake is becoming more polluted and therefore the fish are becoming smaller.

(4)

(8)

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Summary of Length..mm. by year
        Min. 1st Qu. Median  Mean 3rd Qu. Max. Std.dev Sample.Size
year1997 450   487.5    515 520.8   553.5  691  45.780          99
year2011 405   435.0    470 471.9   500.0  625  47.086         100
```

(3)

The middle 50% of spread is about the same for both years. You can see this on the graph because the boxes are about the same length. The table says that in 1997 the box went from

(4)

553.5 to 487.5 which is 66mm and in 2011 it went from 500 to 435 which is 65mm. These are almost the same. So the variation in trout length is about the same for both years. ⑥

Both years have long right tails with a big spread of trout of longer lengths. I can see this as the dots are spread out to the right for both years. In 2011 the right tail goes from 500 to 625 (125mm) and in 1997 it goes from 553.5 to 691 (137.5mm). In both years the spread of trout lengths at the longer end is quite big. ⑥

There is one very long fish in 1997 with a length of 691mm. This is much longer than the next one which is about 620mm. Perhaps someone measured that one wrongly. I don't know whether this is the case or not so I left it in the data set.

In conclusion my sample shows that trout tended to be longer in 1997 than 2011. I can say this of the value of the DBM/OVS ratio. The DBM is 45 and the OVS is 118.5. The ratio is 45/118.5 = 0.38. This is bigger than 0.2. So for all trout in Lake Taupo, they tended to be longer in 1997 than in 2011, so my investigation says the same as the local paper. ⑦ ⑥ ⑤

	Grade Boundary: Low Merit
3.	For Merit, the student needs to investigate a given multivariate data set using the statistical enquiry cycle, with justification.

This involves linking aspects of the statistical enquiry cycle to the context and the population and making supporting statements which refer to evidence such as summary statistics, data values, trends or features of visual displays.

This student's evidence comes from the TKI assessment resource 'Census At School'.

The student has posed an appropriate comparison question (1), selected and used appropriate displays (2), given summary statistics (3), discussed features of distributions comparatively (4), and communicated an informal inference in their conclusion (5).

In the conclusion, the call is justified by considering the position of the sample medians relative to the boxes of the other group, which is appropriate for sample sizes close to 30 (6).

When discussing sample distributions, the student has provided justification for comments made, for example by referring to the visual displays of data (7).

For a more secure Merit, the student would need to strengthen their discussion of the features of the distributions, for example discussing and justifying other features such as the overlap of the middle 50%.

The student could also strengthen the links to the context by ensuring, throughout the response, that the variable is identified as the number of text messages per day. |

Problem

For the participants in the 2015 Census at School do females aged between 13 - 16 years tend to send more text messages per day than Boy aged 13 -16?

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Plan

I will be getting my data from Census at School NZ and it will be a sample and it is from 2015. In the sample there will be 30 females aged 13 – 16 years and 30 males aged 13 -1 6 years and I will be seeing if females tend to send more text messages than boys in New Zealand.
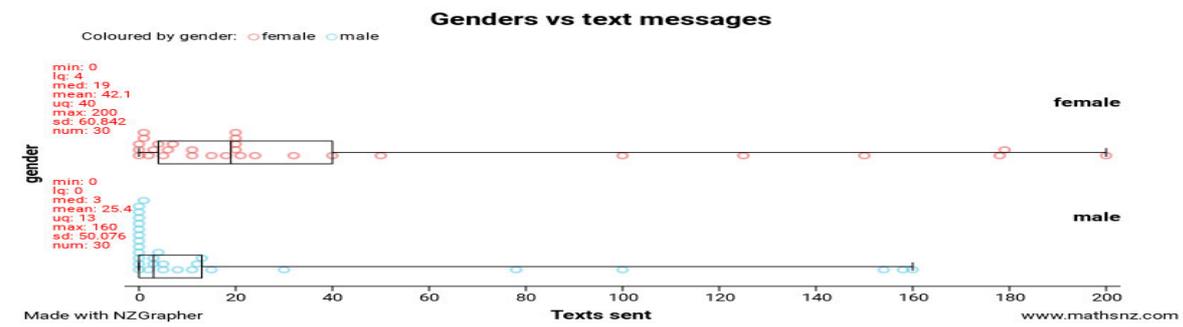
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Analysis

Summary Statistics	female	male
Minimum	0	0
LQ	4	0
Median	19	3
UQ	40	13
Maximum	200	160
IQR	40 - 4 = 36	13 - 0 = 13
Range	200 – 0 = 200	160 – 0 = 160

③

Measure of centre

I noticed that females maximum is a little bit higher (200) than the Boys maximum (160). This means that females do message more than Boys on phone but only a little bit.

I also noticed that the median for females (19) is quite a bit higher than the median for boys (3). This could mean from my sample that females do tend to text message more than boys.

Measure of spread

I can see that the female IQR is more spread out 36 whereas the boys IQR is much more squashed 13. This means that the spread for females is sending text messages in a day is almost 3 times greater than the that of males. I also noticed that both the female and boys graphs have some values that do not match the rest of the data for example, the boys have a person aged 15 who sends 160 text messages and the females have a 15 year old who sends 200 text messages This to me seems a little excessive but I know that this could be correct.

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Measure of features

From my sample I noticed that 50% of boys data is crunched up between 0 and 3 this could be because there are 12 boys that do not text message at all. I can also see that there is a cluster of points around 0 – 4 for females this could be because 5 females that don't text at all the day before.

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Conclusion

The females and boys boxes do overlap but the females median 19 goes past the boys upper quartile13. So more than 50% of females box is outside that of the boys box. Looking at the graphs visually I can make the call that for the participants in the 2015 census at school yes females aged between 13-16 do tend to send more text messages per day than boys aged 13 -1 6 from the participants of Census at School 2015.
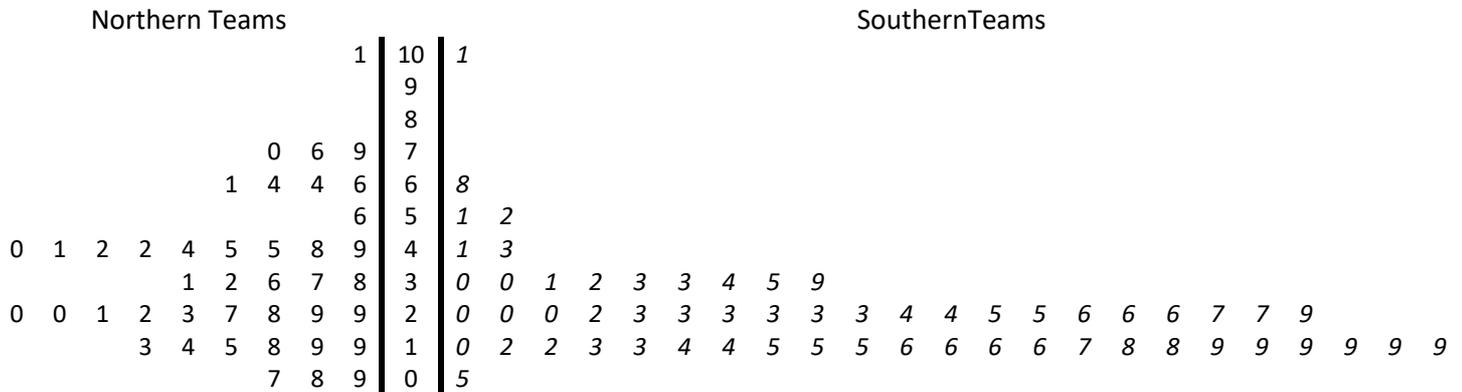
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	Grade Boundary: High Achieved
4.	For Achieved, the student needs to investigate a given multivariate data set using the statistical enquiry cycle. This involves using each component of the statistical enquiry cycle to make comparisons. This student's evidence comes from the TKI assessment resource 'Sporting success'. The student has posed a comparison question (1), selected and used appropriate displays (2), given summary statistics (3), discussed features of distributions comparatively (4), and answered the comparison question (5). To reach Merit, the student needs to provide a clearer discussion of the features of the sample distributions, and justify their comments by referring to evidence in the visual displays. The student would also need to show a clearer understanding about the variable being investigated (points scored in test matches).

I wonder if the All Blacks tend to score more points in test matches against Northern hemisphere teams than Southern hemisphere teams? I am going to look at a random sample of data points taken from all test matches between 1992 to 2011.
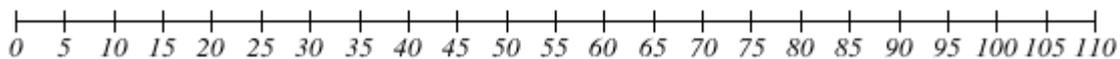
①

Northern Teams   SouthernTeams

```
                      1 | 10 | 1
                        |  9 |
                        |  8 |
              0  6  9   |  7 |
           1  4  4  6   |  6 | 8
                     6  |  5 | 1  2
0 1 2 2 4 5 5 8  9      |  4 | 1  3
           1 2 6 7 8    |  3 | 0  0  1  2  3  3  4  5  9
0 0 1 2 3 7 8 9 9       |  2 | 0  0  0  2  3  3  3  3  3  3  4  4  5  5  6  6  6  7  7  9
         3 4 5 8 9 9    |  1 | 0  2  2  3  3  4  4  5  5  5  6  6  6  6  7  8  8  9  9  9  9  9  9
             7 8 9      |  0 | 5
```

Key: the stem is in tens and the leaf in ones, so 5│1 means 51

②

Northern

7   20   36   48   101

Southern

5   16.5   23   29.5   101

```
 0   5   10  15  20  25  30  35  40  45  50  55  60  65  70  75  80  85  90  95  100 105 110
```

The summary statistics are:

	Min	1st Quartile	Median	3rd Quartile	Max
Northern	7	20	36	48	101
Southern	5	16.5	23	29.5	101

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The average number of points scored against Northern teams (36) is more than the average number of points scored (23) against Southern teams.

The middle 50% of Northern scores is more spread out than the middle 50% of Southern scores.

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I can say that the All Blacks tend to score more points in test matches against Northern teams than Southern teams between 1992 and 2011. This is confirmed by the DBM/OVS ratio which is 0.41 and this is bigger than 0.2.

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	Grade Boundary: Low Achieved
5.	For Achieved, the student needs to investigate a given multivariate data set using the statistical enquiry cycle. This involves using each component of the statistical enquiry cycle to make comparisons. The student has posed a comparison question (1), selected and used appropriate displays (2), given summary statistics (3), discussed features of distributions comparatively (4), and answered the investigative question (5). For a more secure Achieved, the student would need to provide a clearer understanding of the population the investigation is about. The student also needs to strengthen the comparative discussion of the features of the distributions, for example by discussing other features such as the spread and the shape.

I wonder if baby girls tend to weigh less when they are born than baby boys? I have been given a set of weights of 29 boys and 30 girls born at Auckland hospitals recently and I am going to use this data set to answer my question.

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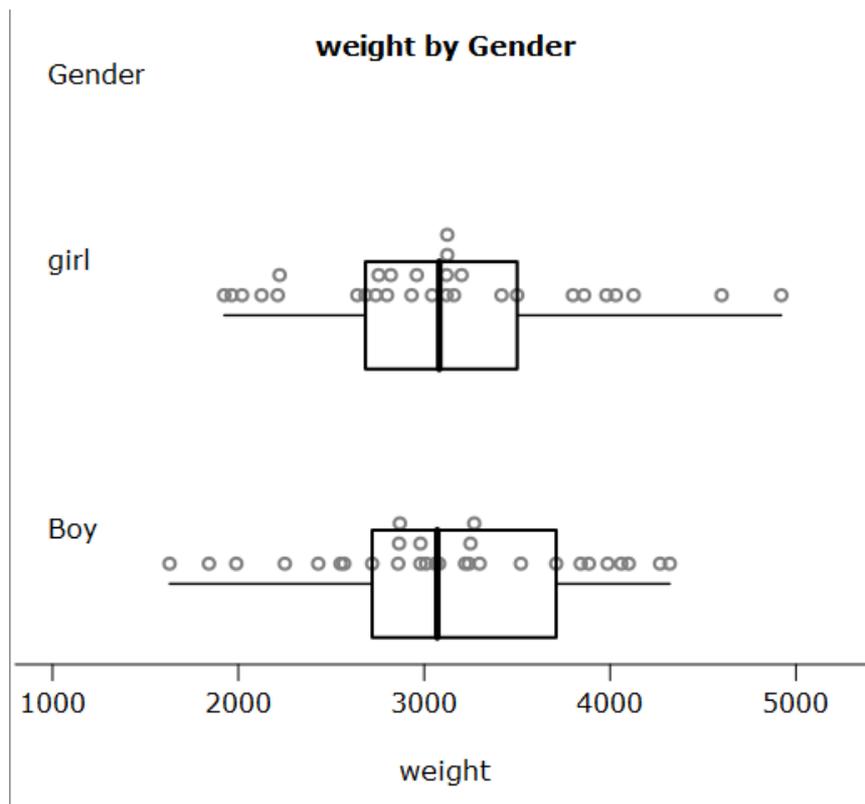
②

The graph shows that the average masses for boys and girls in this data set are about the same. The middle line in the box measures the average mass and they are both just about in the same place on the scale. This is confirmed by the median values in the table below. The median for boys was 3080 grams and the median girls mass was 3081 grams.

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The middle 50% of baby boys weights has almost all of the middle 50% of the baby girls weights inside it.

```
Summary of weight by Gender
      Min. 1st Qu. Median Mean 3rd Qu. Max. Std.dev Sample.Size
Boy  1630    2860   3080 3136    3710 4321  714.85          29
girl 1924    2698   3081 3097    3479 4921  761.25          30
```

③

I can't call that baby girls born in Auckland hospitals tend to weigh less than boys when they are born.

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	Grade Boundary: High Not Achieved
6.	For Achieved, the student needs to investigate a given multivariate data set using the statistical enquiry cycle. This involves using each component of the statistical enquiry cycle to make comparisons. The student has posed an appropriate comparison question (1), selected and used appropriate displays (2), given summary statistics (3), and answered the investigative question (4). To reach Achieved, the student would need to use appropriate evidence to make the call for sample sizes close to 100, and discuss features of the distributions comparatively.

My dad says that the trout have got smaller in Lake Taupo. I wonder if this is correct? I wonder if trout in Lake Taupo tended to be larger in 1997 than 2011? I am going to look at a set of data I was given by my dad from 1997 and 2011

(1)
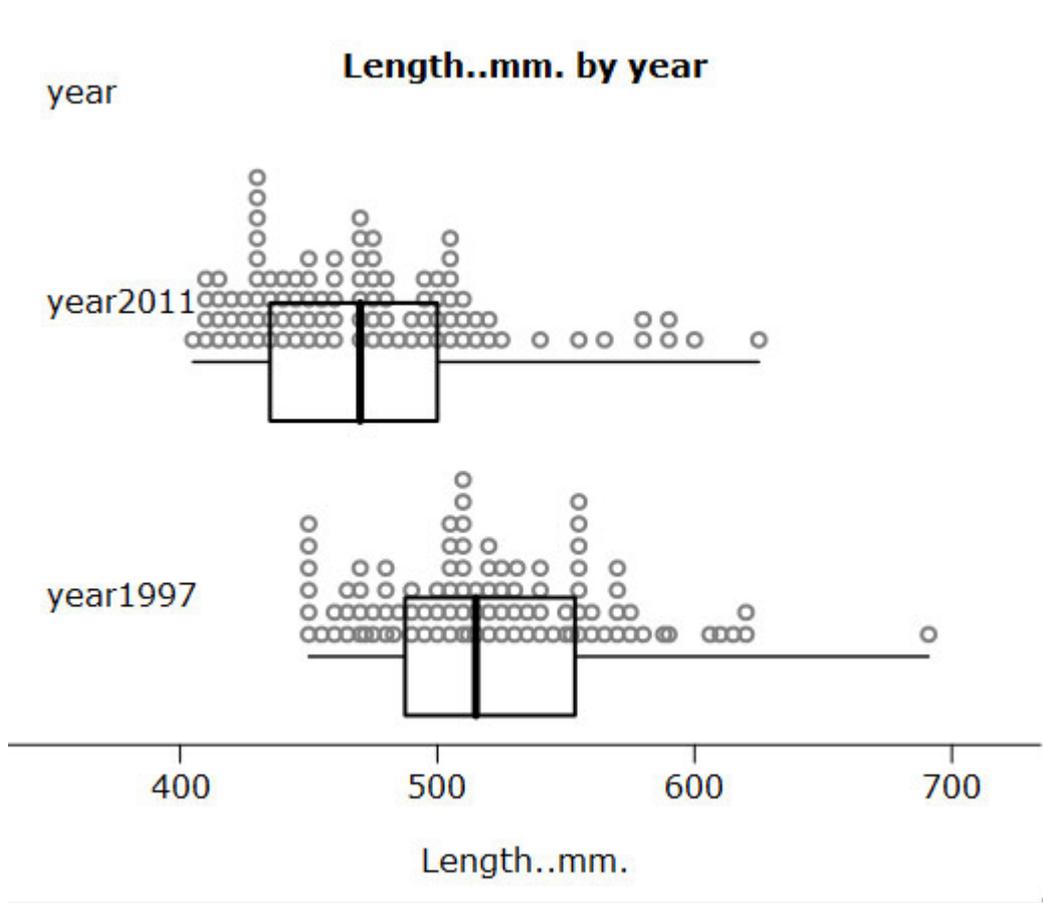
(2)

Trout are longer in 1997 than in 2011 because the median was 515mm and now it is 470mm. The mean also says this as it was 520.8mm in 1997 and 471.9mm in 2011

The longest trout in 1997 was 691mm and the longest in 2011 was 625mm.

This all tells me dad was right this time for once. Lake Taupo trout did tend to be longer in 1997 than in 2011.

(4)

```
Summary of Length..mm. by year
          Min. 1st Qu. Median  Mean 3rd Qu. Max. Std.dev Sample.Size
year1997  450    487.5    515 520.8   553.5  691  45.780          99
year2011  405    435.0    470 471.9   500.0  625  47.086         100
```

(3)